A Wave Based Model to predict the Airborne and Structure-Borne Sound Insulation of Finite-Sized Multilayered Structures

A. Dijckmans[1], G. Vermeir[1], W. Lauriks[2]

[1] *Laboratory of Acoustics and Thermal Physics and Laboratory of Building Physics, Katholieke Universiteit Leuven*
Celestijnenlaan 200D, B-3001 Heverlee, Belgium
Email: arne.dijckmans@bwk.kuleuven.be

[2] *Laboratory of Acoustics and Thermal Physics, Katholieke Universiteit Leuven*

Introduction

Building acoustics and noise control rely mostly on two basic tools: measuring and modeling. Although measurements will always be indispensable, the possibilities of accurate modeling today make it useful for a whole range of applications: to validate measurements, to create prototypes, to make relative comparisons, ... The vibro-acoustic behaviour of multilayered systems consisting of elastic, porous and fluid layers can be modeled in different ways.

The Transfer Matrix Method (TMM) has been widely used to predict the airborne sound insulation of multilayered structures. Basic TMM is a simple method which assumes infinite layers and plane wave excitation. To get a global sound reduction index, the prediction results are integrated over all angles of incidence. The assumed directional distribution of the energy in the incident sound field greatly influences the calculation results, especially for multilayered structures. To take into account the finiteness of the element, a spatial windowing technique can be applied. This technique only deals with the diffraction effect caused by the boundaries. The modal behaviour of the element resulting from standing wave modes in the structure is not incorporated. At lower frequencies, the modal behaviour of both rooms and plates can influence the measured sound reduction index [1]. Numerical, deterministic methods like finite element models (FEM) or boundary element models (BEM) take into account the modal effects. As the number of elements increases with frequency, these models are limited to the lower frequency range. An alternative to FEM is the use of a purely modal approach which in simple cases can give solutions with reasonable computation times. In many cases, the consideration of full coupling between room modes and bending wave modes of the plate is not necessary [2]. For multilayered structures like double walls, the interaction between the vibrations of the panels and the acoustic pressure in the air gap cannot be neglected.

In this paper, a deterministic prediction technique, based on the indirect Trefftz method, is used to calculate the vibro-acoustic properties of finite-sized multilayered structures in a broad frequency range (typically 50-5000 Hz in building acoustics), thanks to the enhanced computational efficiency compared to FEM.

Wave Based Model

The Wave Based Method (WBM) is a Trefftz-based deterministic prediction method for the steady-state dynamic analysis of coupled vibro-acoustic systems. A general description of the method can be found in [3]. In this method, the steady-state dynamic variables (sound pressures and plate displacements) are expressed in terms of a set of acoustic wave functions which are solutions of the homogeneous parts of the governing dynamic equations, completed with a particular solution function of the inhomogeneous equation. Since the functions are exact solutions of the governing equations, the contribution of a certain function in a set is merely determined by the (vibro-)acoustic boundary conditions. As only a finite number of functions can be considered, the boundary conditions can only be satisfied approximately. As many problems in building acoustics can be simplified to a rectangular geometry, the WBM is developed for this specific case.

Problem definition

The geometry of the considered problem is shown in Figure 1. A rectangular multilayered structure with dimensions L_{px} and L_{py}, consisting of N plates separated by air cavities, is placed between two rectangular three-dimensional rooms. The side walls of the rooms and air cavities are rigid. In source and receiving room, the back walls have a normal impedance $\underline{\tilde{Z}}_n$. The plates are simply supported. The air cavities can be filled with a porous material. To calculate the airborne sound insulation, a harmonic volume point source is placed in the source room. To calculate the structure-borne sound sensitivity, plate 1 is excited by a harmonic point source of normal force.

Rooms and air cavities

The source room is divided into two parts by a plane through the point source, parallel to the element. The steady-state acoustical pressure in each (sub)room and air cavity $\underline{p}_{a,i}$ ($i = 0 \ldots N{+}1$) is governed by the homogeneous Helmholtz equation:

$$\nabla^2 \underline{p}_{a,i}(x,y,z) + k_a^2 \cdot \underline{p}_{a,i}(x,y,z) = 0 \tag{1}$$

$k_a = \frac{\omega}{c_{air}}$ is the acoustic wavenumber in air, with ω the circular frequency and c_{air} the speed of sound in air. In source and receiving room, uniform damping is

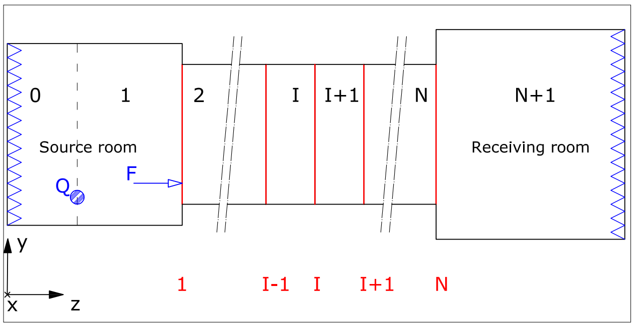


Figure 1: Geometry of the considered problem: a multilayered structure - composed of N rectangular plates, coupled by cavities - between two rectangular 3D rooms with rigid side walls and absorbing back walls. Excitation by volume velocity point source or point force.

introduced by making the acoustic wavenumber complex:

$$\underline{k}_a = k_a \cdot \left(1 - j\,\frac{1}{2}\,\frac{fT}{2{,}2}\right) \tag{2}$$

where T is the reverberation time of the room. f is the frequency, $j = \sqrt{-1}$.

Plates

For acoustically thin plates, the transverse displacement of the plates $\underline{w}_i$ $(i = 1 \ldots N)$ fulfils Kirchhoff's thin plate bending wave equation:

$$\nabla^4 \underline{w}_i(x,y) - \underline{k}^4_{B,i} \cdot \underline{w}_i(x,y) = \frac{F_i}{\underline{B}'_i}\delta(x - x_0, y - y_0)$$

$$+ \frac{\underline{p}_{a,i}(x,y,z_{pi}) - \underline{p}_{a,i+1}(x,y,z_{pi})}{\underline{B}'_i} \tag{3}$$

where the bending wave number $\underline{k}_{B,i}$ and the plate bending stiffness $\underline{B}'_i$ are defined as

$$\underline{k}_{B,i} = \sqrt[4]{\frac{m''_i\omega^2}{\underline{B}'_i}} \quad \text{and} \quad \underline{B}'_i = \frac{E_i h_i^3 (1 + j\eta_i)}{12(1 - \nu_i^2)} \tag{4}$$

with $m''_i = \rho_i h_i$ the surface mass density of plate i, h_i the plate's thickness. The material of plate i has a density ρ_i, a Young-modulus E_i, a loss factor η_i and a coefficient of Poisson ν_i. F_i is the amplitude of the point force acting on the plate at position (x_0, y_0).

Porous Materials

To model the wave propagation through a porous material, two different approaches are used. Open porous materials with a motionless or rigid frame, can be considered to behave as a fluid, with an adjusted density and a higher damping. The steady-state acoustical pressure $\underline{p}_{por}$ in the equivalent fluid is then governed by the homogeneous Helmholtz equation:

$$\nabla^2 \underline{p}_{por}(x,y,z) + \underline{k}^2_{por} \cdot \underline{p}_{por}(x,y,z) = 0 \tag{5}$$

$\underline{k}_{por} = \frac{\omega}{\underline{c}_{por}(\omega)} = \omega\sqrt{\frac{\underline{\rho}_{por,eff}(\omega)}{\underline{K}_{por,eff}(\omega)}}$ is the acoustic wavenumber in the porous material. $\underline{\rho}_{por,eff}(\omega)$ is the effective density, $\underline{K}_{por,eff}(\omega)$ the complex effective bulk modulus of the equivalent fluid. Several theories can be used to calculate the wavenumber and complex density of a porous material with a motionless frame, like the theory of Biot-Johnson-Allard [4], or empirical models like that of Delany and Bazley.

Secondly, a porous material between two plates can be modeled as a locally reacting material. The coupling between the plates is represented by the stiffness per unit area $\underline{s}'' = \frac{E_{por}}{d}(1 + j\eta_{por})$:

$$\underline{s}'' \cdot (\underline{w}_i(x,y) - \underline{w}_{i\text{-}1}(x,y)) = \underline{p}_{por,eq,i}(x,y) \tag{6}$$

The porous material has a Young modulus E_{por}, a thickness d and a loss factor η_{por}. $\underline{p}_{por,eq}$ is the equivalent acoustic pressure in the porous material, independent of z. The shear stiffness and the thickness resonances in the porous material are neglected. This mass-spring-mass model is a good approximation for sandwich panels where the porous material is glued to the plates. In that case the sound transmission is mostly determined by the dynamic stiffness of the frame.

Field variable expansions

The acoustic pressures are approximated in terms of the following acoustic wave function expansion.

$$\underline{p}_{a/por}(x,y,z) = \sum_{m=0}^{M}\sum_{n=0}^{N}\left(e^{-j\underline{k}_{zmn}z}\underline{A}_{mn} + e^{j\underline{k}_{zmn}z}\underline{B}_{mn}\right)$$

$$\cdot \cos\left(\frac{m\pi}{L_x}x\right)\cos\left(\frac{n\pi}{L_y}y\right) \tag{7}$$

where

$$\underline{k}_{zmn} = \sqrt{\underline{k}^2_{a/por} - \left(\frac{m\pi}{L_x}\right)^2 - \left(\frac{n\pi}{L_y}\right)^2} \tag{8}$$

L_x and L_y are the cross-sectional dimensions of the room or cavity. The wave functions are exact solutions of the respective homogeneous Helmholtz equation (1) or (6).

Also for the transverse displacement of the plates, a field variable expansion is used:

$$\underline{w}(x,y) = \sum_{p=1}^{P}\sum_{q=1}^{Q}\underline{C}_{pq} \cdot \sin\left(\frac{p\pi}{L_{px}}x\right)\sin\left(\frac{q\pi}{L_{py}}y\right) \tag{9}$$

Continuity and boundary conditions

The proposed pressure expansions satisfy a priori the rigid wall boundary conditions. The plate displacement expansions satisfy a priori the simply supported boundary conditions. The unknown pressure and plate amplitudes $\underline{A}_{mn}$, $\underline{B}_{mn}$ and $\underline{C}_{pq}$ are determined by the boundary conditions at the back walls of source and receiving room (imposed normal impedance $\underline{\tilde{Z}}_n$), the continuity conditions at the source plane (continuity of pressure $\underline{p}$ and transverse displacement $\underline{w}$) and the continuity conditions at the plates surfaces (continuity of transverse displacement $\underline{w}$ and plate impedance equation (3)).

Results

Numerical validation example

In this section, the above described wave based model is illustrated by means of a numerical example. The sound reduction index of a single plasterboard partition (thickness 0.025 m) with dimensions $L_x = 1.5$ m and $L_y = 1.5$ m is calculated with the WBM. The geometry used in the model is shown in Figure 2(a). Source and receiving room have rigid back walls and the plate is simply supported. A volume point source is placed in the corner of the source room at 0.5 m distance of the walls. The sound reduction index (SRI) is determined by following measurement formula:

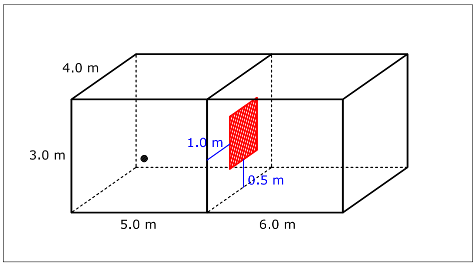
$$SRI = L_{pe} - L_{pr} + 10 \log \frac{S}{A_r} \qquad (10)$$

The sound pressure levels in emitting and receiving room L_{pe} and L_{pr} are calculated by analytical integration of the acoustic pressure over the respective room volumes. S is the surface area of the element and $A_r = \frac{0.16 V_r}{T_r}$ the absorption area of the receiving room, with V_r the volume and T_r the reverberation time. The result is compared with the Transfer Matrix Method in Figure 2(b), for a diffuse sound field excitation - first under the assumption of an infinite wall structure and secondly by applying the finite size correction. The material properties used in the simulations are given in Table 1.

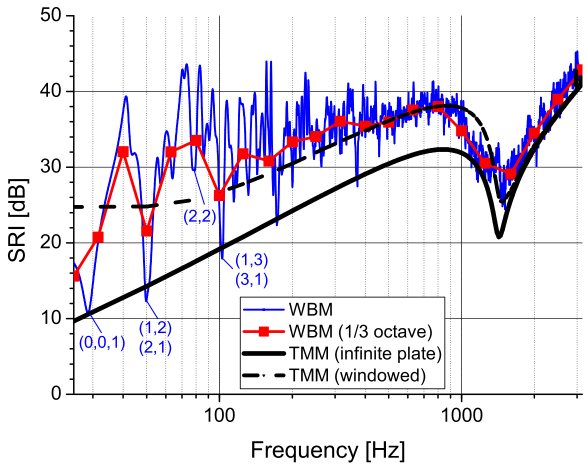
	ρ [$\frac{kg}{m^3}$]	E [Pa]	η [-]	ν [-]
Plaster-board	1200	3530e6	0,03	0,50
Glass	2500	62e9	0,05	0,24
Fibre-board	750	3500e6	0,03	0,46
EPS	-	13e6	0,10	-

Table 1: Material data used in simulations

Below the critical frequency of the plasterboard, the diffraction of the waves, which is taken into account by spatially windowing the results of the TMM, is important. This diffraction effect, caused by the finite size of the element, is also predicted by the WBM. Furthermore, the WBM results show a strong modal behaviour of the sound transmission. Integration over third octave bands smoothes the transmission loss curve, but in the lower frequency range, where the number of modes per frequency band is low, there are still large deviations from the TMM. Dips that are correlated to the plate modes are most profound, for example at 50 Hz (plate modes (1,2) and (2,1)) and at 100 Hz (modes (1,3) and (3,1)). The modal behaviour of the rooms gives additional modulations, for example at 29 Hz (receiving room mode (0,0,1)).



(a) Calculation set up (Wave Based Model).



(b) Sound Reduction Index (Wave Based Model versus Transfer Matrix Method).

Figure 2: Single plasterboard partition with dimensions 1.5 m x 1.5 m and thickness 0.025 m.

Experimental validation examples

The airborne sound insulation of a double window - consisting of a 6/12/8 mm double glazing in an aluminium frame, an air cavity of 300 mm and a single glass pane of 6 mm - was measured in the laboratory. The measurement aperture has dimensions 1.25 m x 1.5 m, the source and emitting room both have a volume of 87 m^3. This system is simulated with the wave based model, in which the reverberant rooms of the lab are approximated by rectangular rooms and the glass panes are assumed to be simply supported. The multilayered system is also simulated by means of the standard TMM, assuming infinite structures. Spatial windowing was also applied. The material data used for the simulations are given in Table 1. The results are shown in Figure 3. The dip at the critical frequencies of the glass panes (approximately 2130 Hz and 1600 Hz for respectively 6 mm and 8 mm glazing) is visible in both measurement and predictions. Below these critical frequencies, the transmission loss is largely underestimated by the TMM. Even after applying the finite size correction, the underestimation is 15 dB and more. The diffuse sound field assumption, used in the TMM simulations, seems incorrect. The WBM gives accurate results below 2500 Hz. It is important to take

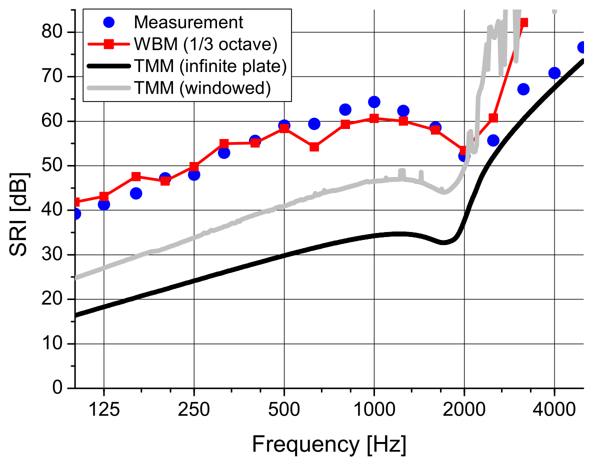


Figure 3: Measured and calculated (Wave Based Method vs Transfer Matrix Method) sound reduction index of triple glazing 6(12)8(300)6 mm with dimensions 1.5 m x 1.25 m.

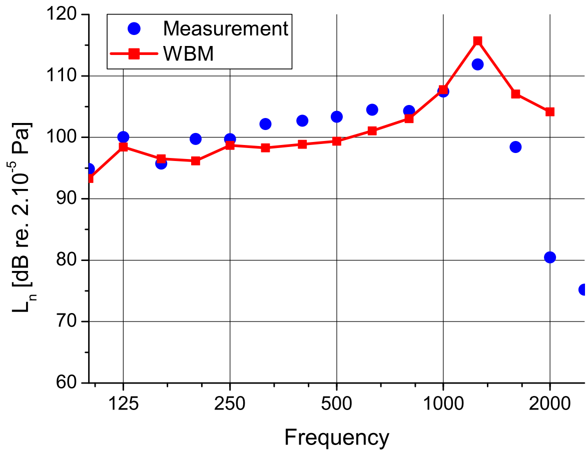


Figure 4: Measured and calculated (Wave Based Method) normalized impact sound pressure level L_n of a sandwich construction (142 mm EPS between two 4 mm fibreboards) with dimensions 1.5 m x 1.25 m.

into account the modal behaviour of the finite sized glass panes and air cavities. In double and triple glazing, the vibro-acoustic coupling interaction between the panels and the enclosed cavities becomes important. Above the coincidence dip, the WBM overestimates the sound reduction index. This can be due to the fact that flanking transmission through the window frame is neglected.

The impact sound pressure level L_n of a 1,5 m x 1,25 m sandwich panel is measured in the lab with a mini tapping machine (Norsonic) . The result is corrected to a standard tapping machine by use of a correction factor K = 21 dB and normalized to a reference absorption area A_0 = 10 m^2.

$$L_n = L_{pr} + K + 10\log\frac{A_r}{A_0} \tag{11}$$

The sandwich panel consists of 142 mm expanded polystyrene (EPS) glued between two 4 mm fibreboard plates. In the WBM, the EPS is modeled as a locally reacting spring layer. The material properties used in the simulation are given in Table 1. Assuming perfect rigid contact, the spectral density of the force signal of a standard tapping machine equals 4 N^2/Hz. The WBM result is compared with the measurement in Figure 4. Regarding the simple model used for the sandwich element, the agreement is good over a broad frequency range (100 - 1250 Hz). The impact level is maximum at the mass-spring-mass resonance frequency around 1250 Hz. The large discrepancies above 1600 Hz can be explained by the fact that the real power injected in the sandwich panel by the mini tapping machine is less than theoretically assumed, because of the non-rigid contact and the limitations of the electromagnetic driving force mechanism.

Conclusions and future work

A Wave Based Method is used to calculate the sound transmission through multilayer constructions containing thin, isotropic plates, air layers and porous layers. The full vibro-acoustic coupling between the room modes, the bending wave modes of the plates and the acoustic modes in the cavities is taken into account.

To better characterize poro-elastic materials, the transfer matrix method - which uses the full Biot-Allard theory [4] - will be incorporated into the model. In a next step the wave based model will be further extended to include orthotropic plates.

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