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Finite element modelling of Japanese *koto* strings

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Abstract

The development of a high resolution finite element model of the Japanese *koto* has been previously reported. The koto is a plucked zither made of paulownia wood and 1.83m in length. Its 13 strings are made of polyester fibre and supported by 13 moveable bridges approximately 6cm high. A functional representation of each string was included in the high resolution model by using a wave form that entered the koto sounding body at a signal point where the bridge would be located for the standard (*hirajōshi*) tuning. Correlation between spectra generated in the model and spectra of an actual instrument as played provided initial observations of string behavior. This study aims to improve the fidelity of the spectral response by directly coupling the string to the resonating body. A beam model and a truss model with the dimensions of a professional string that is pre-tensioned to yield a close approximation of strings on an actual instrument are studied. Initial results from the string models as compared to notes played on the *koto* used as the basis of the finite element modelling are reported.

Keywords: Finite element modelling, Strings, Koto

1 INTRODUCTION

There are few acoustic studies of the Japanese koto (13-stringed plucked zither) and the study of string behavior has yet to be investigated in any detail. A functional representation of the string was included as part of the development of a high resolution finite element model of the koto using COMSOL Multiphysics®. In that model, the artificial stimulation of the sounding body by means of a wave form at a single point where the moveable bridge would be located for the standard (*hirajōshi*) tuning enabled the comparison of spectra generated in the model and those of an actual instrument. This study aims to improve the fidelity of the spectral response of this finite element model. While it is possible to model string behavior independently of the modelling of the sounding body of the instrument, the main difficulty lies in coupling one to the other in a way that realistically transfers the energy and simulates the physical plucking within the finite element model. Two options are investigated: a truss model and a beam model. This paper focuses on the setting up of the model and how to implement the complexities into Comsol from first principles. The development of these models is ongoing.

The science of plucked strings instruments especially those found in Europe such as guitars are well represented in the literature (1). In recent years, studies of instruments such as the Finnish Kantele (zither) (2) using finite element methods have been undertaken. By comparison, research by Ando (3, 4) remains the principal source of information for the acoustics of the koto while other information on the koto is found as in more general reviews of wooden stringed musical instruments (5) including those of Asia (6). Studies of East Asian plucked string instruments such as *qin* (zither) (7) are now available, but only a small number of finite element studies of Asian instruments have been undertaken (8, 9).

More specific studies of the physical modeling of strings can be found in the context of investigation of the full instrument such as the piano (10). Methods for the real-time synthesis of strings (11) have also been developed. Significantly, while some initial research using finite element modelling of strings have been undertaken (12), few are yet to be fully realized. Of relevance to this study, however, are examples of modeling vibrating strings found in the Comsol Model Library (13).

2 A FINITE ELEMENT MODEL OF KOTO STRINGS

This section discusses the development of the finite element model of Japanese koto strings. It presents an overview of the model of the koto's sounding body used for this study. Other components, namely the strings, the bridges and the pluck are then described. This is done first in terms of traditional materials and practices before

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discussion of the ways they are the set up in the finite element model of this study. Finally, the constructed string model and its use to investigate string behavior is presented.

2.1 The model of koto's sounding body

The sounding body of the koto is made of paulownia wood and 1.83m in length. Previous work by this author used a high resolution finite element model of the koto (10). The objective of that model was to make it as realistic as possible. This author's hand-crafted professional grade koto and COMSOL® Multiphysics Versions 5.2 to 5.4a were used. The basis of the measurements for the model was 2300 DICOM images of cross-sections of the koto obtained from a high-resolution computer tomography CT scan that were converted to a Simpleware mesh and then imported into Comsol, the so-called CT model (see Figure 1). The physical properties of paulownia were provided by Christopher Waltham from the University of British Columbia (14). It is important to note that no arbitrarily adjusted constants were incorporated. A number of experiments using equipment such as an acoustic camera, a laser scanning vibrometer (LSV) and techniques such as Chladni patterns were part of a multi-faceted validation process. When appropriate, model results were compared to the actual koto including Fourier Transforms of the waveform as played by the author. In this study the strings were removed from the model and frequency input achieved by generating a sound wave (sine or saw tooth) applied at the point where the bridge touches the sounding board.

Figure 1. Geometry of the high resolution finite element model of the koto

This type of high resolution model has a very high number of mesh elements and consequently a very long running time for each simulation. A simplified box model was therefore constructed for this study (see Figure 2 below). It comprises a hollow box with four internal struts and two round sound holes on the base plate. As described in more detail below, a single string was laid down the middle of it across three bridges and attached at each end.

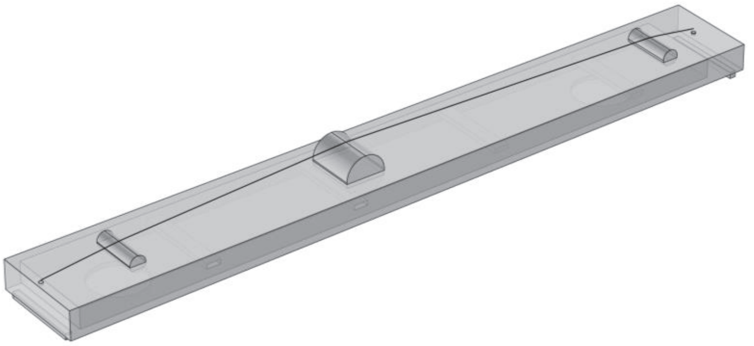

Figure 2. Simplified box model used in this study

2.2 Other components of the box model

In addition to the sounding body, three main components were developed and incorporated into the finite element model for this study. This section discusses these components. It briefly describes historical forms and more recent developments in traditional practice before discussing how the model component was constructed.

2.2.1 The strings

Historically, the thirteen strings of the koto varied in thickness and were made of silk. In the early twentieth century synthetic materials began to be used and the polymer under the trade name Tetron® has been preferred for making strings since the 1950s. All strings are now of equal length, weight and tension. Traditional knots are used to secure the strings at each end and maintain the tension.

The string was constructed for the model by building a four-sectioned Bezier curve whose coordinates touch the end points and the bridges. A cross-section, circular in this case, was then swept along the Bezier curve which acted as a guidepost. Appropriate material, in this case a polymer, was assigned to this new object.

There are two major ways in which string oscillations can be modelled within Comsol: either as a truss or as a beam. Both models are contained within the advanced Structural Mechanics section of Comsol. For this purpose the differences are not great. The truss model is used for modelling slender elements that can only sustain axial forces. It can be used for modelling structures where the edges are straight, but more importantly here, it can be used to model sagging cables such as the deformation of a wire exposed to gravity. The beam model can also be used for modelling slender structural elements, especially those having a significant bending stiffness (15). Both models allow the string to be pre-tensioned as is required here.

The biggest challenge was to couple the string to the sounding body. There are different types of coupling, each with its own characteristics. These include constraints that that allow rotation while others apply constraints in various dimensions. It is assumed for now that no sympathetic coupling of strings occurs, that is, they are considered to be isolated.

The standard (*hirajōshi*) mode based on the pentatonic modal system typical of East Asian zithers is employed in this study. The position of the bridge for each string for an instrument tuned in this way is illustrated in Figure 3. It should be noted that Japanese tuning systems are based on relative pitch and not absolute pitch. The actual position of the bridge may therefore vary, but the relative distance between bridges does not change.

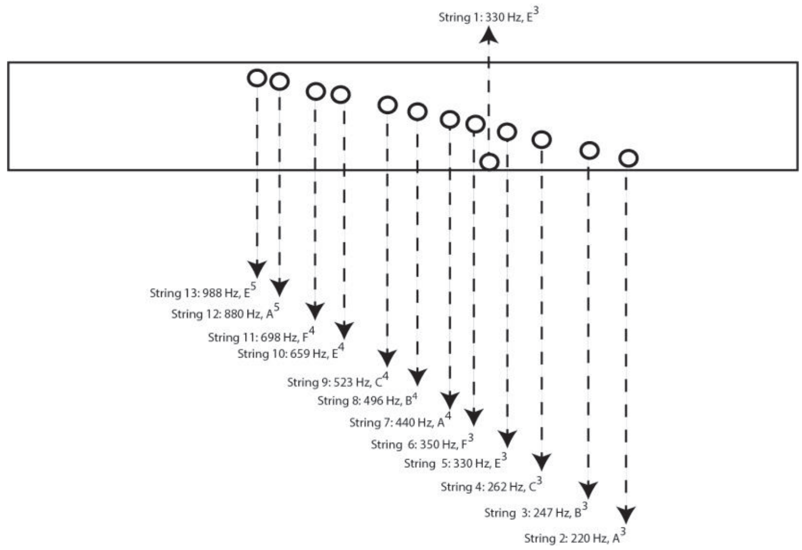

Figure 3. The placement of bridges and frequencies for each string for the standard (*hirajōshi*) mode

2.2.2 The bridges

The koto has both moveable bridges and fixed bridges. There are thirteen moveable bridges. A moveable bridge is placed beneath each string on the top plate and positioned according to the requirements of the tuning for performance. These bridges were historically made of wood or wood with ivory insets in the top segment. In recent times they have been made of ivory or more commonly from plastic. There are three main shapes that are designed to accommodate different positions on the top plate. The standard shape is used for most strings. It is approximately 6cm in height and measures 5cm across the span of the two supporting legs. The bridge supporting the thirteenth string which is closest to the performer typically has an extension on one supporting leg to help prevent it slipping off the edge of the instrument when the highest pitch is played while the second string may require a smaller bridge if a particularly low pitch is required. All bridges have a groove through which the string passes and stabilizes its position on the narrow top. A half cylinder shaped bridge is used in the simplified box model. A realistic model of a bridge, however, has been created using the shape and dimension of the standard bridge for use in future work (see Figure 4).

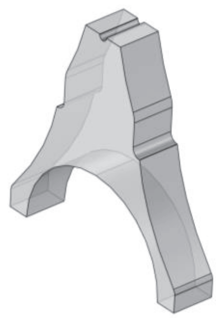

Figure 4. Model of the koto bridge developed for this study

By comparison, there are two fixed bridges located at each end of the koto and secured to the top plate. The strings pass over these bridges and are secured by traditional knots at these locations.

2.2.3 The pluck

Strings are plucked by means of three plectra worn on the player's right hand thumb, index finger and middle finger. The size, shape and materials used in the construction vary among different performance traditions. The two most common are square or rectangular-shaped plectra. The tip of the plectrum historically was made of bamboo, bone or ivory although more recently other materials such as plastic may be used. The tip is glued into a ring typically made of tightly rolled paper that is lacquered or covered with leather. These fit over the end of the finger to just below the player's nail with the plucking edge on the inside of the fingertip.

To model the plucking process, a force, usually a periodic one driven by the exciting frequency, f, is applied across the top face of the instrument in the y-direction. Another equal but opposite force, −f, is applied to the z (downward) direction to give a resultant force at 45 degrees to the top face of the koto.

2.3 Combining the components into the finite element model of the koto strings

The simplified box model was combined with the three other components — a single string was laid down the middle of the box across the three bridges and attached at each end. While the position of the string on the top plate may have an impact on the resonance, this study was concerned with the initial observations with the intention of investigating further details using the knowledge gained for studies using the CT model.

In this study, the koto body in turn required coupling with the surrounding air. The koto was therefore placed along the long axis of a cylinder as shown in Figure 5.

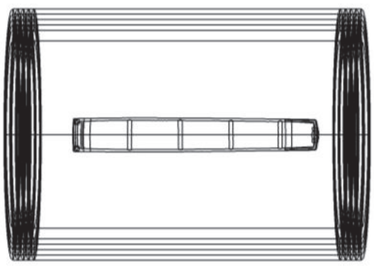

Figure 5. The simplified box model of the koto placed in a cylinder of air

This reduced the degrees of freedom and domain elements of the complex CT model with a proportional reduction in run time. The mesh element size always enclosed a minimum of six elements per wavelength. A sine wave with a frequency of 220 Hz was applied to a point on the string for the studies.

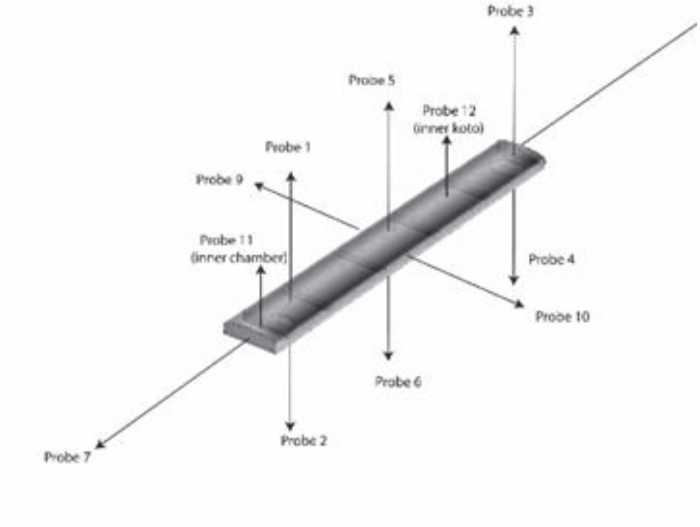

Figure 6. The set of 12 probes distributed around the koto to respond to the total acoustic pressure field

Finally, the response in the surrounding air can be measured through a set of probes distributed around the koto to respond to either the total acoustic pressure field in Pascals or the sound pressure level in decibels (see Figure 6). The advantage of the total acoustic pressure measurement is that the answer is computed as a complex number. Thus, in addition to the magnitude of the signal (at each point as a function of time), the phase angle can thus be readily computed as arctan (imaginary/real) as a useful indicator that each individual peak is indeed resonant and shows the direction of the signal. Solid displacement of the wood elements can be obtained separately.

2.3 Initial results

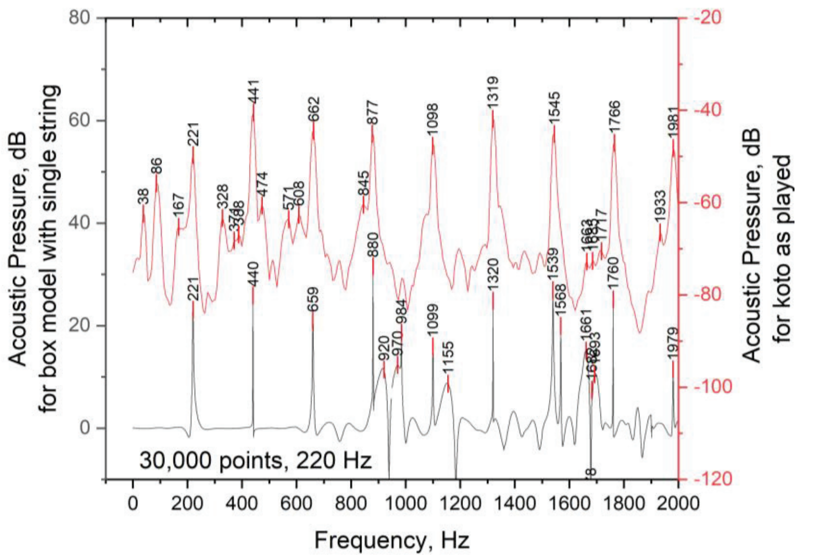

Figure 7. Comparison of the results simple box model of the koto with spectrum of the actual koto as played at 220 Hz

Initial simulations were made. In Figure 7, the lower portion of the graph shows the Fourier Transform of the fluctuating decibel level at Probe 4. The results of the Fourier Transform spectrum of the actual koto as played at 220 Hz are presented above the Probe 4 results. It can be seen that the primary excitation frequency and its harmonics are obtained from the single string box model. The prediction of the intermediate peaks is less successful. This is not surprising since we are not comparing like with like — the box model is an isotropic simplified model of the anisotropic model of the complex geometry of the koto itself. Future work will now attempt to align the two models primarily by using the techniques learned from the simplified box model and applying them to the CT model, then comparing these results with those obtained for the koto as played.

3 CONCLUSIONS

A first step towards creating a finite element model for Japanese plucked stringed instruments and investigati ng string behavior has been undertaken. We have been able to obtain initial results and this will guide ongoing work on modeling the strings of the koto. It is hoped that this work may also contribute to the evaluation of the patterns of sympathetic response across the instrument body for a deeper understanding of the koto's distinctive and tonal coloring.

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