Fundamental sound fields in the core region of curved ear canals

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Introduction

The ear canal works as a transmission-line resonator providing passive pressure amplification in the frequency band 2-4 kHz. Usually it is assumed that the sound wave propagating in the ear canal is a fundamental wave because due to the slenderness of the canal no transversal sound field modes can propagate. The simplest fundamental mode occurs in a straight duct of constant cross-sectional area. In this case the wave is a planar. In a real ear canal, however, the cross section considerably varies from the entrance to the tympanic membrane. In good approximation this can be taken into account using Webster's equation [1] or a stepped-duct approximation [2]. Although in such inhomogeneous ear canals the waves are no longer planar, the sound field widely remains fundamental in the sense that the total sound field can be separated into an incident and reflected wave.

A decisive step towards a more accurate representation of sound fields in real ear canals using fundamental modes has been derived in [3]. The authors introduced a theory taking into account curved equipotential surfaces. In this way they found a modification of Webster's equation which allows sound field computations for new types of curvilinear horns. Later on, the curvilinear horn theory has been extended to curved ducts with general area functions [4]. The usage of an "effective Webster area function" indicates a close relationship to Webster's solution.

Real ear canals differ from cylindrical tube not only with respect to varying cross sections, but also with respect to considerable curvature. Usually two distinct bends can be found. Such general ear canals have been theoretically and experimentally investigated in [5, 6]. It was found that at some locations strong transverse variations of the sound pressure appear which are not in agreement with fundamental waves as usually assumed.

Such disturbances are examined in this paper as well. The sound field in the ear canal is investigated using finite elements (FE). The paper focuses on the sound field in the "core region" of ear canals. The core region is specified by the requirement that the sound field structure is (a) independent of the direction of sound incidence and (b) not influenced by the local sound radiation of the tympanic-membrane. To identify the core region, the sound field at the pinna, in the cavum conchae and close to the tympanic membrane has to be considered. The coupling of the external sound field to the ear canal and the near-field impact of the tympanic membrane is considered in a companion paper [7]. More details can also be found in [8].

It is the main goal of this paper to elucidate the general structure of sound fields in the core region and to provide some theoretical considerations which explain the findings.

Finite element model

An adequate representation of widespread external sound fields around the head cannot be obtained using FE because the number of nodes rapidly increases when the air-filled space around the head is enlarged. To study the sound fields in the ear canal and fairly close to the pinna, a comparably small volume encompassing the pinna, referred to as "pinna-box" [7, 8], is sufficient for the investigations. The anatomical and physiological part of the FE model comprises the pinna including a small part of the surrounding head surface, the ear canal, and a middle ear model, which is necessary to model the influence of tympanic-membrane vibration on the ear canal sound field appropriately. These elements are shown in Fig. 1. The ear canal walls and the pinna are assumed perfectly rigid and clamped. Thus an external source produces sound waves in the ear canal, but no vibrations of the pinna or the ear canal walls. The maximum lateral dimension of the ear canal in the central part is about 7.5 mm. Therefore in the audio frequency range no higher order modes can propagate.

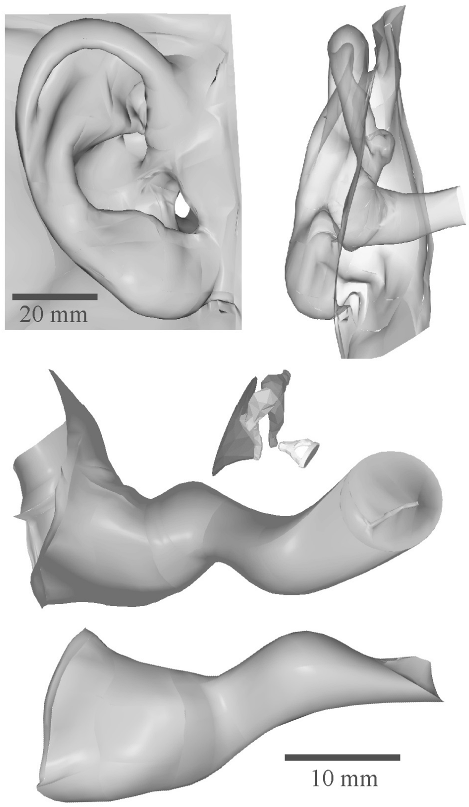

Figure 1: Geometric details of the finite-element model used. The middle ear in the centre of the figure is depicted in the same scale as used for the ear canal.

Details of the FE model concerning mechanic parameters and meshing are provided in [8].

Sound fields in ear canals

The beginning of the core region in an ear canal can be found by comparing the sound fields calculated for sound coming from different directions. Using the pinna-box [7, 8] lateral sound incidence and grazing incidence from front and rear can be approximated.

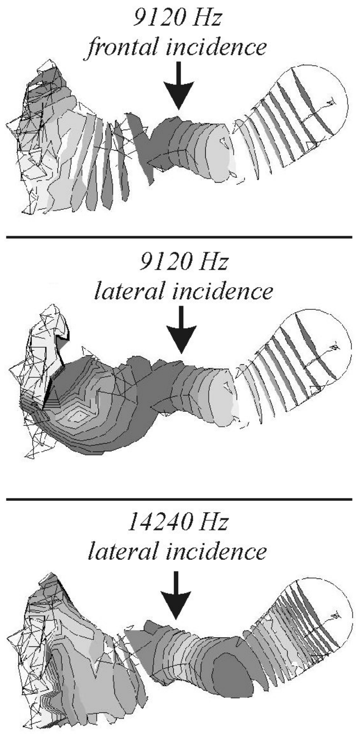

Figure 2: Surfaces of equal sound pressure magnitudes in the ear canal, calculated for different frequencies and directions of sound incidence. Increasing darkness of gray tones indicates increasing pressure magnitude. On the left, the outlines of the cavum conchae can be recognized. On the right, the tympanic membrane seen from above is situated. The arrows indicate the beginning of the core region.

Fig. 2 shows surfaces of equal sound pressure magnitudes which are referred to as "isosurfaces" in the following. At 9120 Hz the sound fields in the cavum conchae differ very much for frontal and lateral incidence. But behind a certain location the structure of the sound field (the shape of the isosurfaces) is virtually identical. The border in between, indicated by an arrow, marks the beginning of the core region. The core region ends somewhere near the tympanic membrane due to radiation of additional sound from the tympanic membrane into the ear canal. However, the posterior end of the core region is not considered in this paper.

The shape of the isosurfaces in the core region observed at 9120 Hz is "regular". This means that the surfaces are not very different from planes. Regular surfaces are slightly curved, but they completely cover the aperture of the ear canal. However, at 14240 Hz a striking deviation from regular surfaces arises. Such dome-like shaped isosurfaces which stick to one side of an ear canal wall are observed at different frequencies and locations. These irregular isosurfaces are referred to as "one-sided" isosurfaces in the following.

In one-dimensional models of ear canals the sound propagation is usually described as a function of location measured along the ear canal middle axis. A reasonable rule of constructing the middle axis is connecting the centroids of adjacent isosurfaces (Fig. 3).

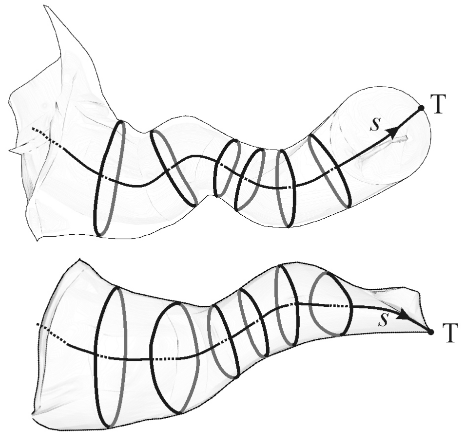

Figure 3: Construction of an ear canal middle axis as connecting line running through the centroids of regular isosurfaces. The middle axis ends in a termination point T in the tympanomeatal corner.

Obviously, in the regions of one-sided isosurfaces the sound propagation is no longer one-dimensional because the structure of the sound field is not oriented along the middle axis. At first sight one would expect that one-sided isosurfaces introduce considerable disturbances of the regular sound field which could not be treated as part of a reasonably defined "fundamental" sound field. However, a closer inspection reveals that the contrary is true.

Fundamental sound fields

The origin of one-sided isosurfaces is easily found. Consider a duct with a single bend. Due to reflections at the wall the sound pressure of a fundamental wave is a little higher on the concave side of the bend than on the opposite convex side. Thus, if a local pressure maximum occurs in the bend, it will be situated on the concave side. The "isosurface" which belongs to this pressure magnitude degenerates to an isolated point. The isosurfaces of slightly smaller pressures cannot reach the other side of the ear canal as well. As a result the isosurfaces must take the shape of domes arching over the location of the maximum. Hence one-sided isosurfaces systematically arise in general curved ear canals. They can occur at arbitrary positions, but are particularly pronounced in bends. The case of maxima arising on a closed circumference, instead at a point on one side of the wall, is an exception that can only occur in ideal straight cylindrical ducts and axisymmetric horns.

To study one-sided isosurfaces in pure form, the sound transmission through a single bend in an otherwise straight duct of constant cross-section has been considered. The upper panel in Fig. 4 shows the isosurfaces for the case of a pressure maximum in the bend. Using an analogous argument, it becomes clear that a local minimum in a bend must evoke one-sided magnitude isosurfaces as well. As expected the minimum appears on the convex side (lower panel of Fig. 4). The conditions at pressure maxima and minima are not uniform. Pressure maxima are always broader than minima, particularly if the incident waves are strongly reflected at the end of the duct like in ear canals.

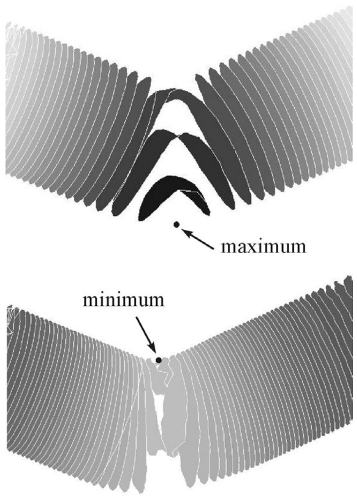

Figure 4: One-sided magnitude isosurfaces for a pressure maximum and minimum occurring in a bend.

In spite of such irregular isosurfaces the sound field in the core region can be interpreted as "fundamental sound field" because, like in the case of ideal plane waves, the sound field transmits acoustic power producing minimum energy density in the duct. This will be shown in the following. The complex power penetrating a surface A is

$$S(\omega) = \frac{1}{2}\int_A p(\mathbf{r}) \cdot \mathbf{v}^*(\mathbf{r}) d\mathbf{A} \qquad (1)$$

If the integral is taken over a regular isosurface, one can conclude that velocity components tangential to the isosurface do not contribute to the power. On the other hand, such components lengthen the velocity vector and therefore increase its RMS value and thus the energy density. Hence velocity vectors in minimum energy sound fields must be normal to the isosurfaces, at least approximately. Near the (rigid) walls this condition is met in any case.

Thus plane waves in straight ducts and regular waves having identical isosurfaces of pressure magnitude and phase transmit acoustic power with minimum energy because the velocity vectors are always perpendicular to the pressure isosurfaces. But how are the conditions in regions of one-sided isosurfaces? It is instructive to calculate the velocities from a given pressure field $p(\mathbf{r})=|p(\mathbf{r})| \exp(j\varphi(\mathbf{r}))$ where $\mathbf{r}$ denotes the position vector. The following expression can be easily derived using Euler's equation

$$\begin{aligned} v(\mathbf{r},t) = &-\frac{1}{\omega\rho}\operatorname{grad}\left\{|p(\mathbf{r})|\right\}\sin\left(\omega t + \varphi(\mathbf{r})\right) - \\ &-\frac{1}{\omega\rho}|p(\mathbf{r})| \cdot \operatorname{grad}\left\{\varphi(\mathbf{r})\right\}\cos\left(\omega t + \varphi(\mathbf{r})\right) \end{aligned} \qquad (2)$$

At the locations of pressure minima and maxima the gradient of the pressure magnitude vanishes. Therefore, due to Eq. (2) the velocity in the extrema is only determined by the phase gradient. In contrast to the magnitude, the phase monotonically decreases from the source to the termination. No local phase extrema exist which could disturb regular phase isosurfaces. Therefore, in order to maintain the orientation of velocity vectors pointing mostly in the direction of propagation, the phase isosurfaces should be regular also in regions of one-sided magnitude isosurfaces. Actually this behavior is fully confirmed by FE calculations. Obviously phase isosurfaces indicate the direction of propagation much better than magnitude isosurfaces since they are not warped near extrema. Surfaces of equal pressure magnitudes and phases near a pressure maximum are shown in Fig 5.

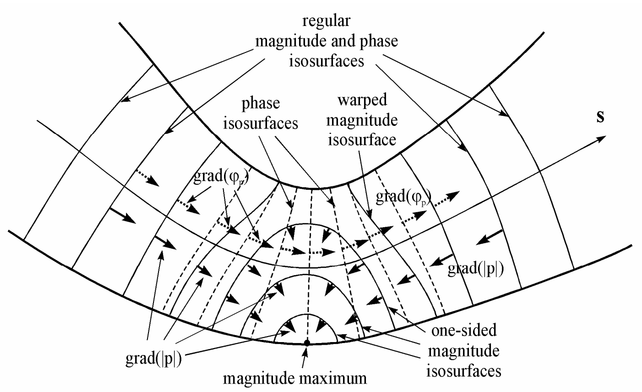

Figure 5: Schematic representation of the isosurfaces of pressure magnitude and phase and the corresponding gradients in the vicinity of a pressure maximum. All the surfaces are perpendicular to the walls.

At sufficient distance from the maximum the isosurfaces of magnitude and phase coincide. However, near the pressure maximum the gradients differ in direction. Therefore, the velocity changes its direction during a cycle of the harmonic sound wave. Since the gradient of the magnitude is small near a pressure maximum, the direction of the velocity is mostly given by the phase gradient which points in the direction of propagation. Just during a short time interval around zero-crossing of $\cos(\omega t+\varphi(\mathbf{r}))$, the small magnitude gradient near the maximum determines the direction of velocity according to Eq. (2).

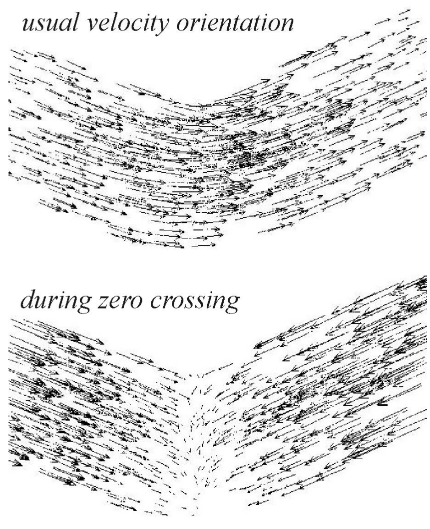

Figure 6: Velocities in a pressure extremum.

The effect of one-sided isosurfaces on sound transmission is low. This is seen when the transmission via a curved section in an otherwise straight duct is examined using finite elements. The tube shown in Fig. 7 consists of a toroidal part and two straight adaptors at both ends. The diameter of the curved section and the adaptors is 8 mm. At the right end a non-reflecting source generates a sound field, whereas at the left end a load impedance is applied. The adaptors are chosen long enough to ensure almost perfect plane waves at the ends of the adaptors in the frequency range of interest. The pressure values p_{in} generated near the source (5 mm from the entrance) and p_{out} near the acoustic load (5 mm from the outlet) are taken from the numerical solution to compute the transfer function p_{out}/p_{in}. This transfer function is compared to the transfer function of a homogeneous straight tube of the same length.

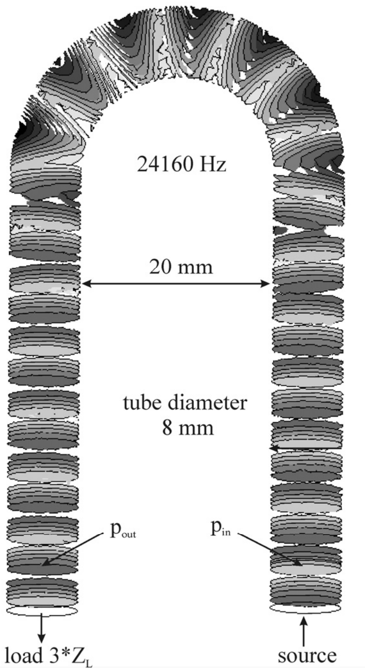

Figure 7: Sound field in a toroidal duct at a frequency slightly below the cut-off frequency of the first higher order mode.

The differences between the transfer functions are surprisingly small in spite of strong one-sided isosurfaces appearing in the bended part of the duct and a little beyond. At high frequencies the sound field structure approaches that of the first higher order mode of the toroid. Fig. 7 represents the sound field at 24160 Hz which is just below the cut-off frequency of the mode. Up to this frequency the deviations of the transfer functions are not greater than 1 dB in magnitude and 12° in phase. Thus, in respect to sound transmission, the one-sided isosurfaces are rather marginal disturbances if they are evanescent.

Conclusions

The core region of an ear canal denotes the central part where the sound field structure is neither influenced by the external sound source nor by the sound radiated from the vibrating eardrum. In the core region the sound field is widely regular. It mainly consists of slightly curved isosurfaces which completely cover the aperture of the canal. However, in addition one-sided isosurfaces arise which stick to isolated points on the ear canal walls. Such disturbances of the regular sound field structure systematically arise in curved ear canals. They occur at all maxima and minima of the pressure.

The principle of minimum energy which governs fundamental waves applies in curved ducts as well, however, in a weakened form. The physical reason is the slenderness of the ear canal which enforces conversion of locally appearing higher order modes to fundamental modes within short distances. Therefore the sound field in the core region is referred to as fundamental sound field including one-sided isosurfaces. The disturbance of the regular sound field is restricted to a section of minimal extension. Moreover time intervals of velocities being not perpendicular to pressure isosurfaces are kept as short a possible.

Surprisingly, one-sided isosurfaces hardly influence the sound transmission through the ear canal. However, further investigations not presented in this paper reveal that the sound field in regions of one-sided isosurfaces is particularly sensitive to disturbances caused by measuring equipment like probe tubes or impedance measuring heads.

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