Comparison of artificial and natural rainfall

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Introduction

There are two important differences between natural and artificial rainfall. (i) While drops of natural rain have reached their terminal velocity, when they hit the ground, they are still in the phase of acceleration in the rain noise laboratory. (ii) The maximum number of drops per unit area and unit time in natural rain is at drop diameters below 2 mm, whereas artificial rain according to IS0 140 [1] consists mainly of 5 mm thick drops. Do these two types of excitation lead to comparable radiation of sound? What are the differences?

Computational model

Impact force

When a raindrop hits a surface, it exerts a force. According to Petersson [2] this force can be calculated by Newton's Law

$$F = \frac{d}{dt}(m\,v) = m\,\dot{v} + \dot{m}\,v \tag{1}$$

with the force F, the mass m, the impact velocity v and the time t. The dot above a symbol denotes the derivate with respect to time. Assuming that there is no change of the velocity during the impact the first summand in Equation (1) vanishes and it can be simplified to

$$F = \rho_{H_2 0}\, v\, \dot{V} \tag{2}$$

with the density ρ_{H_2O} of water and the part $V(t)$ of the drop volume, which is above the surface (V = 0 after the end of the impact). By Fourier transformation of Equation (2) one obtains the force spectrum $\widehat{F}$. Figures 1 and 2 show the force and the force spectrum for different shapes of drops. (For non-spherical drops the diameter stands for an equivalent diameter.) For further details concerning the calculation of impact energy and radiated power see references [2] und [3].

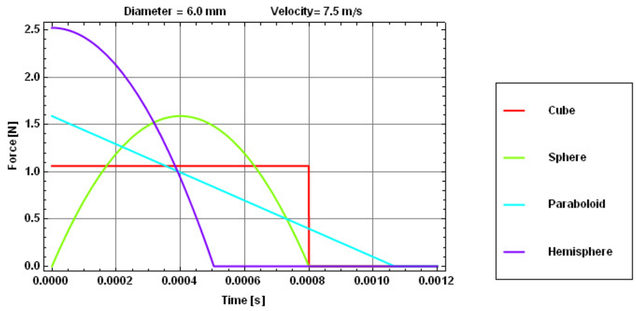

Figure 1: Force of impact for different drop shapes

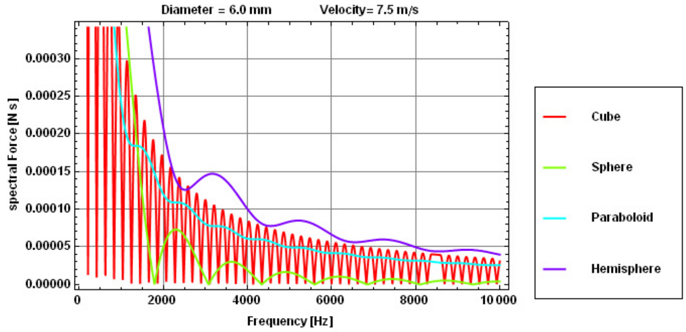

Figure 2: Impact force spectra for different drop shapes

Impact velocity

For natural rain the way from the clouds to the ground is long. Therefore the raindrops have reached the terminal velocity v_t for free fall in air, which can be deduced from the equilibrium between gravitational force and air resistance

$$m\,g = \frac{1}{2} A\, c_w\, \rho_{Air}\, v_t^2 \tag{3}$$

with gravitational constant g, the cross sectional area A orthogonal to the direction of the velocity, the drag coefficient c_w and the density of air ρ_{Air}. The terminal velocity is

$$v_t = \sqrt{\frac{2\,m\,g}{A\,c_w\,\rho_{Air}}}\,. \tag{4}$$

For artificial rain the falling height is only some meters and the terminal velocity is not reached. The movement of a drop is described by

$$m\,\ddot{x} = m\,g - \frac{1}{2} A\, c_w\, \rho_{Air}\, \dot{x}^2\,. \tag{5}$$

The coordinate x is pointing to the ground. For the velocity $v(h)$ after a height h of free fall follows

$$v(h) = \frac{\exp(-B\,h)\sqrt{g\left(\exp(2\,B\,h)-1\right)}}{\sqrt{B}} \tag{6}$$

with

$$B = \frac{A \rho_{Air} c_w}{2m} . \qquad (7)$$

According to Best [4] the terminal velocity of drops is given by

$$v_t = D \exp(bz)\{1 - \exp[-(d/a)^n]\} \quad \text{[m/s]} \qquad (8)$$

with the altitude z in km; for the other parameters see Table 1.

Atmosphere	D	b	a	n
ICAN	9.32	0.0405	1.77	1.147
STA	9.58	0.0354	1.77	1.147

Table 1: Parameters for Equation (8)
(ICAN: Northern Standard Atmosphere, STA: Summer Tropical Atmosphere, for details see references in [4])

With these values and using Equations (4) and (7) a drag coefficient was calculated und inserted in Equation (6). The resulting velocities after a free fall of 3.75 m compared to terminal velocities are shown in Figure 3.

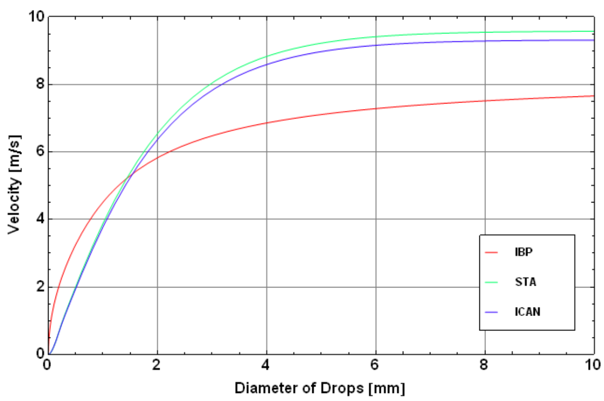

Figure 3: Velocity of water drops after a free fall of 3.75 m (IBP) compared with the terminal velocities of natural rain (STA and ICAN)

Below diameters of 1.75 mm the velocity after 3.75 m become higher than the terminal velocity of Equation (8). This is a contradiction because no drop can be faster than its terminal velocity.

However, according to [5] the drag coefficient is not a constant, but depending on the velocity (Figure 4).

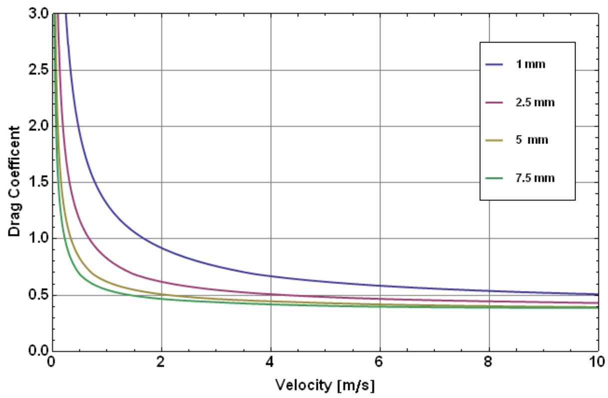

Figure 4: Drag coefficient for several spherical drops as a function of velocity

Introducing this dependence into Equation (5) and solving it numerically removes the contradiction. Results for the impact velocity for several falling heights are shown in Figure 5. In computations of rain noise the impact velocity has to be modelled carefully.

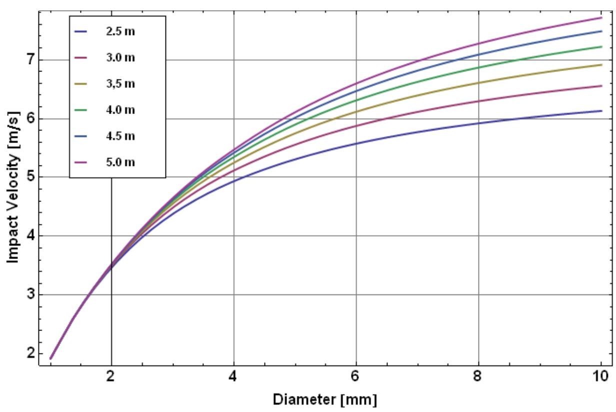

Figure 5: Impact velocity for spherical drops for several falling heights as a function of diameter

Natural and artificial rainfall

According to Marshall and Palmer [6] the distribution of raindrops is given by

$$N(d) = N_0 \, e^{-\Lambda d} \qquad (9)$$

with

$$\Lambda = 41 R^{-0.21} \quad [\text{cm}^{-1}] \qquad (20)$$

where $N(d)$ is the number of drops per mm diameter interval and m³ of air and R is the rainfall rate in mm/h. For a rainfall on 18 September 1969 in Locarno-Monti Waldvogel [7] measured the values for the parameters of Equations (9) and (10) listed in Table 2.

Rain number	Time (CET)	N_0 [m^{-3} mm^{-1}]	Λ [mm^{-1}]	R [mm/h]
1	1430-1500	6347	2.97	5.6
2	1500-1520	6571	3.35	2.6
3	1520-1545	16523	3.51	5.7
4	1545-1620	3804	2.60	5.0
5	1620-1700	22791	4.16	3.5

Table 2: Parameters for natural rain according to [7]

In our rain noise test facility we measured the distribution of diameters by collecting and weighting the drops. It can be described by the normal distribution

$$N_a(d) = \frac{N_{0,Lab}}{\sqrt{2\pi}\,\sigma} \exp\left(\frac{-(d-d_{mean})^2}{2\sigma^2}\right) \qquad (31)$$

with $N_{0,Lab}$ = 195 (mm s)$^{-1}$, d_{mean} = 5.87 mm and σ =0.34.

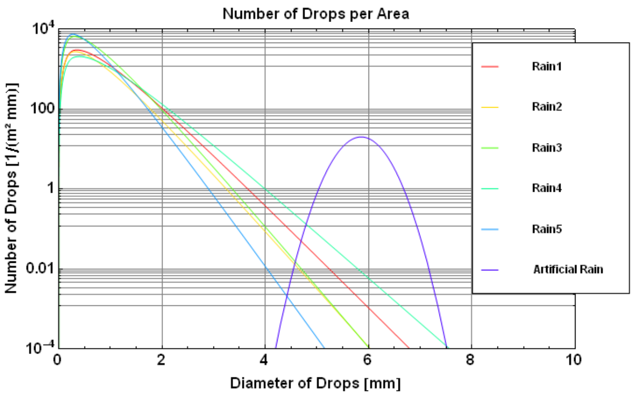

Figure 5: Number of drops per unit area as a function of drop diameter for natural and artificial rains

While the number of drops per square meter has a maximum at diameters below 1 mm, the maximum for artificial rain is around diameters of 5.8 mm. There are also differences in the shape of the distribution.

Validation

To validate the model we mounted a glass pane in our rain noise test facility and measured the rain noise for different falling heights. For these different heights the radiated power was calculated using different shapes of drops. According to [8] a raindrop with diameter 5.4 mm, falling with terminal velocity, looks like a hemisphere, while a drop with diameter 0.8 mm looks spherical.

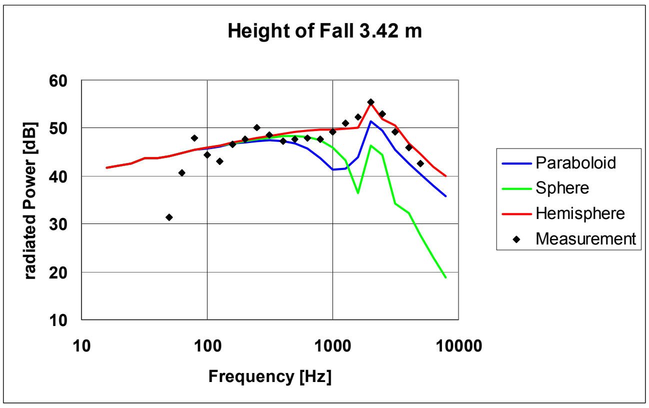

Figure 6: Comparison of measured and computed intensity for a glass pane excited by artificial rain

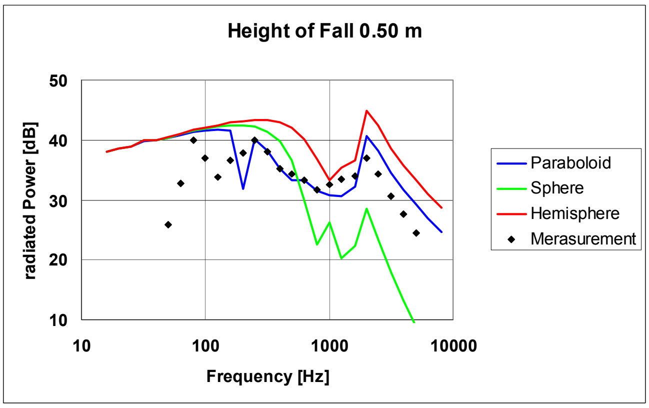

Figure 7: Comparison of measured and computed intensity for a glass pane excited by artificial rain

For a falling height of 3.42 m the measured and calculated intensities are in good agreement for hemispherical drops (Figure 6), whereas for the falling height of 0.50 m (Figure 7) the measurement lies mostly between the computations for paraboloid and spherical shape. It is not clear, whether the deviations are due to the drop shape or due to vibrations excited during the dripping.

The glass pane properties used in the calculations are as follows: Young's modulus: 62 GPa, density: 2500 kg/m³, Poisson's ratio: 0.22, internal loss factor: 0.001, thickness: 6 mm.

Comparison of artificial and natural rainfall

After the successful validation of the computational model for a falling height of 3.42 m this model was used for a comparison of the computed artificial rain to five phases of a natural rain impinging on the same glass pane (Figure 8). Obviously, the artificial rain produces more sound power than any phase of the natural rain. The largest differences between the five phases of the natural rain are around 10 dB.

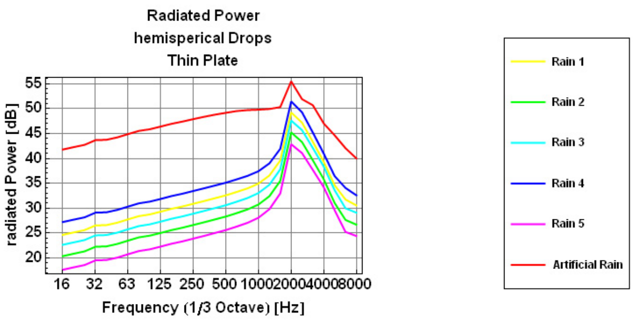

Figure 8: Radiated power for a glass pane excited by artificial rain and five natural rains

Conclusions

Using the model proposed by Petersson [2] the sound power radiated by a glass pane in the rain noise test facility could be predicted in good agreement with the measurement. For the usual falling heights in the test facility the drops are hemispherical at impact. Application of the model to natural rainfall indicates that artificial rainfall generates more sound power.

Notwithstanding there remains a lot to be done for a detailed understanding of rain noise. For example, the shape of drops shortly before impact is not known exactly. Furthermore, there is also only an estimate for the impact velocity. Possibly high-speed photography will give some more information.

Acknowledgments

The measurements of rain noise and drop diameter distribution in the rain noise test facility were carried out by J. Seidel.

References

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