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Navier-Stokes-based modeling of the clarinet

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Abstract
Results are presented from a modeling study of the clarinet in which the air flow through the instrument is calculated using the Navier-Stokes equations. The reed is modeled as an Euler-Bernoulli beam with damping whose motion is driven by the pressure in the mouthpiece. Damping of the reed due to its contact with the lip is studied and shown to be crucial to achieve oscillations in which the reed vibrates at the lowest resonant frequency of the instrument, producing sound at that frequency. This finding is consistent with previous studies in which a clarinet is excited with an artificial blowing machine.

Keywords: Navier-Stokes, Clarinet, Modeling

1 INTRODUCTION

Modeling of musical instruments has yielded many important insights into a variety of different instruments including string instruments (pianos, guitars, and violins), percussion instruments (drums and cymbals), and wind instruments (recorders, trumpets, and clarinets). Physics based modeling strives to apply the fundamental equations of mechanics to understand the vibrations of the instrument and the resulting sound production. This is perhaps most challenging for wind instruments, since these instruments require the application of the Navier-Stokes equations, a set of nonlinear partial differential equations that are notoriously difficult to deal with even numerically. However, available high performance parallel computers are now able to obtain solutions of the Navier-Stokes equations for the air flow through and around wind instruments for fairly realistic instrument geometries. In recent work our group has reported results for the recorder, flute, and trumpet [1,2,3,4]. In this paper we report new results for the clarinet.

2 THE MODEL

Our clarinet model consists of two main components, one that computes a solution of the compressible Navier-Stokes equations for the air velocity and pressure as functions of time, and a second component that calculates the motion of the reed as it is driven by the pressure at its surface as derived from the Navier-Stokes solution. The Navier-Stokes equations are solved using a direct numerical simulation with a predictor-corrector algorithm as described in Ref. 1, while the reed motion is described using the beam model studied by Avanzini and van Walstein [5] including damping internal to the reed and from contact with the lips. Both are explicit, finite-difference-time-domain algorithms; in the future we plan to implement an implicit algorithm for the reed calculation, which should improve the accuracy of that part of the model. The Navier-Stokes calculation uses a nonuniform Cartesian grid with a spacial grid size of 0.1 mm near the reed and in the direction of the reed vibration. The reed moves continuously through this fixed grid according to the immersed boundary method [6], so the resolution for the reed motion is not limited by the Navier-Stokes grid. Other details of the calculation are given in Refs. 1 and 3. The time step for the calculations shown below was 2×10^{-7} s and the total number of grid points was $\sim 1 \times 10^{7}$.

The model geometry was a simplified version of a real clarinet, to reduce the required computational time. The resonator was a tube with a square cross-section (5×5 mm) and approximately 7.0 cm long. The reed was 9 mm long with a width of 0.3 mm. The Young's modulus was 150 $\mathrm{N/m^2}$ and density of 150 $\mathrm{kg/m^3}$; both of these are not typical of a real reed, but were chosen to give a resonant frequency and compliance that would yield reasonable oscillations for the chosen resonator dimensions. The fundamental frequency of the instrument was approximately 1.1 kHz, consistent with a closed-open tube.

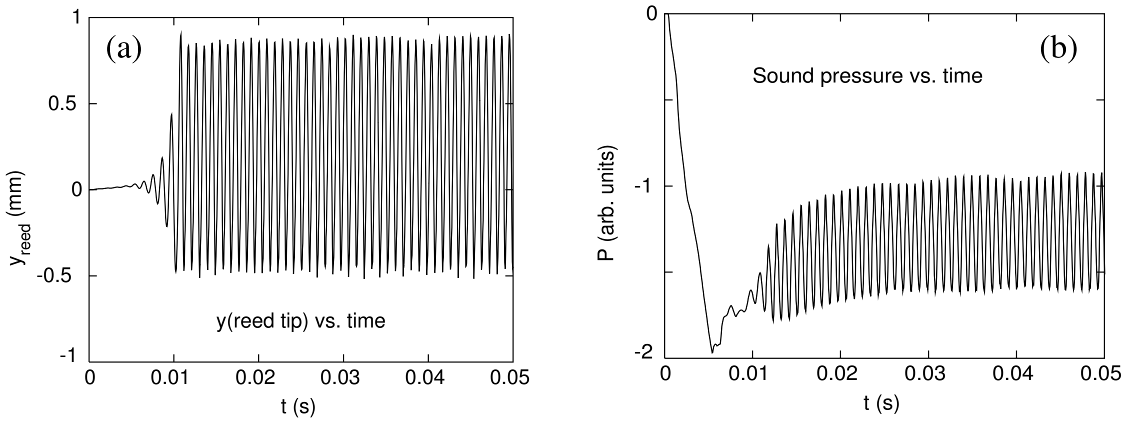

Figure 1. (a) Left: Position of the reed tip as a function of time. $y_{\mathrm{reed}} = 0$ corresponds to an undisplaced reed tip. (b) Right: Sound pressure outside the instrument as a function of time. The air speed in the mouthpiece was 5 m/s and the dimensionless lip-reed damping was $R = 450$ (to be compared with other results below).

Our results are broadly consistent with recent studies of the clarinet using the lattice Boltzmann method to treat the Navier-Stokes equations [7-9], although that work employed a two dimensional model.

3 BASIC REED MOTION AND SOUND PRODUCTION

Figure 1 shows typical results in the parameter regime that produces a steady reed oscillation and an approximately pure tone. Figure 1(a) shows the position of the reed tip as a function of time while Fig. 1(b) gives the sound pressure outside the instrument; this is the sound that would be heard by a listener. In this calculation and in others shown below, the blowing velocity in the mouthpiece was increased linearly from zero to a final value at $t = 5$ ms and then held constant.

After an initial transient period the reed motion reaches a steady oscillation amplitude at about $t = 10$ ms while the sound pressure does not reach steady state until somewhat later, about 30 ms in this example. The lengths of these transient periods depend on how hard the instrument is blown, with larger blowing speeds/pressures giving somewhat shorter transient times.

Figure 2 shows the sound pressure from Fig. 1(b) on an expanded scale. The waveform is approximately sinusoidal at the fundamental frequency of the instrument (about 1100 Hz) although some small contributions from higher frequencies are also evident.

4 EFFECTS OF REED DAMPING

There are several sources of damping that are commonly discussed when describing the dynamics of the reed (see, e.g., [5]). These are damping internal to the reed itself, damping due to contact with the player's lips, and what is sometimes termed "fluid" damping to account for energy loss to the surrounding air. In our model energy loss to the air is accounted for through the interaction of the reed and air, so that effect is included in a rigorous way. We have included damping internal to the reed using a value of the damping parameter suggested in Ref. 5. The lip damping is included in our model using the functional form described in Ref. 5. Since the dimensions of our model instrument and reed are smaller than those of a real clarinet and given the uncertainties in modeling real lips, we have treated this damping as an adjustable parameter we denote as R. A number of experiments with blowing machines (e.g., [10,11]) have reported that if the lip damping is zero or too small,

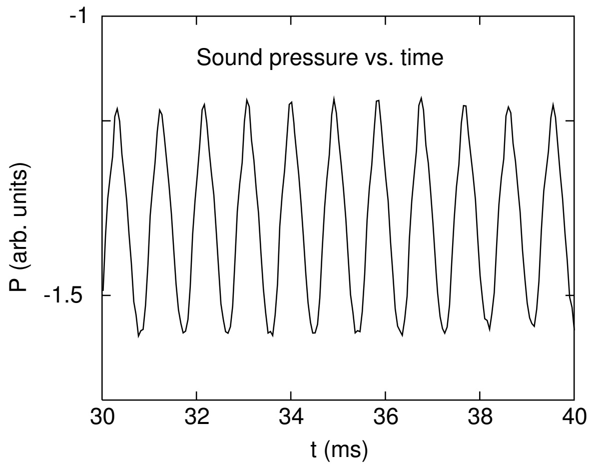

Figure 2. Sound pressure results from Fig. 1(b) on an expanded scale.

the sound produced is a "squeak" rather than a realistic musical tone. Our initial results also suggest that the location of the lip damping, i.e., the place along the reed where the lip contacts the reed, has a noticeable effect on the behavior. For all of the results in this paper the lip contacts about a third of the reed with the center of the contact region about half way along the reed. Here we illustrate the effect of lip damping by repeating the simulation from Figs. 1 and 2 but with different values of the lip damping R strength, and the results are given in Fig. 3. There we show results for the both reed oscillation and the sound pressure. All parameters, including the blowing pressure, were the same as in Figs. 1 and 2, only the damping parameter R was varied. We found that with $R = 0$ (no lip damping) there was no reed oscillation at all (and hence no sound) after a short (less than a few milliseconds) transient period. When R was increased, that is, by *increasing* the damping to a value about half the value in Figs. 1 and 2, a good reed oscillation was found with the expected behavior of the sound waveform.

5 CONCLUSIONS AND FUTURE WORK

This paper describes first results for a "toy" model of the clarinet in which the Navier-Stokes equations are used to compute the air velocity and sound pressure, and the Euler-Bernoulli beam equation is used to describe the dynamics of the reed. Our model of the clarinet differs from a real instrument in several ways; e.g., the dimensions are smaller than those of a real clarinet and the values of several parameters associated with the reed required a corresponding adjustment. Even so, our results indicate that a simulation of a fairly realistic clarinet in three dimensions is now feasible.

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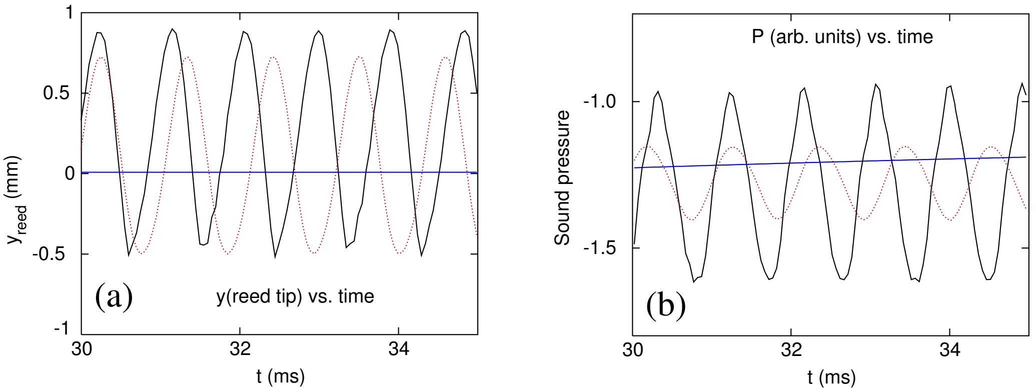

Figure 3. Effect of reed damping. (a) Left: Position of the reed tip as a function of time. (b) Right: Sound pressure as a function of time. Black curves: $R = 450$; Red curves: $R = 250$; Blue curves: $R = 0$. There was no oscillation with $R = 0$.

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