Non-destructive measurement of the pressure waveform and the reflection coefficient in a flue organ pipe

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Abstract
A multiple-microphone method employed for in-duct acoustics applications is adapted for a flue organ pipe geometry. In order to make the measurement, non-destructive the microphones are replaced by a pressure probe synchronized with a reference signal. Limits of the method regarding the pipe geometry are verified by the numerical calculations. Acoustic pressure waveforms are reconstructed and a framework for the reflection coefficient calculation is designed and tested.

Keywords: flue organ pipes, measurement methods, pressure waveform, reflection coefficient

INTRODUCTION
At present, it is safe enough to state that the qualitative nature of the sounding mechanism of a flue organ pipe is known (see e.g. [4, 1, 3]). However, many questions regarding the subtleties of the pipe design remain not fully resolved. The general problem lies in the inherent nonlinearity of the self-sustained system’s dynamics. In order to deal with these questions properly, good and detailed measurements of various aspects of the pipe design are needed. In-depth knowledge of the internal pressure field is among them without any doubt.

Although there were measurement methods approaching this topic [5] it is highly convenient for practical reasons to have a non-destructive technique. To this purpose, we alter the classical multiple microphone method [8] and test it on a pipe well-known from our previous experiments.

The paper is organized as follows. First, the multiple microphone method is briefly reviewed along with its prerequisites (a plane wave region span, a suitable wave number). Then the measurement setup is introduced followed by the experimental results (the pressure waveform reconstruction, the reflection coefficient evaluation). The text is completed with the discussion and some conclusions are finally given.

THEORY
Suppose that plane waves are propagating through the pipe. The axis along the pipe is labeled x and its origin is placed at the open end of the tube (i.e. all pipe interior points have negative x coordinate). The acoustic pressure field is conveniently represented as \( p(x, \omega) e^{i\omega t} \). Next, replace the angular frequency \( \omega = 2\pi f \) with the frequency \( f \) and split the forward and backward propagating waves to obtain

\[
p(x, f) = P_+(f) e^{-i\Gamma x} + P_-(f) e^{i\Gamma x},
\]

where \( \Gamma \) is the wave number introduced below and \( P_+ \), \( P_- \) denote the complex amplitudes of the forward and backward propagating waves respectively. When the acoustic pressure is known at multiple locations \( p(x_1, f), p(x_2, f), ..., p(x_n, f) \) we may use Eq. (1) to form the set of linear equations

\[
A p = b,
\]

with
Figure 1. Isolines of the acoustic pressure in the lowest eigenmode (up) and the last eigenmode before the transversal patterns occur (down). The mouth and the open end are marked grey.

\[
A = \begin{bmatrix}
e^{-i\Gamma x_1} & e^{i\Gamma x_1} \\
e^{-i\Gamma x_2} & e^{i\Gamma x_2} \\
\vdots & \vdots \\
e^{-i\Gamma x_n} & e^{i\Gamma x_n}
\end{bmatrix}, \quad p = \begin{bmatrix} p(x_1, f) \\ p(x_2, f) \\ \vdots \\ p(x_n, f) \end{bmatrix}, \quad b = \begin{bmatrix}
p_{+}(f) \\
p_{-}(f)
\end{bmatrix}.
\]

The set in Eq. (2) is overdetermined. Therefore the Moore–Penrose pseudoinverse \((\cdot)^+\) is employed and the complex amplitudes are obtained as follows

\[
p = A^+ b = (A^HA)^{-1} A^H b ,
\]

where \((\cdot)^H\) denotes the conjugate (Hermitian) transpose. Note that this approach corresponds to the least squares fitting of the Eq. (1) right-hand-side to the experimental data. The reflection coefficient of the open end \(R\) is then straightforwardly calculated as

\[
R(f) = \frac{P_-(f)}{P_+(f)}.
\]

The described method is applicable only to the plane waves. When considering the organ pipe, the nonplanar regions are inevitable, at least at the pipe mouth. In order to assess the spatial applicability of the method, the eigenfrequencies and corresponding eigenmodes of the pipe are calculated. Directly above the flue the relation between the acoustics and the turbulent flow field is strongly nonlinear. However, the turbulent velocities quickly decay leaving much less perturbed flow. Generally, the governing equations linearized around the mean flow would be required. Nevertheless, as it is commented below, the mean flow Mach number, as well as its gradients, are very low. Since we are interested just in an indicative assessment of the wavefront shapes, the simple Helmholtz equation \(\nabla^2 p + k_{\text{eig}}^2 p = 0\) is numerically solved employing our own code with the finite volumes discretization. The open end and the inner surface of the mouth region are treated as ideal pressure release surfaces. The full three-dimensional calculation has been conducted, the 2D results from the center cut plane are depicted only for the sake of simplicity. Based on the wavefronts depicted in Fig. 1 we assume that the plane wave region spans from the \(x = -0.55\) m to the pipe open end \((x = 0)\). The full geometrical length of the pipe is 0.71 m. Therefore we expect to observe the acoustic pressure node at the open end and follow the waveform a bit beyond the antinode of the first resonance.

The least squares fitting that results from Eq. (4) demands knowledge of the wave number \(\Gamma\) relevant for the studied case. Although there is a nonzero mean flow through the pipe, its Mach number (see below) is so low
that we do not have to split the wavenumber into the upstream and the downstream part. For the same reasons we neglect the turbulent losses outside the mouth region. Hence, we are left only with the thermoviscous losses which are expressed in the usual manner [8, 2]

\[ \Gamma = \frac{\omega}{c_0} - i\delta, \tag{6} \]

where \( c_0 \) is the adiabatic speed of sound and the attenuation parameter \( \delta \) is defined as

\[ \delta = \frac{1}{a} \sqrt{\frac{\omega \nu}{2 c_0^2} \left( 1 + \frac{\gamma - 1}{\text{Pr}} \right)}, \tag{7} \]

where \( a, \nu, \gamma \) and \( \text{Pr} \) denote a ratio of the pipe cross-section to its perimeter, the kinematic viscosity of air, ratio of specific heats and the Prandtl number respectively.

**EXPERIMENTAL SET-UP**

A transparent open flue organ pipe of the fundamental frequency 208 Hz has been used for measurements. For the above described procedure the transparency is not required but we benefit from our previous Particle Image Velocimetry measurements with the same pipe [7]. Based on them we assess the mean flow through the pipe to have the Mach number \( \text{Ma} \approx 10^{-3} \), which allows for neglecting the convective effects on the sound field inside the pipe.

Two major issues have to be overcome when designing an experimental set-up suitable for the above-described procedure: the pressure field inside the pipe should not be distorted by the presence of the microphones and the microphones (or probes in general) must endure strong sound field. The acoustic pressure inside the pipe exhibits amplitudes well over 130 dB, as it is shown below in detail. Hence, an array of common microphones cannot be employed, at least not without significant waveform distortion. A miniature pressure transducer (Kulite XCQ–080) capable of capturing the high amplitudes has been used instead.

In order to keep the internal pressure field as undistorted as possible, just one probe is employed and the measurement is repeated for multiple probe positions. Such procedure relies on the fact that a "well-behaved" organ pipe is constructed to produce the same sound repeatedly. The necessary synchronization is provided by an auxiliary microphone (Sennheiser MKE 2 P–C) placed at half of the pipe length. Its waveform has been used only for determining an instant from which the transducer signal is further analyzed (retaining the correct phase information).

A holder fixes the position of the thin metal tube terminating in the pressure transducer (see Fig. 3). Its legs covered less than 5% of the open end surface. The metal tube occupied 0.3% of the pipe cross-section. Since we have limited our measurements to the plane wave frequencies and the mean flow is too slow to produce significant vortex shedding around the obstacle we expect the effects of the measurement construction presence to be negligibly low.

The measurements took place in an anechoic chamber. The measurement block scheme is depicted in Fig. 2. Two pipe set-ups have been investigated. For the sake of brevity, only the labels S1 and S2 are used henceforth. The voicing parameters are summarized in Tab. 1.

<table>
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<th>Table 1. Voicing parameters of the investigated pipe set-ups.</th>
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<td>Foot pressure [Pa]</td>
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<td>Cut-up length [mm]</td>
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<td>Flue width [mm]</td>
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Figure 2. The block scheme of the measurement set-up.

Figure 3. The holder of the pressure transducer tube (left), the transparent organ pipe in the anechoic chamber (middle), the pressure transducer inside the pipe (right).
RESULTS

Eighteen positions inside the pipe (x = −540 mm, −510 mm, ..., −30 mm) were chosen for measurements and long time-averaged spectra of the pressure signal are used henceforth. The cut-off frequency of our rectangular pipe is slightly above 3 kHz.

Spatial waveforms of the reconstructed acoustic pressure are depicted in Fig. 4. The setup S1 exhibits sine-like shape, only little distorted by the relatively weak contribution of other harmonics. On the other hand, the setup S2 is noticeably influenced particularly by the 2nd harmonic that is not in phase with the 1st one. It results in a quite asymmetric waveform. Even though there are no additional local minima or maxima, several inflection points that would not occur on a pure sine wave have taken place.

In accordance with theoretical predictions the acoustic pressure node is not situated at the interface between the pipe interior (marked by the black background in Fig. 4) but it is slightly protruded outside the pipe (the white background). The end correction Δe = 0.34√S_e (with S_e being the open end cross-section = 2475 mm²) is marked in Fig. 4 as well. Since the open end has a nonzero radiation impedance the curves do not intersect precisely at a single point but evidently the Δe provides a very good approximation.

The reflection coefficient of the open end evaluated by Eq. (5) could be expressed as \( R = |R|\exp(i\phi_R) \), where the magnitude \(|R|\) and the phase \( \phi \) depends on frequency. Both parameters are plotted in Fig. 5. In the low
frequencies, the $|R| \to 1$ and the reflected wave travels with the opposite phase regarding the incident one, which is the textbook case. It suggests that the measurement and the data treatment were conducted properly and the method does not interfere with the nature of the pipe. It is known (see e.g. [9]) that when the mean flow is present the reflection coefficient might even exceed 1 due to the flow-acoustic interactions at the open end. However, given our Mach number, it is not likely to observe such behavior.

**DISCUSSION**

The reflection coefficient was presented only on the frequencies corresponding to the strong spectral lines and the pressure distribution data were obtained the same way. The reason why all the FFT bins are not used lies in the musically obvious fact that the majority of the pipe acoustic energy is stored in the harmonics. The rest of the spectrum is relatively weak and incoherent between individual spatial measurement points. It follows that the ratio $P^-/P^+$ resembles a 0/0 case, which results in considerably uncertain reflection coefficient values. An example of this behavior is depicted in Fig. 6. The broad resonance of the noise on the eigenfrequency of the pipe interferes with the 3rd resonance of the oscillating jet driving, thus causing the spectral peak broadening and weakening (cf. e.g. [3] for more details). The fit according to Eq. (4) becomes less reliable and the resulting value (marked red) is evidently wrong. The same, yet not so pronounced effect takes place on the 6th, 9th and 12th harmonic as well.

Although the pressure amplitudes inside the pipe (ca. 250 Pa $\sim 140$ dB) would suggest that some finite-amplitude nonlinearities could be present, our measurements have not directly proven any. Very probable reasons lie in the fact that the nonlinear effects do not build up significantly if there is even small detuning of the driving from the resonator eigenfrequencies (see e.g. [6] or when the significant dispersion and dissipation takes place (which would be the case within the generally nonlinear flow-acoustic interactions in the pipe mouth).

For the sake of necessary brevity, a detailed analysis of the measurement errors due to the probe positions (see [8]) is omitted here.

**CONCLUSIONS**

A non-destructive method of the pressure waveform and the reflection coefficient measurement for flue organ pipes has been tested and discussed. The results exhibit the key features required by theory and verified by other approaches (such as the validity of the end correction or the low limit behavior of the reflection coefficient regardless of the voicing parameters). Therefore we can conclude that the above described method is promising for future application on cases that are “textbook” to a less extent.
Figure 6. Magnitudes of the reflection coefficient for the S2 setup. The doubtful values are marked red.

Apart from dealing with the issues outlined in the Discussion, future research is intended to cover the comparison with theoretical predictions [9] and focus on the more complex situation regarding the pipes with non-negligible mean flow.

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REFERENCES