PROCEEDINGS of ISMA 2019
International Symposium on Music Acoustics

13 - 17 September 2019 in Detmold, Germany

Optimization of marimba bar geometry by 3D finite element analysis

Douglas BEATON [*], Gary SCAVONE [†]

Computational Acoustic Modeling Laboratory
Centre for Interdisciplinary Research in Music Media and Technology
Schulich School of Music, McGill University, Canada

Abstract

The natural frequencies of marimba bars are tuned by removing material from the bottom of the bar. Three partials are typically tuned, though less are tuned for higher notes. With only three or fewer partials to tune, many different bar geometries may produce the desired frequencies. As a result, bar geometries can show significant variation between brands, with each manufacturer employing their own tuning approach, honed over many years of experience.

This work uses 3D finite element analysis to investigate tuning marimba bar geometry, with an aim to inform manufacturing methods. Optimization techniques, including genetic algorithms, are employed to evaluate and improve bar geometries. The preferred geometries tune the desired frequencies, while also scoring well on secondary evaluation criteria. These secondary criteria include: separating the frequencies of torsional modes from those of the tuned modes, prioritizing symmetry, and producing shapes similar to professional marimba bars.

Models are developed using the open-source finite element program *Calculix*. Optimization routines are written in Python. The programs are interfaced to coordinate model execution. Functions are created to identify mode shapes based on displaced geometry. These functions provide resilience against any modal reordering, allowing the optimization routines to run unsupervised over significant changes in geometry.

Keywords: Marimba, Finite Element Analysis, Geometry Optimization

1 INTRODUCTION

The objective of this ongoing work is to produce computer algorithms capable of tuning 3D finite element models of marimba bars for desired qualities. Tuning the nominal overtone ratio of $(1:4:10)$ is of primary importance. Secondary objectives include: separating the frequencies of tuned overtones from those of untuned torsional modes, and producing smooth, largely symmetric geometry. These secondary objectives are under investigation at the time of writing.

Previous reports of percussion bar shape optimization in the literature have focussed on 1D models with abrupt changes in bar thickness [1] [2]. Some works have considered 3D models but did not seek to produce optimized geometry [3], [4]. This work seeks to employ 3D finite element models with smooth surfaces to optimize the modal behaviour of marimba bars.

2 METHODOLOGY

2.1 Geometry definition

Structured finite element meshes were generated parametrically and used in tuning the models. Inputs include the number of elements along each axis and their type (eight-node or twenty-node). Bar geometry was defined in one of two manners, as described in the following sections.

2.1.1 Cross-section interpolation

In this approach, bar geometry is controlled by interpolation between defined cross-sections along the bar's longitudinal axis. Figure 1 gives an example. For any position across the bar's width, a cubic spline is used to

[*] douglas.beaton@mail.mcgill.ca
[†] gary.scavone@mcgill.ca

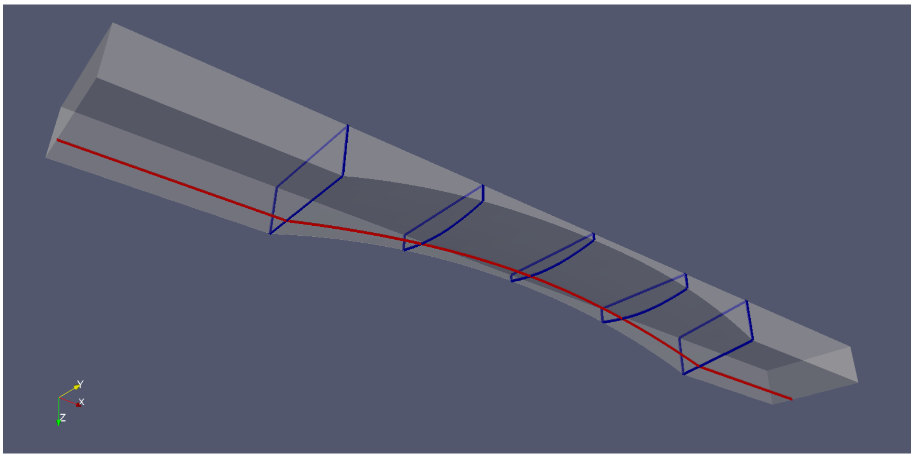

Figure 1. Bar geometry with cross-section interpolation. Cross-sections are outlined in blue. Red line exemplifies spline interpolation for bar thickness between cross-sections.

interpolate the bar's thickness between the cross-sections. Two types of cross-sections were considered: simple rectangular cross-sections (as shown at the ends of the cutaway in Figure 1), and cross-sections with a bottom surface defined by a parabola (e.g. the three middle cross-sections in Figure 1). The ability to rotate these cross-sections provided additional versatility.

2.1.2 Point grid surface interpolation

For this approach, a bar's cutaway geometry is defined by specifying thicknesses over a grid of points. A two-dimensional interpolation scheme determines thicknesses between the grid points. A uniform thickness is defined for locations outside of the cutaway. Figure 2 gives an example.

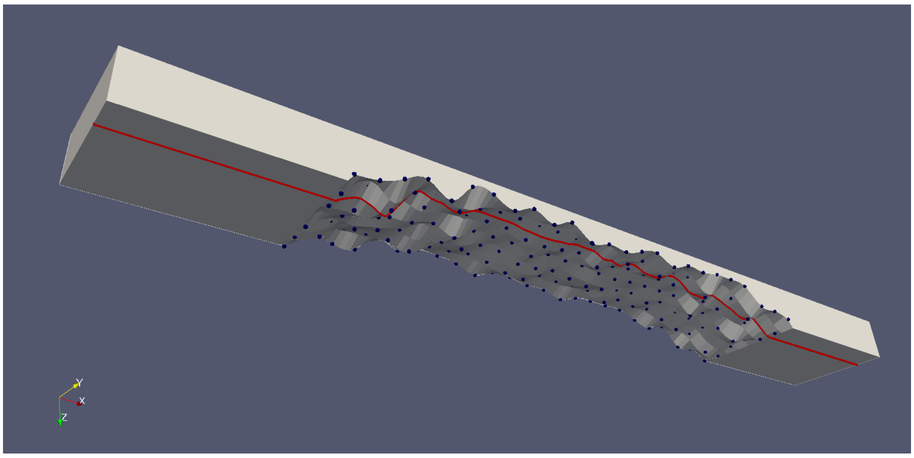

Figure 2. Bar geometry with point grid surface interpolation. Blue dots indicate surface definition points. Red line exemplifies interpolation at a given lateral coordinate. The model shown has exaggerated surface roughness for illustrative purposes.

2.2 Tuning approaches

Two approaches to tuning bar geometry are under investigation, as described below.

2.2.1 Multidimensional Newton-Raphson iteration

In this approach, bar geometry is defined using cross-section interpolation. A fixed number of cross-sections are defined. The thicknesses of these cross-sections serve as the inputs to be adjusted. Frequencies of the bar's fundamental mode and tuned overtones are the outputs to be tuned. If the number of cross-sections is greater than the number of tuned modes, multiple cross-sections may be constrained to have equal or proportional thicknesses. With this constraint in place, the Newton-Raphson iterations determine the next set of input value estimates as:

$$\mathbf{t}_{i+1} = \mathbf{t}_i - \mathbf{J}_i^{-1}\mathbf{f}_i, \tag{1}$$

where $\mathbf{t}$ is a vector of cross-section thickness inputs, i is the iteration number, $\mathbf{f}$ is a vector of modal frequencies compared to their target values and:

$$\mathbf{J} = \begin{bmatrix} \frac{\partial f_1}{\partial t_1} & \frac{\partial f_1}{\partial t_2} & \frac{\partial f_1}{\partial t_3} \\ \frac{\partial f_2}{\partial t_1} & \frac{\partial f_2}{\partial t_2} & \frac{\partial f_2}{\partial t_3} \\ \frac{\partial f_3}{\partial t_1} & \frac{\partial f_3}{\partial t_2} & \frac{\partial f_3}{\partial t_3} \end{bmatrix}. \tag{2}$$

Note that the partial derivatives in $\mathbf{J}$ are estimated numerically for each iteration.

2.2.2 Genetic algorithm

In this approach, bar geometry is defined using point grid surface interpolation. A vector of the thicknesses at each grid point forms the chromosome for a given candidate model. An initial population of candidate models is generated. The cutaway geometry of each candidate is set to a shape roughly approximating a typical marimba bar. Independent random perturbations are added to each grid point in each candidate. The random nature of these adjustments makes every candidate in the initial population unique.

The fitness of each candidate is evaluated as:

$$F = -\sum_{h=1}^{H}\left(\frac{100(f_{h,t} - f_{h,m})}{f_{h,t}}\right)^2, \tag{3}$$

where F is the fitness score, H is the number of modes being tuned, $f_{h,t}$ is the target frequency for mode h and, $f_{h,h}$ is the frequency of mode h from the candidate model under evaluation. The factor of 100 is applied to avoid excessively small fitness magnitudes.

Once the fitness of each candidate is calculated, candidate models are paired for crossover reproduction using tournament selection. New child candidates are produced by selecting a random number of grid point thicknesses from each of the parent candidate's chromosomes. Random mutation is applied to a small portion of the new candidate models. A portion of the current generation of candidate models with top fitness scores are combined with the new child candidate models to form the next generation. The process is repeated for a selected number of generations, or until the top performing candidate in a generation satisfies some established convergence criteria.

3 RESULTS

3.1 Suite of bars tuned by Newton-Raphson iterations

Measurements of bar outer dimensions were taken from a Yamaha 5.0 octave marimba. Models with these measured outer dimensions were generated for notes C2 through C4. Bar cutaways were set to be 64% of the overall bar length, centred at the midpoint. This suite of bar models was tuned to have appropriate modal frequencies using Newton-Raphson iterations. Figure 3 plots the resulting frequency ratios for the first nine modes of each model.

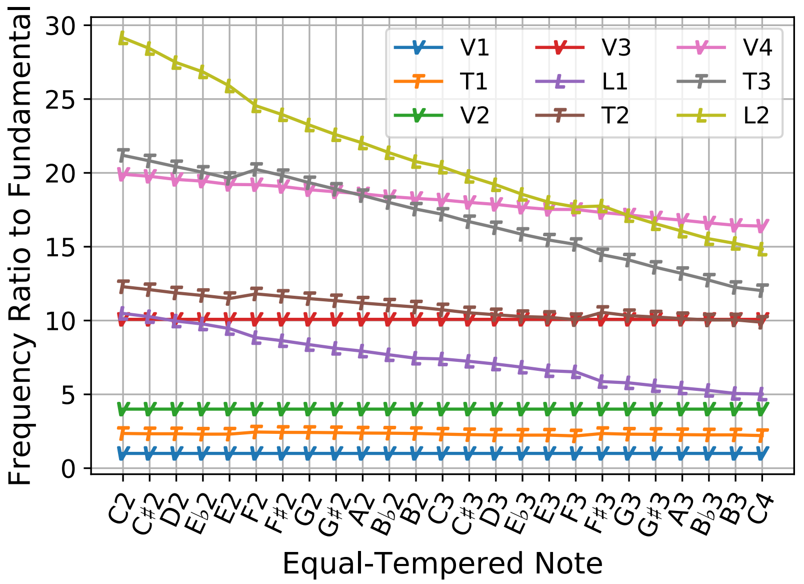

Figure 3. Modal frequency ratios from the suite of bars tuned using Newton-Raphson iterations. Symbols: V - flexural mode in a vertical plane; L - flexural mode in a lateral plane; T - torsional mode.

Note that measured bar widths decreased between notes E2 and F2, as well as between F3 and F♯3. As shown in Figure 3, the second torsional mode moves closer in frequency to the third vertical mode as the musical note increases from C2 to C4. The two modes appear quite close over the range D3 to C4. These results were used to determine notes to model when investigating separating torsional mode frequencies from tuned flexural mode frequencies.

3.2 Bar tuned via genetic algorithm

A bar of note C3 was selected for investigation using a genetic algorithm. Outer bar dimensions and material properties were set equal to those from Bork et al. [3]. A population of 50 candidate models was generated. The algorithm was arbitrarily set to run for over 500 generations. Table 1 provides the errors in modelled modal frequencies after multiples of one hundred generations. Figure 4 compares cutaway geometry for the fittest model of the intial population, and the fittest model from 400 generations onward.

As shown in Table 1 the maximum tuning error of any of the three tuned modes is around one cent after 100 iterations. After 300 iterations this error is well below one cent. No further improvement is observed between 400 and 500 iterations. Comparing parts (a) and (b) of Figure 4, it is observed that surface roughness along the fittest candidate bar's cutaway increased from the initial population to the final population.

4 SUMMARY

Newton-Raphson and genetic algorithm techniques have been implemented with the goal of tuning virtual marimba bars for their desired fundamental and overtone modal frequencies. Both techniques successfully tune the three vertical flexural modes of interest to within reasonable tolerances of their target values. At the time

Table 1. Modal tuning errors in fittest candidate models from specified generations.

Generation Number	Modal Tuning Error (cents)		
	Fundamental	Overtone 1	Overtone 2
0	53.9	122.6	47.8
100	-1.09	1.26	-0.32
200	-1.05	0.29	-0.68
300	0.13	0.12	-0.24
400	0.10	0.02	-0.23
500	0.10	0.02	-0.23

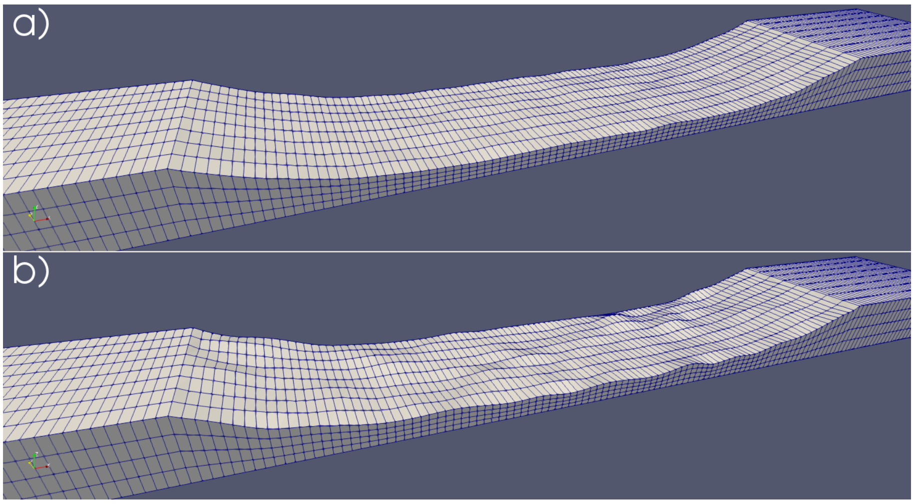

Figure 4. Geometry of bar tuned via genetic algorithm. a) cutaway of fittest candidate from initial population. b) cutaway of fittest candidate from final population.

of writing, work is underway to investigate separation of torsional mode frequencies from tuned flexural mode frequencies using both methods. Work is also underway investigating fitness functions that favour smooth, symmetric cutaway geometry with the genetic algorithm technique.

ACKNOWLEDGMENTS

This work was made possible through financial support from the Vanier Canada Graduate Scholarships Program, the Schulich School of Music, and the Centre for Interdisciplinary Research in Music Media and Technology (CIRMMT). The authors would also like to thank Fabrice Marandola, Martin Daigle and David Brongo at the Schulich School of Music for their insights into marimba performance and access to instruments.

REFERENCES

[1] Petrolito, J.; Legge, KA. Optimal undercuts for the tuning of percussive beams. The Journal of the Acoustical Society of America, 102 (4), 1997, pp 2432-2437.

[2] Henrique, LL.; Antunes, J. Optimal design and physical modelling of mallet percussion instruments. Acta Acustica United With Acustica, 89, 2003, pp 948-963.

[3] Bork, I.; Chaigne, A.; Trebuchet, LC.; Kosfelder, M.; Pillot, D. Comparison between modal analysis and finite element modelling of a marimba bar. Acta Acustica United With Acustica, 85, 1999, pp 258-266.

[4] Bretos, J.; Santamaría, C.; Alonso, MJ. Finite element analysis and experimental measurements of natural eigenmodes and random responses of wooden bars used in musical instruments. Applied Acoustics, 56 (3), 1999, pp 141-156.