Optimization of multilayered porous acoustic absorbers

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Introduction

Nowadays for noise reduction, i.e. for the reduction of the sound pressure level, porous materials, like fiber absorbers and poro-elastic foams, are predominantly used. The acoustic effectiveness of porous absorber layers in components or assembly groups mainly depends on its effective material properties and its assembly.

We will show, how the effective material parameters can be determined by simulation. The basis is a stochastic model which represents the microstructure of the material realistically. Depending on the used acoustic model, different sets of effective material parameters are needed. Whereas the models of Delany & Bazley [3], [9] and Allard & Johnson [2], respectively, used for highly porous and stiff absorbers only depend on the geometrical information of the microstructure, the model of Biot [4] additionally needs the effective elastic behaviour. By taking into account the viscoelastic behaviour of the frame material of the absorber, also the loss factor of the model of Biot can be computed as a function of the frequency of the acoustic wave.

As an example we will study the acoustic behaviour of closed-cell foams. For this type of absorber the equations of Biot can easily be reduced to the equation of motion for an elastic solid.

In the last section we will study the headliner of a modern car and thereby show that even for highly porous materials it is necessary to regard the solid-borne sound for low frequencies (Figure 6).

The advantage of our method compared to all the other methods currently available for poro-elastic absorbers is that one can predict the behaviour completely without the production of any blank parts or prototypes.

Effective material parameters

For any porous media we first derive a model for the microstructure. Depending on the type of absorber, these models include different information: While the models for fiber absorbers depend on the fiber diameter and their orientation, foam models rely among other things on the number of cells and the total volume [10]. In all cases the model parameters can be obtained for existing absorbers by means of 3D image analysis.

Based on the model parameters and a stochastic process, different realizations of the microstructure model can be generated. The effective material parameters for the absorber are then calculated by solving PDEs on the microstructure models.

Effective geometrical parameters

The effective geometrical parameters of the models of Delany & Bazley and Allard & Johnson defined in [2] can be determined by solving the cell problems of the Stokes' equation and the diffusion equation, respectively. For the case of identical parallel cylindrical pores the parameters can also be analytically calculated [2]. The discretization error made by determining them numerically, using the algorithms described in [12] and [13], is exemplarily shown for the flow resistivity in Figure 1.

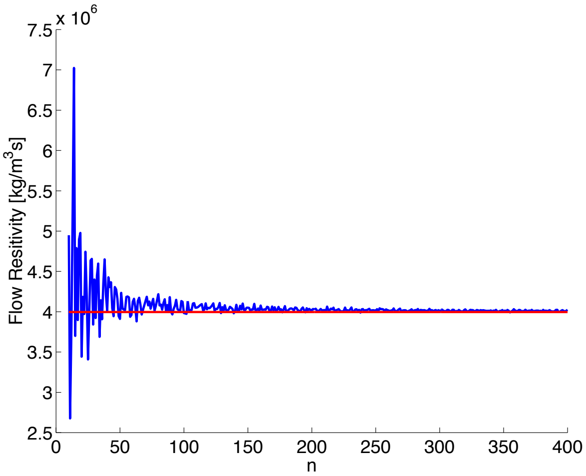

Figure 1: Numerically and analytically determined flow resistivity in the case of cylindrical pores with n being inversely proportional to the mesh size.

Effective viscoelastic parameters

The experimental estimation of frequency dependent effective elastic parameters for porous viscoelastic materials is impossible for the whole set of parameters. Numerically they can be determined in three steps:

- First of all the frame material is modelled as a generalized Maxwell-element by fitting the complex Young's modulus

$$E(f) = \mu_0 + \sum_{j=1}^{N} \mu_j \frac{\omega^2 \tau_j^2}{1+\omega^2\tau_j^2} + \mathrm{i} \sum_{j=1}^{N} \mu_j \frac{\omega \tau_j}{1+\omega^2\tau_j^2} \quad (1)$$

  of the generalized Maxwell-element to the measured Young's modulus (of the frame material) using the routine [11] (Figure 2).
- Secondly, using the algorithm described in [8], a relaxation test is numerically performed for the porous absorber.
- Thirdly, using again the routine [11], the obtained normalized storage modulus is fitted by the normal-

ized characteristic relaxation function

$$\gamma(t) = 1 + \sum_{j=1}^{N} \frac{\mu_j}{\mu_0} \exp\left(-\frac{t}{\tau_j}\right)$$

of a generalized Maxwell-element (Figure 3).

Since the fluid does not contribute to the shear restoring force, the shear modulus of the porous absorber coincides with the shear modulus of the frame material

$$N(f) = \frac{E(f)}{2(1+\nu(f))}$$

where ν is the Poisson's ratio.

This way we obtain a model for the viscoelastic behaviour of the porous absorber from which the effective Young's modulus can be computed according to Equation (1) and the loss factor of the porous absorber is simply defined as ratio of the imaginary and the real part of the complex Young's modulus

$$\eta(f) = \frac{\Im(E(f))}{\mathrm{Re}(E(f))}.$$

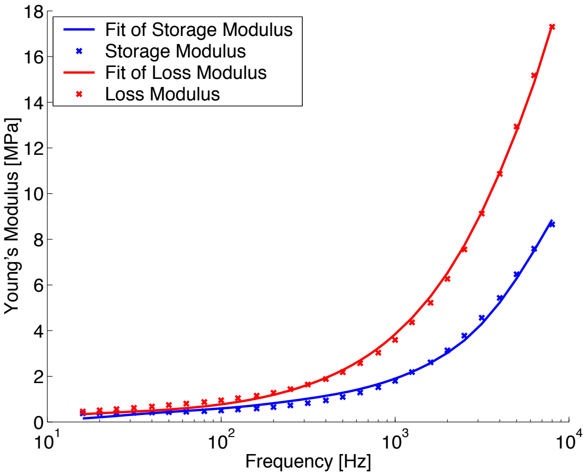

Figure 2: Fitting of the measured complex Young's modulus of the viscoelastic solid.

Closed-cell foams

Closed-cell foams are the typical example for a highly porous absorber, whose acoustic behaviour can not be predicted by the models of Delany & Bazley and Allard & Johnson, respectively, because both models essentially depend on the flow resistivity.

On the other hand also the model of Biot, derived in [2] for open-cell absorbers, can not be applied. Therefore we first derived an acoustic model for closed-cell foams from the equations of Biot.

Acoustic model

Using the equality

$$u^s \equiv u^f =: u$$

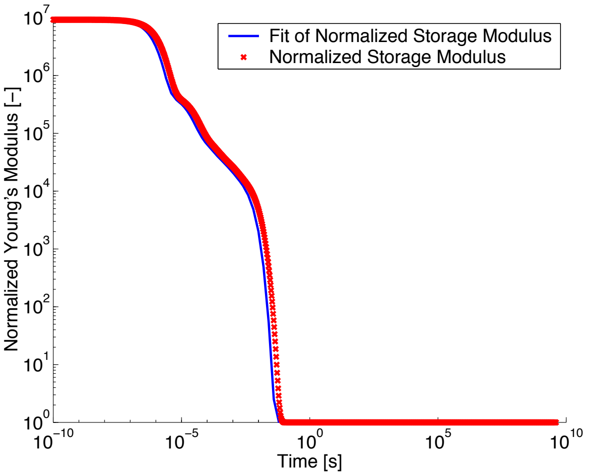

Figure 3: Fitting of the normalized storage modulus of the isotropic closed-cell foam for a relaxation test.

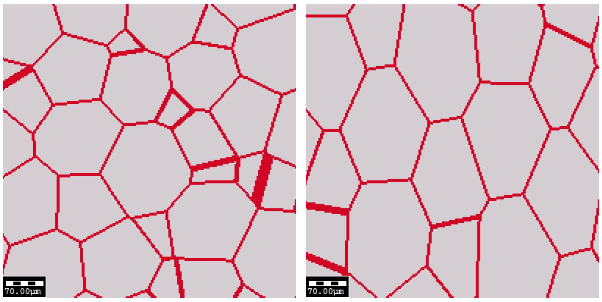

Figure 4: Isotropic (left) and anisotropic (right) closed-cell foam.

of the (average) displacements of the solid and fluid part, derived in [5] for closed-cell absorbers, the equations of Biot [4] (for isotropic materials) reduce to

$$(\rho + \phi\rho_0)\ddot{\vec{u}} = (P - N + 2Q + R)\,\mathrm{grad}\,(\mathrm{div}\,\vec{u}) + N\Delta\vec{u}, \quad (2)$$

in which ϕ denotes the porosity, ρ the density, N the shear modulus and P, Q, R the Biot parameters of the closed-cell foam and ρ_0 the density of the fluid in the pores. Since Equation (2) has the form of the equation of motion for an elastic solid, the acoustic behaviour of closed-cell foams can be predicted, except for the determination of the parameters, the same way as for elastic solids [7].

Determination of the effective viscoelastic parameters

By fitting a generalized Maxwell-element to the measured complex Young's modulus of the viscoelastic solid (a polymer), we determined as a first step the Maxwell parameters of the viscoelastic solid, shown in the first column of Table 1. Using these parameters for the solid skeleton material of the microstructure models shown in Figure 4, allowed us to determine the Maxwell parameters of the closed-cell foams (Table 1).

Acoustic absorption

In Figure 5 we compare the acoustic absorption at normal incidence for a 10 cm thick layer of viscoelatic solid and closed-cell foam, respectively, backed by a rigid wall. Therefore we additionally use the density $\rho_s = 900\ \mathrm{kg/m^3}$

Parameter	Solid	Isotropic Foam	Anisotropic Foam
$\nu[-]$	0.4	0.3658	0.3637
μ_0 [MPa]	2.218e-05	9.5541e-06	9.4393e-06
μ_1 [MPa]	1.34383	83.45239	82.44990
τ_1 [s]	1.439e-04	8.4558e-07	8.4558e-07
μ_2 [MPa]	193.68936	3.75925	3.71403
τ_2 [s]	1.340e-06	1.6789e-05	1.6788e-05
μ_3 [MPa]	8.72564	0.57894	0.57204
τ_3 [s]	2.6608e-05	9.0830e-05	9.0802e-05
μ_4 [MPa]	0.57352	0.24706	0.24416
τ_4 [s]	7.8205e-04	4.9348e-04	4.9335e-04
μ_5 [MPa]	0.49526	0.21337	0.21081
τ_5 [s]	6.5728e-03	4.1472e-03	4.1471e-03

Table 1: Maxwell parameters of the viscoelastic solid and the closed-cell foams.

of the frame material and the density $\rho_f = 1.204$ kg/m^3 and the bulk modulus $K_f = 0.142$ MPa of the fluid (air at 20°C) in the pores.

For the high frequency range the anisotropic closed-cell foam shows only a slightly better acoustic absorption.

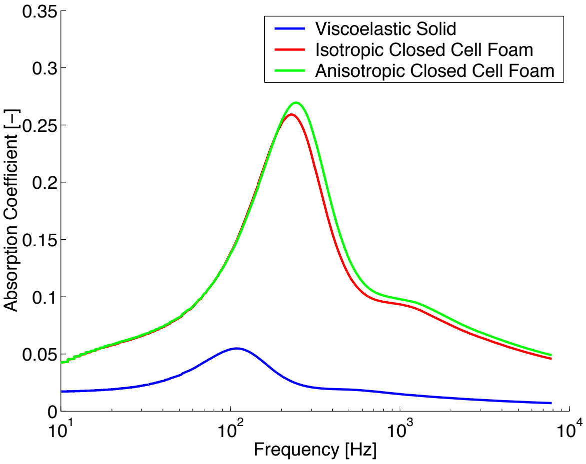

Figure 5: Comparison of the acoustic absorption at normal incidence of the viscoelastic solid and the closed-cell foams. As an approximation, the isotropic part of the effective elasticity tensor was also used for the anisotropic closed-cell foam.

Multilayered absorbers

In practical applications layered porous materials are used. To predict the acoustic behaviour of these multilayered absorbers, we relate the velocities and stresses near the interface of two layers according to [6].

By predicting the acoustic absorption coefficient of the multilayered headliner described in Table 2 with the different acoustic models and comparing the predictions with measurements, we will show, that it is necessary to regard the solid-borne sound for low frequencies.

Simply plugging the parameters of Table 2 into the acoustic models of Delany & Bazley, Allard & Johnson and Biot for open-cell absorbers gave the predictions shown in Figure 6. Comparing them with the measurements done in [1] reveals, that for frequencies below 1 kHz only the model of Biot can predict the acoustic absorption correctly, and only the model of Biot takes into consideration the solid-borne sound.

Parameters	Blanket	Screen	Polyurethane Foam	Polycarbodiimide Foam
Thickness d [mm]	4	0.8	5	16
Porosity ϕ [%]	98	80	97	99
Flow resistivity σ [kg/(m^{3}s)]	34000	3200000	87000	65000
Tortuosity α_∞ [-]	1.18	2.56	2.52	1.98
Viscous characteristic length Λ [μm]	60	6	37	37
Thermic characteristic length Λ' [μm]	87	24	119	121
Density ρ [kg/m^3]	41	125	31	16
Young's modulus E [kPa]	286	2600	143	46.8
Shear modulus N [kPa]	110	1000	55	18
Loss factor η [-]	0.0145	0.1	0.0545	0.1

Table 2: Effective acoustic parameters of the headliner described in [1].

Summary and outlook

Our method for predicting the acoustic behaviour of porous absorbers can be applied to a wide variety of materials: fibrous absorbers, open and closed-cell foams, viscoelastic solids or viscoelastic composites. For foams and composites the acoustic models have to be extended to the anisotropic case.

By varying the parameters of the microstructure, it is possible to optimize the microstructure with respect to a desired acoustic absorption behaviour. Thereby one should apply constraints on these parameters which assure that the corresponding configurations can be

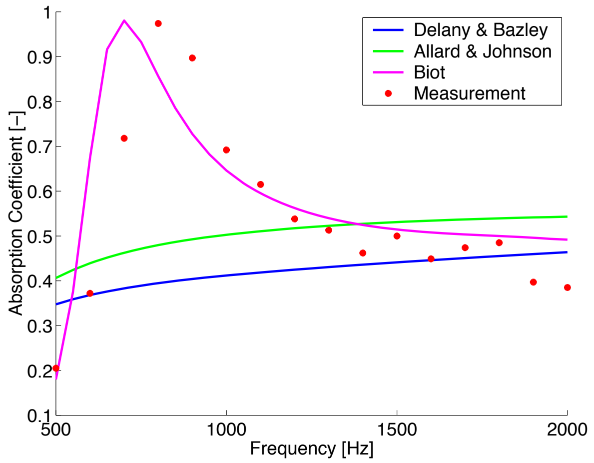

Figure 6: Predicted acoustic absorption for the multilayerd absorber with the parameters of Table 2.

manufactured in reality.

Apart from optimizing single materials, it is also possible to optimize multilayered absorbers by changing their layer design and comparing the predicted acoustic behaviour. As we have shown, the choice of the correct acoustic model for each layer, can be of crucial importance.

Acknowledgement

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