Vibroacoustics and sound emission characteristics of thin-walled, oil-immersed transformer vessels

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Introduction

The main sources of noise in power transformers are vibrations of the transformer core cause on magnetostriction and vibrations of the windings cause on Lorentz forces. They originate in oscillating magnetic fields at the line frequency of 50Hz. Since both physical effects are unidirectional, the typical humming noise of transformers have spectral components at twice the line frequency (100Hz) and higher harmonics.
The vibration sources are located within a oil-immersed, rib-stiffened transformer vessel made of thin, flexible steel plates with typical thickness of $10\,mm$. The vibrations generate sound pressure waves which are muliply reflected from structural boudaries and discontinuities. They form standing patterns of vibrations - structural modes of the vessel and standing waves inside the fluid-filled acoustic cavity. If the corresponding natural freqencies are located next to forcing frequencies, unexpected high sound levels are regularely observerd. In the following sections, the vibro-acoustics and sound emission characteristics of power transformers are investigated by numerical methods. The characteristic features of the interaction between the elastic vessel structures and enclosed volume of fluids are analysed.

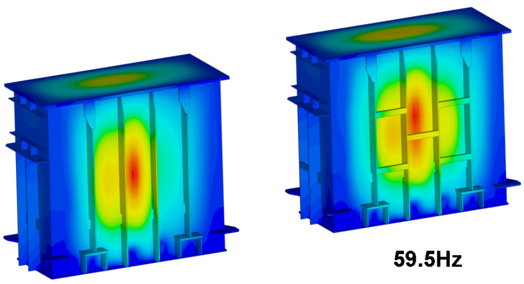


Figure 1: Low order breathing modes: Rise of natural frequencies by additional cross-stiffening.

Rib-stiffened vessels

In order to increase the static stiffness of the transformer tank, the thin wall plates are reinforced by welded parallel or grid-like arrays of stiffeners (Fig. 1). This stiffening also effects the dynamic and acoustic properties of the vessel, depending on the dimensions, geometric form and number of stiffeners, and the (temperature-dependent) material properties. The stiffeners further introduce a coupling between in-plane and bending waves, affecting the flexural wave dispersion characteristics and thus altering the ratio of structural to acoustic wavenumbers. (For details, see e.g. [3]). But this ratio fundamentally determines the radiation efficiency of the complete vessel. Thus, the stiffening introduces a large number of additional parameters which dominate the vibroacoustic behaviour of the transformer.

Low frequency range

For low frequencies the plate bending wavelength is much larger than the distance between adjacent stiffeners. Bending waves can propagate free across the stiffeners, which do not pose any spatial constraints. The bending stiffness of discrete stiffeners has the same effect as a spatially homogeneous increased stiffness across the complete structural continuum. In this frequency range, typically *breathing modes* of entire side walls of the vessel are observed. Their vibration pattern involves several adjacent vessel plate sections across the stiffening ribs. These low order mode shapes are unaltered by additional stiffenings, but their natural frequencies increase (Fig. 1).

Transition frequency range

Any mechanical stiffening structure added to the vessel constrains the rotation and normal displacement of the wall plates. Incident bending waves are partially reflected at the stiffeners and cause bending near fields next to the constraining stiffeners. The bending wave propagation is affected significantly by repeated scattering at multiple stiffeners. In the transition frequency range where the bending wavelength is in the same order of magnitude as the vessel dimensions or the distance between adjacent stiffeners, no clear generic vibrational behaviour can be identified (Fig. 2). Some mode shapes are mainly limited to the plates between parallel stiffeners. Other mode shapes with complicated forms arise and the distribution of natural frequencies can be very irregular. Any additional (cross-) stiffening will reduce the number of modes in this frequency range.

High frequency range

In the high-frequency range the bending wavelength is less than the stiffener distance. The inter-stiffener plates can be treated as individual clamped plates if the stiffeners have very high bending and torsional stiffness relative to the plate bending stiffness. In lightly damped structures the wave propagation is almost completely blocked by the stiffeners and in-plane vibration modes of the fluid-loaded wall plates are formed. But the I-shaped flat stiffeners itself show resonant vibrations in

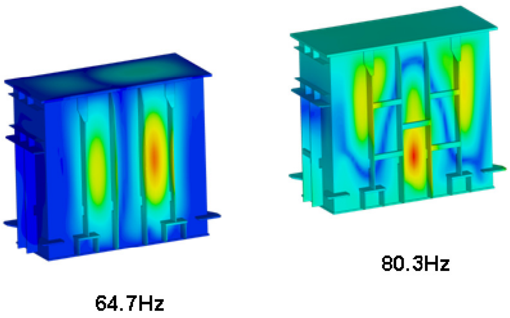


Figure 2: Single field- and cross-stiffening modes in the transition frequency range.

form of lateral bending modes (Fig. 3). In this case, the stiffeners behaves as frequency dependent boundary conditions for any individual plate because the constraints are determined by the dynamic properties of the bounding stiffeners and the whole of the rest of the vessel structure. But any frequency-dependent boundary conditions exclude the existence of an orthonormal set of natural modes.

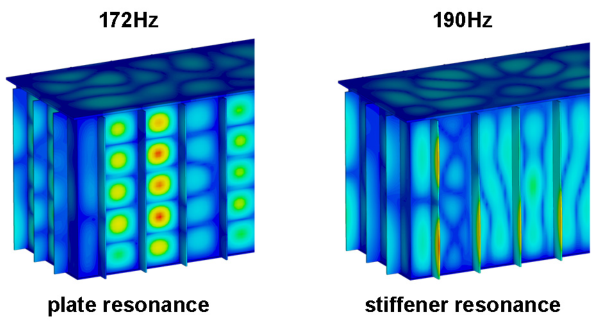


Figure 3: Resonant vibration patterns in the frequency range of audible transformer noise

Structural modes of one-side fluid-loaded plates

Pressure variations in the fluid adjacent to the vessel will accelerate the wall and does result in structural vibrations and vice verca. Reflected sound waves on the surface of the vessel produce a fluid load to the solid. The primary effect of the fluid load is an increased effective mass and damping of the solid structure. The added mass accounts for the inertia of the fluid next to the vibrating structures and decreases their natural frequencies. Especially enclosed finite volumes of liquids exert large reaction forces on the surface of the enclosing physical boundaries. Since the density of the oil is of the same order of magnitude as the density of steel, the fluid load significantly changes the vibrational behaviour of the vessel. Due to geometric unsymmetries (e.g. plates at the edge of a transformer vessel) and the vicinity of adjacent solid structures inside the vessel, the fluid load at power transformers is unsymmetric in general. As a result, fluid forces will couple translation and rotation due to the added mass. Thus the natural frequencies of the fluid-loaded structure cannot be considered by simply increasing the structural mass - the added mass is a function of (1) the geometry of the surface of the structure (including the geometry and relative position of proximate structures), (2) the amplitude and the direction of vibration (mode shape) and (3) a Reynolds-number-like parameter [3]. The added mass has to be measured experimentally or calculated theoretically [1]. For geometric complex setups like transformers, the natural frequencies of fluid-loaded structures can be found from a coupled fluid-structural analysis.

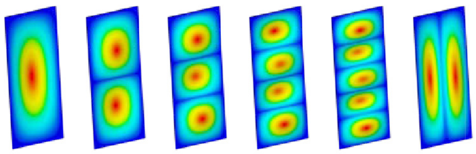
In the frequency range between $100\,Hz$ - $800\,Hz$, the steady-state structural mode shapes formed on partitions of the vessel are mainly based on resonant vibration patterns of the stiffeners and on structural modes of the fluid-loaded wall sections which are separated by the stiffening bars. Some dominant modes can be clearly related to peaks in the measured and calculated sound power emission spectra of the transformer (Fig. 7) and correlate with an increased sound power level if the natural frequency is next to a forcing frequency.

To estimate the natural frequencies of vessel partitions, we consider the inter-stiffener vessel wall sections as individual clamped rectangular plates which are fluid-loaded at one side. Hereby we have to keep in mind that the one-sided fluid-load still allows the propagation of vibrational energy propagation in the liquid, thus by-passing the displacement constraints imposed by the stiffeners. Further, the type of clamping has a large impact to the mode frequencies and shapes, but the clamping conditions are unknown in advance except in frequency bands free of resonances where the stiffener can be considered as infinite stiff.

Since no closed analytical solution is availible for the natural frequencies of rectangular plates, frequency response analyses are applied to a steel plate with dimensions $10\,mm \times 600\,mm \times 1500\,mm$. The mode shapes and frequencies of the clamped plate without fluid load fits exactly to the values stated in [1]. Adding a fluid-load at one side will result in the same mode shapes but the natural frequencies are lowered up to a factor of 2.1 for the lowest mode (Fig. 4). This inertial reaction of the fluid decreases at higher modes, following the a modified approximation formulae for fluid immersed plates [2].

$$f_{(fluid)} \approx f \left(1 + \frac{\rho_{(fluid)} \,/\, 2}{\rho_{(plate)}\; t\; k_i}\right)^{-1/2} \qquad (1)$$

Hereby, $f_{(fluid)}$ denotes the natural frequency of the fluid-immersed plate, f the natural frequency of the unloaded plate, ρ the density of the medium, t the thickness of the plate and k the primary modal wavenumber. For one-side fluid loaded plates, the mass effect is lowered by a factor of 2 compared with a fluid-immersed plate. This is considered by the reduced fluid density (factor 1/2) in equ. (1).



mode	11	21	31	41	51	21
f in Hz (no load)	166	195	250	332	440	455
f in Hz (fluid load)	77	112	156	222	310	302

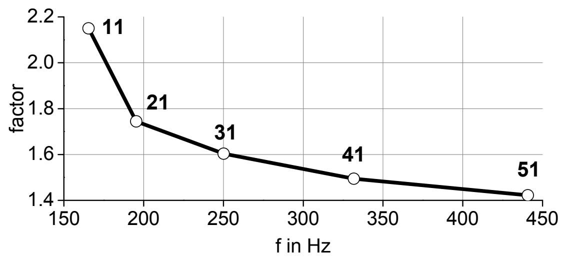


Figure 4: Reduction of natural frequencies of a clamped rectangular plate by one-side fluid-loading.

Cavity modes and coupled vibration response

The oil inside a power transformer represents an acoustic cavity which is bounded by the vessel, forming the fluid-structure-interface oil-steel. Pressure waves which traverse the cavity at the speed of sound, get (partly) reflected at the boundaries, interfere between intersecting waves and forming acoustic cavity modes. The corresponding natural frequencies of the coupled fluid-structure system are determined beside the material properties mainly by the geometry of the physical boundaries and the spatial characteristics of the interference pattern. The interface media oil and steel having different acoustic impedances $Z = \rho\ c$, where ρ and c denote the density and (longitudinal) speed of sound in the medium, respectively (Table 1). With increasing difference of the acoustic impedances, the intensity reflection coefficient $r = (Z_{Oil} - Z_{Steel})/(Z_{Oil} + Z_{Steel})$ of acoustic waves rises. In case of perpendicular incidence, $r = 89\%$ at the oil-steel-interface. Thus, sound waves get multiply reflected between opposite walls of the transformer vessel, increasing the sound energy inside the cavity until a steady state condition with constant sound power emission towards the environment is established. Standing waves inside fluid-filled cavities are formed if the distance d between opposite boundaries is a multiple of half of the wavelength λ of the acoustic wave

$$d = n\,\frac{\lambda}{2}\,. \tag{2}$$

Hereby, n is an integer describing the mode number and the corresponding natural frequency of the standing wave $f_n = c/2 \cdot n/d$ Accordingly, in three dimensions the natural frequencies for a rectangular cavity are given by

$$f_{i,j,k} = \frac{c}{2}\sqrt{\left(\frac{i}{L_x}\right)^2 + \left(\frac{j}{L_y}\right)^2 + \left(\frac{k}{L_z}\right)^2} \tag{3}$$

Media	Z in $Pa\ s/m$
Air	$4.1 \cdot 10^2$
Oil	$1.5 \cdot 10^6$
Steel	$4.1 \cdot 10^7$

Table 1: Acoustic impedances of media at the fluid-structure interfaces of a transformer vessel

where L_x and L_y and L_z are the dimensions of the cavity, and $i,\ j,\ k\ =\ 0,\ 1,\ 2,\ 3,\ \ldots$ the corresponding mode numbers. Inside the fluid cavity, spectral sound components of transformer noise below 1000Hz have wavelength which are in the range of typical dimensions of power transformer. Thus, cavity resonance problems with largely increased vibration amplitudes of the vessel arise regularely at power transformers if a natural frequency of a standing wave is close to any forcing frequency.

The flexible steel plates of the thin-walled vessel represent acoustic boundaries which are neigher open (sound pressure $p = 0$) nor closed (normal fluid velocity $\vec{v}_n = 0$) but adding an additional acoustical terminating impedance to the coupled fluid-structure system. Thus, equ. (3) cannot be used to determine the resonance frequencies of the acoustic cavity in the coupled system of a fluid-filled transformer vessel. Instead, the actual boundary conditions defined by the vessel wall structures surrounding the acoustic cavity have to be considered.

In case that the acoustic cavity inside a oil-filled

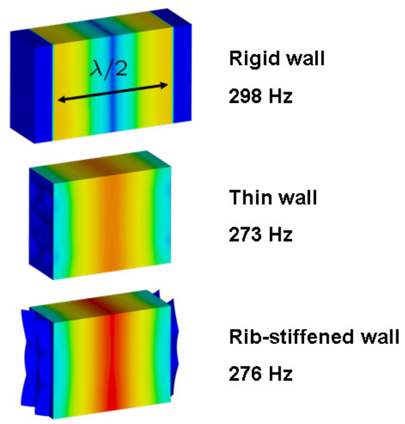


Figure 5: Frequency of the fundamental cavity mode for certain boundary conditions

transformer vessel is subject to fixed boundary conditions, the sides of the vessel behave like rigid walls having vibration minimum and sound pressure maximum located next to the wall. But typical thin-walled transformer vessels show acoustic boundary conditions which are next to the free boundary type (stationary minimum of sound pressure next to the boundaries). Since the wall plates are flexible, the acoustic cavity acts as a amplifier and filter, similar as it is the case in a music instrument. The mass of the plate and additional stiffeners mounted on the vessel significantly change the acoustic impedance and therefore the natural frequencies.

As an example, a simplified transformer vessel with dimension of the oil volume of $d = 2.3m$ is considered. Using mineral oil as the media in the acoustic cavity ($c \approx 1400m/s$), the fundamental mode $d = \lambda/2$ at ideal open or closed boundary conditions is at $f = c/d \approx 300Hz$ (Fig. 5). Including a thin-walled transformer vessel with typical wall thickness of $10mm$ in the model, the additional mass largely reduces the natural frequency of the cavity mode to $273Hz$. Any additional stiffening structures welded on the vessel affecting the acoustic impedance, thus increasing the resonance frequency. The fluid cavity and wall panel undergo a coupled vibration response.

Influence of the wall thickness

To quantify the lowering of natural frequencies of cavity modes by the mass of the vessel, its thickness is varied in the coupled model. In case of very thin wall plates (e.g. plate thickness of $1mm$ as used for distribution transformers), the fluid filled vessel will nearly match the ideal case of free boundary condition. For thicker wall plates, the additional mass of the panels gets significant and lowers the resonance frequency up to 70Hz (Fig. 6). Quantifying the effect of a cavity resonance with respect to the overall sound power level of a power transformers, raised levels up to 7dB have been observed by the author. Regarding transformer noise, cavity resonances can thus be more critical than structural modes. The later one are related to local increased vibrations of vessel plates, whereas a cavity mode involves increased vibrations of large sections of the vessel.

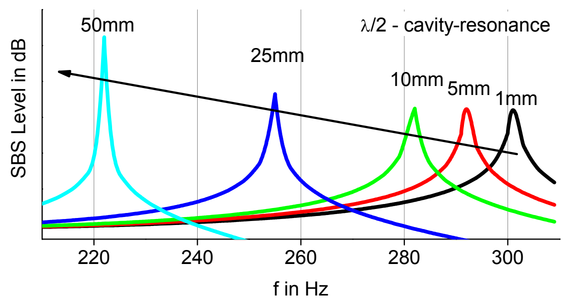


Figure 6: Lowering of the fundamental cavity mode frequency due to the additional mass of thicker walls

Sound emission characteristics

This geometric setup and the spatial distributed diffuse excitation of the vessel results in a complex vibration behaviour. The associated sound emission characteristics results in distinct frequency-dependent sound emission lobes in the acoustic near field. Fluid-structure-interactions (FSI) dominate the vibrational and sound transmission behaviour - amplifying or reducing spectral components of the original sound. For $f < 500Hz$ the acoustic wavelength in air is larger or equal than typical dimensions of transformer vessel sections. In this case, the radiated sound power to the surrounding air is determined mainly by the net normal surface velocity of the vessel. Any local increased (resonant) vibration patterns contributes to the overall sound power by their relative proportion of the total surface area.

High mode densities in the audible frequency range are measured and calculated. Hereby, resonant vibrations of the vessel stiffenings turn out to be more critical than resonances of the fluid-loaded wall plates ((Fig. 7). Lateral vibrations of the stiffeners transfer a bending moment to the adjacent wall sections, thus increasing the area of normal surface vibration.

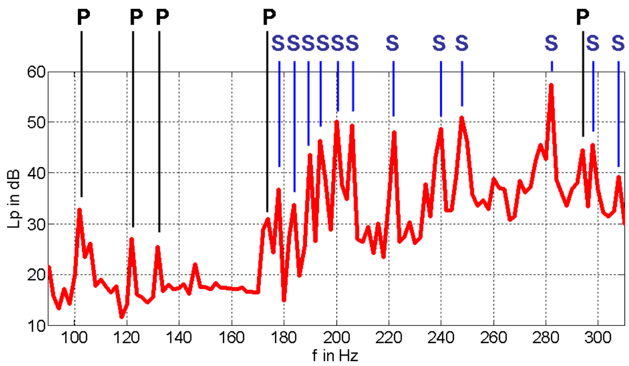


Figure 7: Sound transmission spectra of a transformer vessel: In-plane modes of wall plates (P) and lateral bending modes of stiffeners (S)

Conclusions

The sound radiation characteristics of parallel stiffened vessels of oil-immersed power transformers are extremely complex. Structural and cavity modes cause a strongly frequency-dependent vibrational response of the vessel with high mode density in the frequency range of audible transformer noise. It is impossible to deduce generalalised design guidelines for a vibro-acoustic optimised stiffening design of power transformer vessels. Merely qualitative accounts and general interrelations can be supplied for a more specific desing process. Instead, the complex geometric setup of oil-filled transformer vessels asked for a detailed numerical model and a coupled fluid-structural analysis for a realistic evaluation of the vibroacoustic properties.

References

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