Evaluation of the sound field in small fitted enclosures in the modal range

F. B. Konkel, B. A. T. Petersson

Institute of Fluid Mechanics and Engineering Acoustic, TU Berlin, Germany, Email: frederic.konkel@tu-berlin.de

Introduction

This paper deals with the sound field in small fitted enclosures that are filled to different degrees with rigid obstacles. In a previous work the sound field is divided into three distinguishing ranges [1].

The range at low frequencies is the quasi statistic range, where the system is controlled by the mass or the stiffness of the air. The second established range is the modal range encompassing the first few well separated modes. Finally, the third range is the multi-resonant range.

This paper presents and considers a new calculation model for the modal range of the sound field. Therefore, wave guides, which vary in cross section and length, are modelled. The given free volume is filled with these different wave guides. The transfer impedance in the small fitted enclosure between the source and the receiver is solved by the utilization of a Monte Carlo Simulation. In parallel, the acquired calculated results are compared with measured quantities.

Theoretical background

The sound field varies radically with frequency because the wavelength is either bigger, in the same dimension or smaller than the characteristic dimension of a small enclosure. Supplementary, the small enclosure is filled with rigid obstacles.

The common calculations in the intermediate frequency rang are only valid for empty rectangular enclosures with uniform wall impedances. This is also valid for mirror source modelling.

The modelling with ray- or beam- tracing is impossible because these methods are limited to wavelengths which are much smaller than the characteristic dimension of an enclosure [2].

The variable used to describe the sound field in the enclosure is the transfer impedance Z. Accordingly, Z is given by the ratio of receiver pressure p and the source velocity flow q.

With all walls rigid this enclosure represents an engine compartment as well as white goods. In engineering applications the exact positions of the objects are not known in small enclosures at an early point of design. Therefore, a novel approach will be established to predict the sound field in the modal range for an early design stage.

In this paper the model concentrates on wavelengths which are in the same dimension of the enclosure. Therefore, a transmission line formalism with two ports is generated and a probabilistically distributed approach will be established.

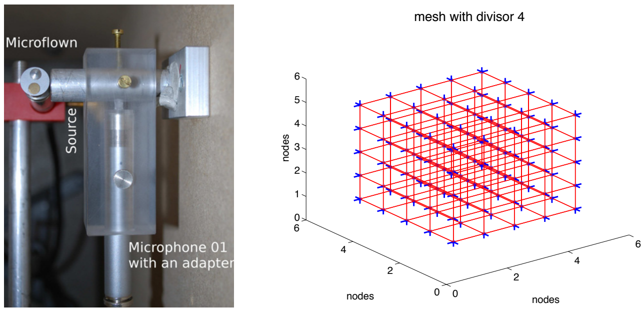

Figure 1: left: Velocity flow investigation, right: six-port 3-D rectilinear mesh with 125 nodes, 300 tubes and 125 terminations

Two port and termination

A two port / fore-pole description is used to describe a system of coupled wave guides. The element matrix for a tube coupled at both ends is given by:

$$\begin{pmatrix} p_1 \\ q_1 \end{pmatrix} = \begin{pmatrix} \cos(kl) & j\frac{\rho c}{S}\sin(kl) \\ \frac{jS}{\rho c}\sin(kl) & \cos(kl) \end{pmatrix} \begin{pmatrix} p_2 \\ q_2 \end{pmatrix} \quad (1)$$

The element matrix for a one-side terminated tube is given by:

$$\begin{pmatrix} p_1 \\ q_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{jS}{\rho c}\tan(kl) & 1 \end{pmatrix} \begin{pmatrix} p_2 \\ q_2 \end{pmatrix}, \quad (2)$$

where l is the length of the tube, and S is the cross section. [3]

The transfer matrices can be transformed easily to impedance matrices. The transformation of the transfer matrices to the impedance matrices for each element e is given by:

$$\mathbf{p}_e = \mathbf{Z}_e \mathbf{q}_e. \quad (3)$$

By using a topology matrix the transfer impedance can be calculated by assembling the four-pole matrices of each point together:

$$\mathbf{q}_e = \Phi_e \mathbf{q}, \quad (4)$$

where Φ describes the position of the element in the assembly. The summation over all N elements results the global impedance matrix $\mathbf{Z}$:

$$\mathbf{Z} = \sum_{e=1}^{N} [\Phi_e]^T \mathbf{Z}_e [\Phi_e]. \quad (5)$$

An average transfer impedance is given for a statistical description of a population of realizations by:

$$\overline{Z} = \frac{1}{N} \sum_{i=1}^{N} Z_i. \quad (6)$$

Equation 6 specifies a transfer impedance for a medial source-receiver-distance. For the determination of the average transfer impedance of the system the system has to be averaged over a large number of randomly created realizations.

Monte Carlo Simulation

The free fluid volume, the maximum length, and the maximum cross section of the wave guides in the set are the common linking parameters.
The Monte Carlo Simulation is required because the influence of the parameters cannot be described analytically [4]. Therefore, different sets of waveguides are simulated. In [5] the analyses of the influence of different length and cross section is given, only one parameter is varied at one time.

Experimental examination

The enclosure is filled with different degrees of fittings. Therefore, the first investigation concentrates on a 30 % filled enclosure. The maximum filling is given by 60 % (see figure 2). The fittings are differently shaped rectangular rigid wood logs and one rectangle of perspex.

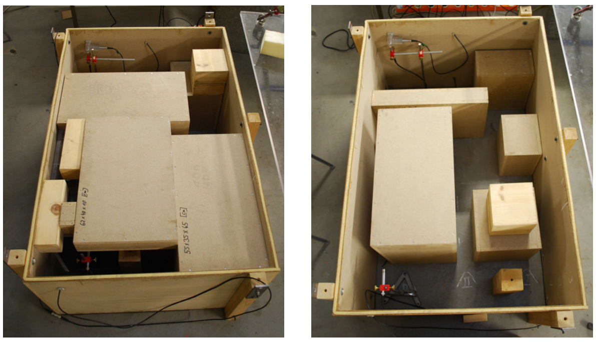

Figure 2: left: test rig 60 % filled, right: test rig 30 % filled with rigid objects without top perspex boundary

The measured transfer impedance Z between the volume velocity source and the pressure receiver is graphically displayed for lucidity in third octave band as thick curve. The source signal is given as white noise.

One of the most important parameters in the measurement is the correct velocity determination.
For the velocity determination two different methods are used. A review of different methods is given in [6].
For the first investigation a velocity sensor based on a hot wire anemometer is used [7].
The second established method is the two microphone method with an adapter respectively.
In figure 3 the measured results are pictured form 30 to 3000 Hz for comparison as a thin curve for a 60 % filled enclosure.
The two microphone method is restricted to the region $0.1\pi < k \cdot \Delta x < 0.8\pi$. Therefore, the calculated error will be of the same order of magnitude [8]. In [9] a method is given to calibrate two microphones to measure acoustic impedance in a wide frequency range.

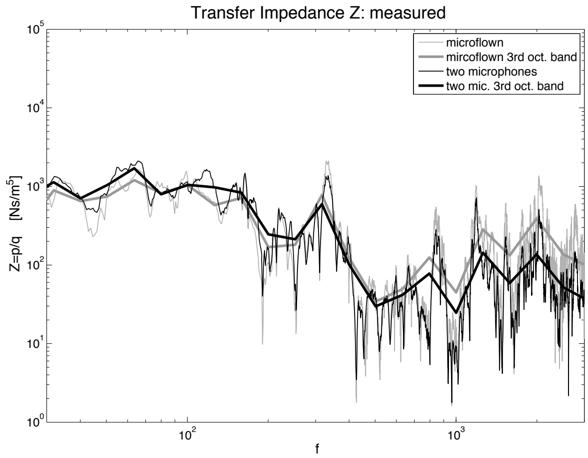

Figure 3: Measured transfer impedance with velocity sensor (grey) and two microphones (black). 40 % free space.

A multiplicity of measurements with a filled enclosure of the same percentage is displayed by grey lines in figure 4 to 6. Consequently, the fittings are installed in different positions within the closed enclosure. Therefore, the maximum, minimum and median value are displayed in solid grey lines.

Results of the velocity determination

The amplitude of the two microphone probe (black curve) in comparison with the velocity sensor (grey curve) is obviously lower above frequencies of 500 Hz. This phenomenon is the expected amplitude decay of a microphone distance of 70 mm.
For the investigation the amplitude of the intermediate frequency range is most important. Hence, the two microphone probe is used for the comparison with the model.

Results of the measurement

The higher the fill rate of the enclosure the longer the way between the source to the receiver for the acoustic wave. Therefore, it can be assumed that the fittings build a kind of labyrinth and it can observed that the transfer impedance will tend to an infinite tube(see figure 5).
It can also be seen that because of non-ideal rigid obstacles the dissipation will rise by the degree of filling.

Modelling of a probabilistically distributed approach

The method uses a two port description to calculate the transfer impedance between the volume velocity source and the pressure receiver with a six-port 3-D rectilinear mesh topology (see the right hand side of figure 1).

Geometries of the model

The script for the rectangle wave guides is based on the definition of a *divisor* N. This divisor divides each length of the volume in N parts. The cross section of each tube per line is equal. This is necessary in order to disable cross section leaps for the description of the first essential modes of the system.

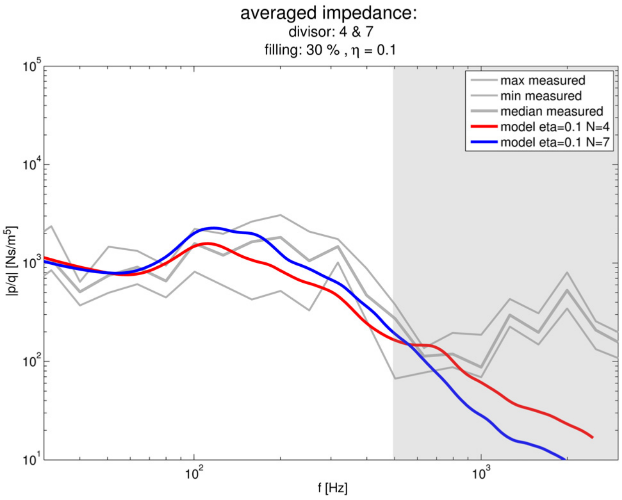

Figure 4: 70 % of free space in the enclosure

In the next step the free volume is filled with the generated tubes. Afterwards the nodes are created. For each node a matrix is generated with the information of the three dimensions and the two kinds of orientations. This matrix also entails the information whether the node is in the middle or at the border of the mesh and the given cross section of the available border tubes.
The number of tubes N_{Tu} is given by $N_{Tu} = 3 \cdot (N+1)^2 \cdot N$ and the number of terminations N_{Te} is given by $N_{Te} = 6 \cdot (N+1)^2$.
To allocate the cross section per each line it is necessary to know the neighbours of each node and for this reason the connecting tubes. Hence, the neighbours are calculated and stored. In the next step the cross sections are distributed to the neighbour nodes in the three dimensions.
The transmission line matrix is filled with this information. With the source and receiver distance the nearest nodes are calculated and the system of equations will be solved. A Monte Carlo Simulation is used to get results for varying parameters which are graphically displayed in figure 4 to 6.

Variation of geometries and parameters

The parameters that control the variation in this model are the free volume space in percent (fVP) and the maximum set of divisors N per length. Therefore, the maximum cross section of the wave guides is defined by:

$$S = \frac{fVP}{(N+1)^2 \cdot (l_x + l_y + l_z)}. \qquad (7)$$

The maximum cross section S in the model used varies between 5 cm (N=7 and 60% filled) and 20 cm (N=2 and 0% filled). Hence, the number of nodes varies between 27 nodes and 512 nodes. The outcome of this is the range of tubes between 54 and 1344 tubes and 54 to 384 terminations.
In all calculations the used divisor varies between 2 and N to picture higher cross sections as well.
Thus, the varying parameters are the free fluid volume, the maximum length and the maximum cross section of the wave guides. The cross section influences the amplitude of the modes and the maximum length should control the frequency of the lowest mode.
The Cartesian orientation of the fittings is only mapped by the random generated cross sections of the tubes in the enclosure. Hence, in this realization there are no terminations within the modelled enclosure. Each tube between the boundaries and additionally the terminations have always equal cross sections. The model is queried for directional organisation of the fittings for further calculations.
The model is limited to a Helmholtz number smaller than 1 ($He < 1$). Hence, for higher frequencies the calculated results are incorrect because the plane wave propagation condition for the wave guides is violated.

Losses

As the measurements show, the losses vary by the percentage of the enclosure is filled with obstacles. This is the case because the wood logs are not perfect rigid obstacles. Therefore, the model is calculated with three different loss factors like $\eta = 0.15, 0.10, 0.05$.

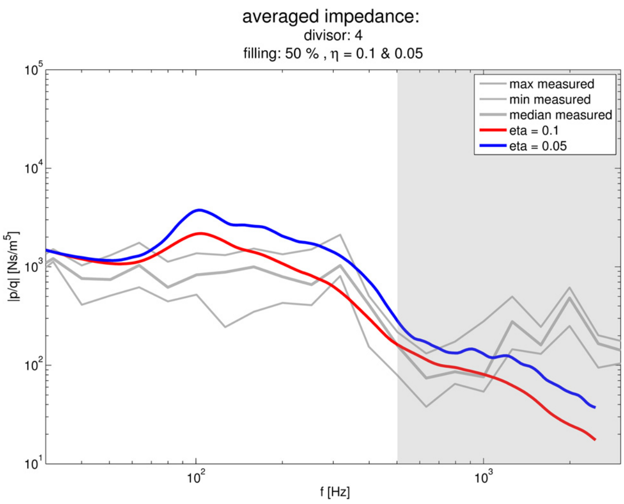

Figure 5: 50 % of free space in the enclosure

Comparison of the model and the measured results

The comparison between the measured results and the calculation for a filling of 30 % is graphically displayed in figure 4 (compare to the test rig in figure 2). It can be observed that the results of the six-port 3-D rectilinear mesh calculation with $k = k \cdot (1 - i \cdot 0.1)$ correspond well with the measured results. The first increase of the model is given by the length between the source and the receiver position with $\frac{c}{2 \cdot l}$. The calculation with a divisor $N = 7$ shows a heightening of this first mode because the cross section S decreases with the number of N. Therefore, the amplitude rises by $\frac{\rho \cdot c}{S}$ [1].
It can be seen that a damping with $\eta = 0.05$ is too low for 50 % of filling. The amplitude of the model is in all reagions too high. Therefore, the chosen η is set to 0.1. Hence, the model corresponds well with the measurements with the higher η (see figure 5). Naturally, the same increasing as in the 30 % filled calculation is obvious.

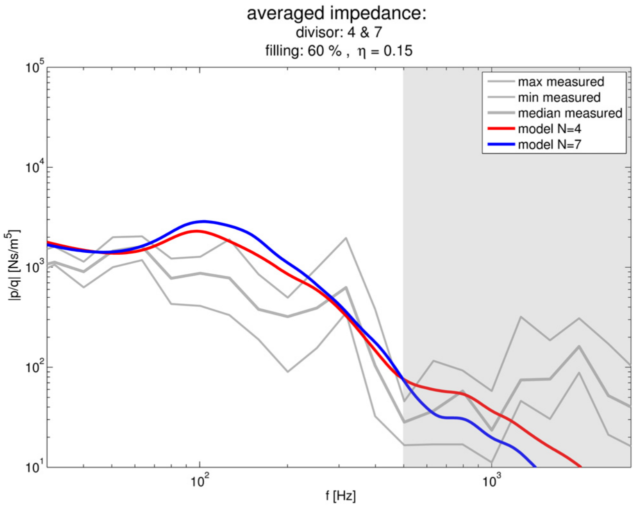

Figure 6: 40 % of free space in the enclosure

By a degree of filling with 60 % the number of N is too high to embrace the claim of a flat characteristics of the transfer impedance Z in lower frequencies. Also, the length of the composite tube is too short because it is limited by the maximum legth of the enclosure. By the use of more divisors N, the decay in the frequency range of 300 Hz to 500 Hz is higher because the length between the source as well as between the receiver to the terminations rises. The reason of this lenght change is the calculation of the best position of the source and reciver node (see figure 4 and figure 6). So the positions jump from a boundary node to a node within the enclosure. η is set to 0.15 because of the higher percentage of filling and therefore the higher damping.

Concluding remarks

In this model the boundary terminations are well organized at each boundary node of the system. Therefore, the source and receiver position always represent the distance to the boundary terminations as well.
It can be observed that the length between the source as well as between the receiver to the terminations are crucial for the characterisation of the decay in the range of 300 to 500 Hz. Also, it can be seen that the amplitude of the model is controlled by the cross section of the tubes and the first increasing is controlled by the maximum length of the composed tubes.
The model uses a Monte Carlo Simulation to generate the mean transfer impedance Z with the varying parameters like the free volume space fVP and the divisor N. Therefore, the cross section S and the maximum length l_{max} of the waveguides changes.
A disadvantage of the well organized model is that the cross section changes always with the length (*divisor N*). Therefore, the amplitude rises for the lower eigen modes of the wave guides.
It can be observed that the impedance leaps for higher filled volumes and lower frequencies tends to an infinite tube. Therefore, the results show that the model is not valid in this frequency range.
Furthermore, the decay between 300 Hz and 500 Hz has to be analysed, as well as the scatter in higher frequencies for higher degrees of filling. The model has to be adapted to these conditions and to the fact that the transfer function tend to an infinite tube with losses.
In engineering applications the exact positions of the objects are not known in a small enclosures at an early point of design. With this novel approach it is feasible to predict the sound field at the modal range for an early design stage.

Acknowledgements

The authors gratefully acknowledges the financial support received from the German Research Foundation (DFG) through Grant PE 1155/5-1.

References

[1] Jens Prager. *On the acoustics of small partially open enclosures densely packed with active and passive installations.* PhD thesis, Technischen Universität Berlin, Februar 2008.

[2] Frank H. Brittain and Marlund E. Hale. On the acoustics of small partially open enclosures densely packed with active and passive installations. *NOISE-CON*, July 2008.

[3] M. L. Munjal. *ACOUSTICS OF DUCTS AND MUFFLERS.* John Wiley & Sons., 1987.

[4] David P Landau and Kurt Binder. *A Guide to Monte Carlo Simulations in Statistical Physics.* CAMBRIDGE UNIVERSITY PRESS, second edition, 2005.

[5] Jens Prager and Bjoern A. T. Petersson. Modelling of sound fitted enclosures using probabilistic approaches. *ACTA ACUSTICA UNITED WITH ACOUSTICA*, 93:750–759, 2007.

[6] J.-P-Dalmont. Acoustic impedance measurement, part i: A review. *Journal of Sound and Vibration*, 243(3):427–439, 2001.

[7] Microflown Technologies. Po box 300, 6900 ah zevenaar, the netherlands.

[8] Hans Bodén and Mats Åbom. Influence of errors on the two-microphone method for measuring acoustic properties in ducts. *J. Acoust. Soc. Am.*, 79(2):541–549, February 1986.

[9] R. Boonen, P. Sas, W. Desmet, W. Lauriks, and G. Vermeir. Calibration of the two microphones transfer function method to measure impedance in a wide frequency range. *ISMA Conference on Advanced Acoustics and Vibration Engineering*, pages 4501–4512, September 2006.