PATH ANALYSIS

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Introduction

The title of this paper is Path Analysis, and not Transfer Path Analysis, because the latter name has been assigned to the Forces method which, as it is used, is a contribution analysis method, more than a path method.

The origins of the method lie in the need to solve two different problems. The first problem consists in quantifying the contribution of each part of a vibrating system to the total noise measured at a given location. This problem will be called problem A. The second one, called problem B, consists in determining the noise produced by each one of the forces acting on a mechanical system.

In the 60's the method used to solve the problem A was called the "Strip" method. In this method the noisy object was totally covered with insulating blankets in order to attain a very reduced noise. Then the surfaces were uncovered one by one and the contributions of each surface deduced from measurements. The "Strip" method has been applied to motors, whole cars or even to whole train coachs, and it is still being applied today.

A typical case of problem B was to estimate the contributions to interior noise of each one of the engine supports on a car. In order to solve this problem, the practical method was to unlink the engine from the car and then to attach the supports one by one.

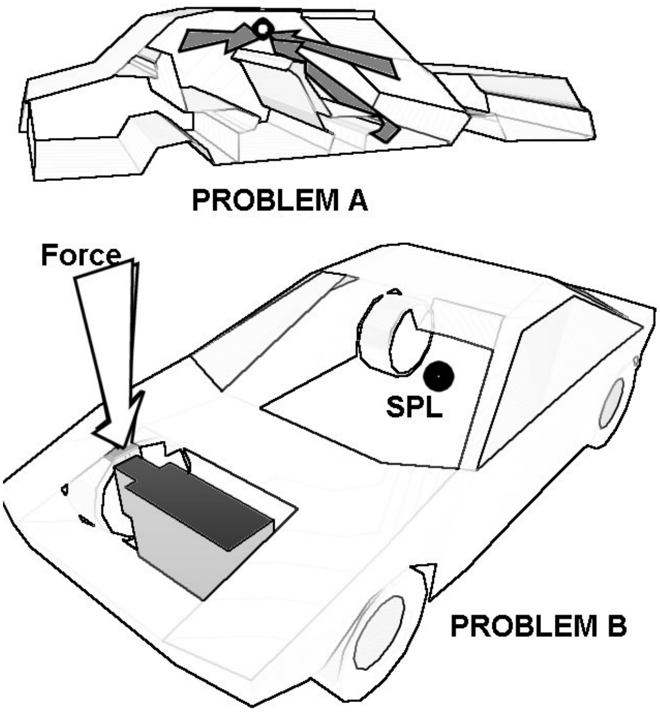

Figure 1: Schematic presentation of problems A and B.

Multiple Coherence

Between the years 1971-77 the next step was done by J. Bendat, L. L. Koss, R. J. Alfredson, C. J. Dodds and R. Potter [1-7].

The problem "A" was posed in terms of finding the best approximation to the coefficients H_i of the next equation:

$$p = \sum_{i=1}^{n} H_i a_i \tag{1}$$

where p stands for the noise acoustic pressure and a_i for the acceleration of each part of the system. In order to do this, they assumed that the "output" p and the "input" a_i were known for a set of linearly independent conditions issued from measurements in running conditions. The obtained system of linear equations (1) is overdetermined and can be solved in a least square error sense to obtain the unknowns H_i .

The important points of this method are:

-It is operational because the results are obtained from real working conditions. In this paper we will call this method Operational TPA.

-Its application needs a multichannel equipment because all the accelerations a_i have to be measured simultaneously.

-Multiple coherence function is used to know and separate the different sources of excitation. Recently, it has been shown that the solution of the least square problem (1) by the multiple coherence method is exactly the same as a well known method to solve a linear system of equations called LDLH matrix factorization [18].

-Coherent sources can not be solved using this method.

-Although there is not an explicit mathematical definition of paths, the path concept is used. Therefore the connectivity problem is not presented.

[3] and [8] are real applications of this Operational TPA.

Transmission Paths Analysis (GTDT)

In 1981 the author gave the next step [9] focusing on the paths. This means to recognize that the H_i from the Operational TPA can not be measured directly but they can be computed from measurable transfer functions and has a useful physical meaning.

In [9] the H_i are called Direct Transfer Functions (DTF) T^D_{ij} and the measurable ones are called Global Transfers Functions (GTF) T^G_{ij}. Besides, it must be remembered that "signal" means linear or angular acceleration, displacement, velocity or pressure.

The GTF T^G_{ij}, between two subsystems i and j, is defined as the quotient between the signal at j, s_j, and the signal at i, s_i, when only a nonzero force f_i is being applied to i. These transfer functions are measurable but we can not superpose their effects to obtain the total noise because the signal has been transmited along all paths, or in other words, because the subsystem i is not the only one with a non null signal and in consequence it is not the only one giving its contribution to the signal at j.

The DTF T^D_{ij} is defined as the quotient between the signal at j, s_j, and the signal at i, s_i, when a nonzero force f_i is applied to i and the signals at all other points different from j are forced to be null.

The signal at k produced by the force applied on k is called “external signal”. This external signal s_k^e is then the direct result of the force f_k, but it is also made up of the contributions of the other signals, s_i, with $i \neq k$, that have been indirectly excited by the force on k.

An important remark is that transfer function signals do not have to be homogenous. Transfer functions can be acceleration / acceleration, acceleration / displacement, velocity / force, etc. This means that this method can be applied with or without measuring forces.

Relation between DTF and GTF

The basis of GTDT is the existing relation between direct and global transfer functions. The DTF set T^D_{ik} can be obtained from the measured Global Transfer Functions [9] by:

$$\left[T^G_{ij}\right]_k T^D_{ik} = T^G_{ik} \tag{2}$$

where $[T^G_{ik}]_k$ is the GTF matrix without the row and the column k and T^D_{ik} is the vector containing all the DTF between k and the other points.

Physical meaning of the DTF

The Direct Transfers quantify the paths linking the degrees of freedom (called points from now) of any linear problem.

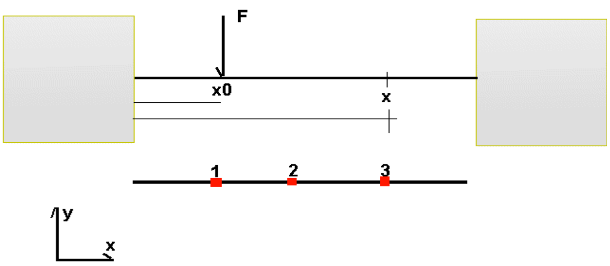

Figure 2: Fixed-fixed string, set of 3 subsystems, points 1,2 and 3 in displacenment

An easy example is given by the transverse displacement of a string of length L, fixed at both ends, and harmonically excited at the point x_0. Its Green function is [21]:

$$G(x \mid x_0) = \frac{c/\omega}{\sin(\omega L/c)} \sin\frac{\omega(L-x_0)}{c} \sin\frac{\omega x}{c}, \quad 0<x<x_0<L \tag{3a}$$

$$G(x \mid x_0) = \frac{c/\omega}{\sin(\omega L/c)} \sin\frac{\omega(L-x)}{c} \sin\frac{\omega x_0}{c}, \quad 0<x_0<x<L \tag{3b}$$

where c is the characteristic wave velocity, which depends on the tension T and the density ρ of the string : $c= (T/\rho)^{1/2}$.

In order to apply TPA we choose 3 points x_1, x_2 and x_3. The DTF between x_1 and x_3, T^D_{13}, is expected to be null because when a force is acting on x_1, if a zero displacement is imposed at x_2, the displacement at x_3 should be also zero. This can be computed from (2) using the GTF given by the Green Function as follows:

$$T^G_{12} = \frac{G(x_2 \mid x_1)}{G(x_1 \mid x_1)} = \frac{\sin(\omega x_2 / c)}{\sin(\omega x_1 / c)}, \tag{4a}$$

$$T^G_{13} = \frac{G(x_3 \mid x_1)}{G(x_1 \mid x_1)} = \frac{\sin(\omega x_3 / c)}{\sin(\omega x_1 / c)}, \tag{4b}$$

$$T^G_{23} = \frac{G(x_3 \mid x_2)}{G(x_2 \mid x_2)} = \frac{\sin(\omega x_3 / c)}{\sin(\omega x_2 / c)}, \tag{4c}$$

$$T^G_{21} = \frac{G(x_1 \mid x_2)}{G(x_2 \mid x_2)} = \frac{\sin(\omega(l - x_1) / c)}{\sin(\omega(l - x_2) / c)}, \tag{4d}$$

$$\Rightarrow T^D_{13} = \frac{T^G_{13} - T^G_{12} T^G_{23}}{1 - T^G_{12} T^G_{21}} \equiv 0. \tag{4e}$$

Applying (2) the obtained DTF T^D_{13} (4e) is then really null.

A more detailed presentation is given in [19] where the same concepts are applied to the flexural vibrations of a beam.

A final and more complex example is given by applying the method to the acoustic wave equation on a discretized 2D rectangular space. Fig.4 shows the computed T^D at a given frequency for 3 points against all other points of the space.

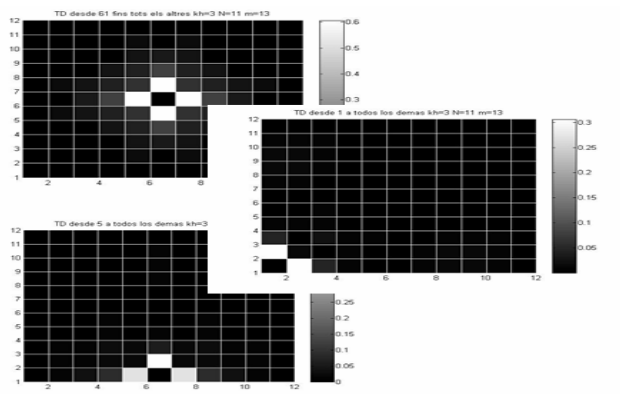

Figure 3: Calculated DTF between a middle, a corner and a middle side point against all other points of the 2D space. The DTF is non null only for the adjacent points.

As shown in Fig.3, only the points on the contour of the tested one have a T^D value different from zero. The value at the other points is almost zero instead of zero as a consequence of the discretization.

Finding a solution to problem A

As shown in [9], the signal in each point can be computed as the sum of the contributions of all the other points through all possible paths on the structure.

$$s_k = s_k^e T_{kk}^D + \sum_{i \neq k}^{n} s_i T_{ik}^D \tag{5}$$

Equation (5) shows how to find the signal in one point of the system from the values in the rest of them when the DTF are known.

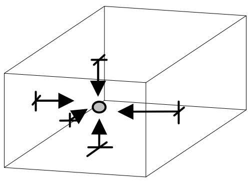

Figure 4: The contributions of the each face vibration to the total noise in the microphone are given by (5). Even if the forces are not applied in all faces, each face is excited by all forces.

From an experimental perspective this means that it is possible to find the contribution $s_j T^D_{jk}$ of each one of the panels of a vehicle on the measured pressure s_k at a point inside the vehicle. Another application of (5) is to quantify the contributions of each one of those parts to the vibration of one panel, spliting in each case the part directly produced by some external excitation.

Another possibility is to split the signal going from a point to another in the part going through the direct link or through other points of the system.

An example of this is to find the signal from a damper that arrives to a microphone but passing before through a window, or to split the signal from a window that arrives to a microphone in the part that has arrived to the window through the air and the part that has arrived to the window through the structure. Fig. 5 shows the contributions to the noise pressure in a train coach obtained by this method.

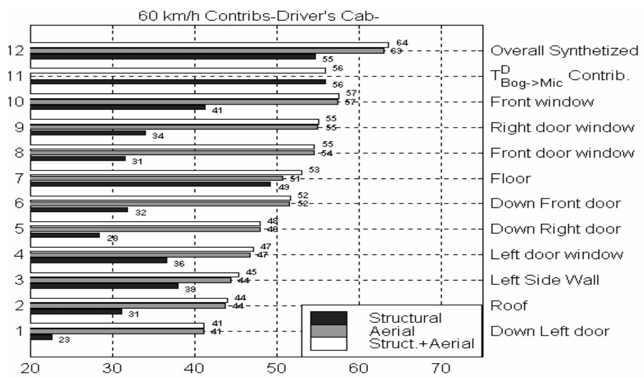

Figure 5: Noise contributions of several subsytems to the interior noise in a railway coach. Each subsystem contribution is divided in contribution due to aerial and contribution due to structural excitation of the subsystem.

Finding a solution to problem B

-Also in this framework we can obtain the forces, or their equivalent that are fractions of the signals from these external forces called here external signals.

$$s_k = \sum_{i \neq k}^{n} s_i^e T_{ik}^G \tag{6}$$

For the special case of acceleration and force signals, (6) can be written with a_k and F_i instead of s_k and s_i^e:

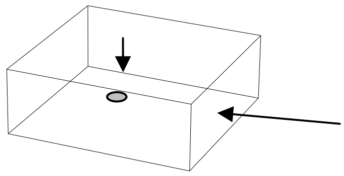

Figure 6: Equation (6) give the contributions of each one of the forces, but each force contribution is transmitted through all the faces. There is not information about the path of the contributions.

Equation (6) corresponds to the TPA method [9]. It allows obtaining the signal as a result of the superposition of the external signals (or forces). It is important to insist on the fact that (6) does not take into account each one of the signals contributions but the external signals contributions, which are the forces or a direct consequence of them. Additionally, only the Global Transfer Functions appear on (6) and not the DTF.

Equation (6) has been the more used result of [9] because of the report [12] done by the Swiss society Keller, working on the automotive industry. This report was based on [9] and it was the starting of the TPA method.

In effect [13], [14], [15], from Keller, Ford, Porsche and Fiat, present some initial works aimed at developing the method in order to identify the forces.

Later in the 90's, LMS was working in a European project in with Keller and Fiat [16]. This partnership is at the origin of the commercial products of the TPA method, equation (5), and developments on the PCA method.

Industrial applications

The author has been applying the methods of equations (4) and (5) for the last 28 years and without using force measurements.

A part of this work has been done in the railway sector in the frame of a technology transfer to the world's bigger train builder company, which has been using this technology for the last 8 years. Another important fact is that the GTDT methodology has been adapted to other fields like industrial machinery, windmills, buildings or civil engineering. For this purpose it has been necessary to develop a theoretical and experimental framework. Specially:

-Definition of finite subsystems.

-Definition of methods to choose these subsystems.

-Application of this method to the vibroacoustic energy under the Statistical Energy Analysis hypotheses [17] and outside of them. In the initial works the method was applied only taking into account the coherent form (low frequency) and latter it has been developed the energetic part (mid and high frequency).

-Integration of supplementary tools in the frame of the EOF (Empirical Orthogonal Functions)

Figure 7: Some GTDT real Industrial applications.

Relation with the numerical methods

In [20] the possibilities of (2) have been explored to deal with numerical calculus exploding de conectivuty role of the DTF..

The first example of [20] consists in solving the inhomogeneous Helmholtz equation in a 2D square domain. After discretization it is shown that the DTFs among the mesh nodes are very similar to those arising from the stencils of several numerical methods (some finite difference and finite element methods are considered). As we can found the DTF from the GTF it is possible to obtain the coefficients in numerical form.

The second example of [20] involves a free field radiation example from a source to a receiver. The source has been encircled with a set of points at a certain distance, showing that no signal can be transmitted from the source to the receiver if the signal at these points is blocked. The DTF from the source to the receiver numerically expresses this fact and consequently has an almost zero value. The DTFs from the circle elements to the receiver are also calculated and compared to their analytical counterparts given by [21].

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