Simulation of transient sound radiation using the Time Domain Boundary Element Method

M. Stütz[1], M. Ochmann[2]

[1] Beuth Hochschule für Technik Berlin, 13353 Berlin, Germany, Email: stuetz@tfh-berlin.de

[2] Beuth Hochschule für Technik Berlin, 13353 Berlin, Germany, Email: ochmann@tfh-berlin.de

Introduction

The boundary element method (BEM) is a widely used numerical tool. Calculations may be performed in the frequency domain (FD) or in the time domain (TD). The fundamentals of the TD-BEM were developed in 1960 from Friedman and Shaw [1] and later in 1968 from Cruse and Rizzo [2]. The FD-BEM is limited to the simulation of stationary processes. Instationary processes like moving sources, engine acceleration, impulses etc. must be simulated in the time domain. Mansur [3] developed one of the first boundary element formulations in the time domain for the scalar wave equation and for elastodynamics with zero initial conditions. The extension of this formulation to non-zero initial conditions was presented by Antes [5]. Later Jäger [6] and Antes and Baaran [7] extended the time domain formulations to analyze 3D noise radiation caused by moving sources. Since then this method is subject to continuous enhancement, whereby however the frequency range algorithms exhibit a clear lead in development.

The main reason that the TD-BEM-Method is not being commercially used till now is the instable behavior that may appear. Ergin et al [8], Chappell et al [9] and Stütz and Ochmann [10] showed numerical evidence that the instable behavior is caused by the eigenfrequencies of the structure. Ergin and Chappell used the Burton-Miller-Method [11] to prevent instabilities. In this paper a different approach, the use of the so-called CHIEF Method, will be presented.

Numerical model

A detailed derivation of the boundary integral equation used here can be found e.g. in Meise [4].

$$4\pi d(\vec{x}_0)\, p(\vec{x}_0, t_0) = -\int\limits_{\Gamma} \left[\frac{q}{r} + \frac{\partial r}{\partial n} \left(\frac{p}{r^2} + \frac{1}{cr}\frac{\partial p}{\partial t} \right) \right]_{ret} d\Gamma \quad (1)$$

p denotes the sound pressure, r the distance between the observation point and source point and c the speed of sound. Γ is the boundary surface and q is the flux, following the relationship derived from Eulers equation

$$q = -\varrho_0 \frac{\partial v_n}{\partial t} \quad (2)$$

The index ret in Eq 1 states that all variables are taken with respect to the retarded time $t_{ret} = t - \frac{r}{c}$. Discretizing time itself in equidistant time steps $t_i = i\Delta t$ with $(i = 1, 2, ...)$ we get $t_{ri} = i\Delta t - \frac{r}{c}$ for the retarded time. The boundary surface Γ is divided into N planar elements with a uniform spatial pressure and flux distribution. For q a constant approach and for p a linear approach in time are chosen. A more detailed description can be found in [10] and [13]. With these approaches one can derive the following discretized integral equation

$$-2\pi p_i(\vec{x}_0) = \sum_{n=1}^{N}\sum_{m=1}^{i}\Bigg[\int_{\Gamma_n} \frac{1}{r} q_m^n \Psi(t_{ri}) d\Gamma_n + \int_{\Gamma_n} \frac{\partial r}{\partial n}\frac{1}{r^2}\left[(i-m+1)p_m^n - (i-m)p_{m-1}^n\right]\Psi(t_{ri})d\Gamma\Bigg] \quad (3)$$

$$\Psi(t_{ri}) = \begin{cases} 1 & ; t_{ri} \in [t_{m-1}, t_m) \\ 0 & ; else \end{cases} \quad (4)$$

Using the Collocation method the following corresponding linear system of equations can be derived

$$-2\pi \hat{p}_i = \sum_{\mu=1}^{\mu_{max}} G_\mu \hat{q}_{i-\mu+1} + \sum_{\mu=1}^{\mu_{max}} H_\mu \left[\mu \hat{p}_{i-\mu+1} - (\mu-1)\hat{p}_{i-\mu}\right]. \quad (5)$$

G_μ and H_μ are square matrices describing the system at a relative time step $\mu = i - m + 1$. The matrix entries are calculated in the following way

$$g_{ab}^{\mu} = \int\limits_{\Gamma_b} \frac{1}{r_{ab}} \Psi(t_{r\mu}) d\Gamma \quad (6)$$

$$h_{ab}^{\mu} = \int\limits_{\Gamma_b} \frac{\partial r_{ab}}{\partial n}\frac{1}{r_{ab}^2} \Psi(t_{r\mu}) d\Gamma\,. \quad (7)$$

Compared to the FD-BEM the resulting matrices are very sparse, because for each matrix with index μ the integration must only be performed within the integration limits of an inner radius $(\mu-1)c\Delta t$ and an outer radius $\mu c\Delta t$. A difficulty that arises from that condition is that one must not integrate over the whole element but over the part that lays within the integration limits. The sparsity of the matrices is explained due to the fact that most elements are not within the integration limits and therefore have a zero entry. Eq. 5 can be written as

$$
-2\pi \begin{bmatrix} \hat{p}_1 \\ \hat{p}_2 \\ \vdots \\ \vdots \\ \hat{p}_i \end{bmatrix} = \begin{bmatrix} G_1 & 0 & \cdots & \cdots & 0 \\ G_2 & G_1 & 0 & \cdots & 0 \\ G_3 & G_2 & G_1 & & \vdots \\ \vdots & & & \ddots & \\ 0 & \cdots & G_{\mu_{max}} & \cdots & G_1 \end{bmatrix} \begin{bmatrix} \hat{q}_1 \\ \hat{q}_2 \\ \vdots \\ \vdots \\ \hat{q}_i \end{bmatrix} +
$$

$$
\begin{bmatrix} H_1 & 0 & \cdots & \cdots & 0 \\ 2H_2 & H_1 & 0 & \cdots & 0 \\ 3H_3 - H_2 & 2H_2 & H_1 & & \vdots \\ \vdots & & & \ddots & \\ 0 & \cdots & -(\mu_{max}-1)H_{\mu_{max}} & \cdots & H_1 \end{bmatrix} \begin{bmatrix} \hat{p}_1 \\ \hat{p}_2 \\ \vdots \\ \vdots \\ \hat{p}_i \end{bmatrix} \tag{8}
$$

to solve the time response at once. This results in 2 Block-Toeplitz-Matrices that are not easy to handle because of the relatively big size of (number of elements x time steps)^2. Thats why the time response is solved in an iterative way, starting with solving the first row of Eq. 8 ($-2\pi\hat{p}_1 = G_1\hat{q}_1 + H_1\hat{p}_1$) and proceeding till the last row. This approach is numerically much easier to handle.

Accuracy and stability

A common problem of the TD-BEM reported in the literature is the occurrence of spurious instabilities. In case of instability the results start to oscillate at high frequency with exponentially rising amplitude. To look at the stability of the method the iterativ solving process can be written as

$$
p_i = Tp_{i-1} + \phi_i \tag{9}
$$

with T being the system operator. If all eigenvalues of T lay within the unit circle the system can be considered as stable, because p can not grow with proceeding time if $\phi_i = 0$.

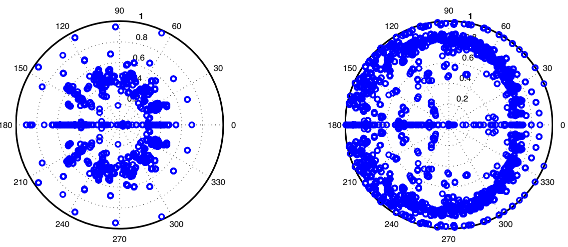
We are now looking at the simple example of a sphere with $r = 1m$ consisting of 384 flat rectangular elements. Figure 1 shows that for this given example the eigenvalues lay all within the unit circle. But with decreasing time step the absolut values of the eigenvalues are increasing, making the system less stable.

Using the sphere to simulate a monopol the results are stable as shown in Figure 2. But apparently the sound radiated by the sphere is influenced by the eigenfrequencies of the structure. This connection however is still mathematically unproven. In Figures 2, 3 and 5 the eigenfrequencies are marked by the vertical dotted lines. The influence of the eigenfrequencies depends strongly on β, which describes the ratio of time step size Δt and element size h.

$$
\beta = c\Delta t/h \tag{10}
$$

Small changes of β can change the influence of the eigenfrequencies substantially. Nevertheless in none of the examined cases an unstable behavior, like an exponential rise of the sound pressure, could be observed.

With decreasing β the evaluation of the matrix entries is getting more complicated, because the intersection of integration area and element is becoming much smaller than the element itself. The integrals are evaluated numerically using a Gauss Quadratur. By increasing the numbers of Gauss points the accuracy can be increased which will decrease the influence of the eigenfrequencies to a certain degree, as can be seen in Figure 3. The influence does not vanish but the aberration from the analytic solution does not exceed 1 dB.



(a) Δt=1.1 ms β=1.9 (b) Δt=0.38 ms β=0.66

Figure 1: Example of eigenvalues within unit circle of system operator T for different time step sizes (a) and (b)

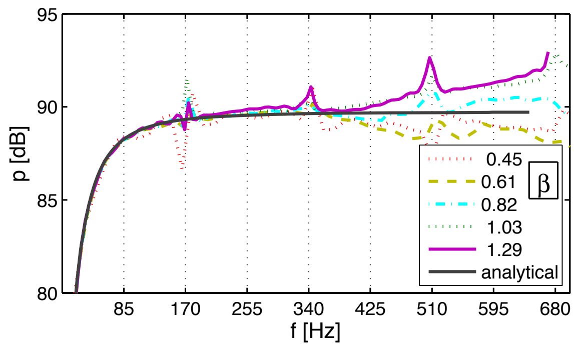


Figure 2: Influence of eigenfrequencies for different β on sound radiation of test sphere)

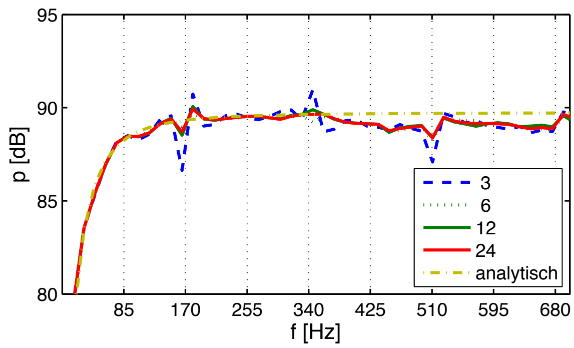


Figure 3: Influence of eigenfrequencies depends on accuracy of integration, results for different numbers of Gauss Points are shown

CHIEF Method

The problem of the natural frequencies is well known in frequency domain BEM calculation, also called the nonuniqueness problem. Various methods have been derived to find a modified equation with a valid solution for all frequencies. Well known approaches are the combined field integral equation by Burton and Miller [11] and the combined Helmholtz integral equation formulation (CHIEF) by Schenk [12]. For time domain calculations first Ergin [8] in 1999 and later Chappell et al [9] in 2006 used a Burton-Miller approach to avoid the above described problem.

The principle of the CHIEF method is very simple. As shown in section "Numerical model" a linear system of equations can be derived using the collocation method. The collocation points are located on the boundary surface. Placing some additional points inside of the structure we can derive a second linear system of equations. But in this case we set the pressure in this points to zero to force the inner sound field to zero.

$$0 = \sum_{\mu=s}^{c_{max}} G^c_\mu \hat{q}_{i-\mu+1} + \sum_{\mu=s}^{c_{max}} H^c_\mu \left[\mu \hat{p}_{i-\mu+1} - (\mu-1)\hat{p}_{i-\mu}\right] \tag{11}$$

If we combine both linear systems Eq. 5 and Eq. 11 we get an overdetermined system. As an example here the resulting matrix containing all H_μ and H^c_μ matrices is shown

$$\begin{bmatrix}
H_1 & 0 & \cdots & \cdots & 0 \\
2H_2 & H_1 & 0 & \cdots & 0 \\
3H_3 - H_2 & 2H_2 & H_1 & & \vdots \\
\vdots & & & \ddots & \\
0 & \cdots & -(\mu_{max}-1)H_{\mu_{max}} & \cdots & H_1 \\
\hdashline
sH^c_s & 0 & \cdots & \cdots & 0 \\
(s+1)H^c_{s+1} - (s-1)H^c_s & sH^c_s & 0 & \cdots & 0 \\
\vdots & & & \ddots & \\
0 & \cdots & -(s_e-1)H^c_{s_e} & \cdots & sH^c_s
\end{bmatrix} \tag{12}$$

The matrix containing the G_μ and G^c_μ matrices is set up in a similar way. To the authors knowledge the use of the CHIEF method in time domain BEM calculations was never shown before. This may have different reasons. One reason is that the CHIEF method does not work using the iterativ solution process. Applying the method to the non-iterativ one-step solution makes it necessary to solve the huge rectangular matrix (Eq. 12). The solution process is not an easy task because of the immense size of the matrices involved. To deal with this problem, the special symmetry of the matrices must be used. Using an iterativ solver that can handle rectangular matrices, like the LSQR method, only one line of the matrix has to be build at a time. The results in Figure 5 show that the CHIEF method successfully regularizes the solution at the inner eigenfrequencies of the structure.

Test cases - circular piston and railway wheel

A good test case to see if this method can simulate the sound radiation of railway wheels is a circular piston.

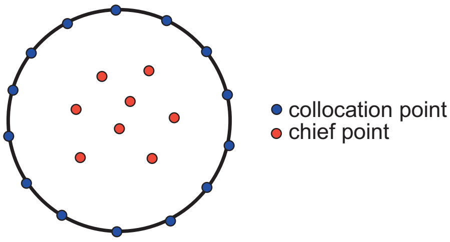


Figure 4: CHIEF points are located inside of the structure and collocation points are located on the surface

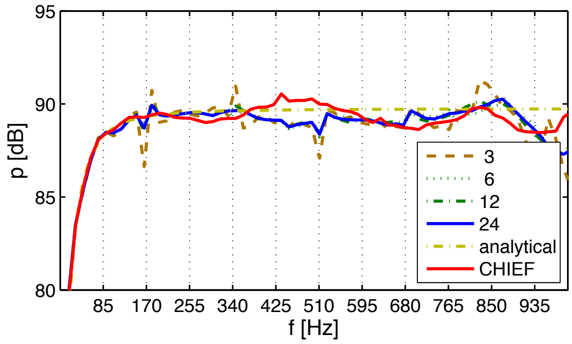


Figure 5: The use of CHIEF points prevents the influence of eigenfrequencies.

The analytical far field approximation for a circular piston in an infinite wall can be found in the literature:

$$p(r,\alpha) = i\omega\varrho v_n R^2 \frac{e^{-ikr}}{r} \cdot \frac{J_1(kRsin\alpha)}{kRsin\alpha} \tag{13}$$

Figure 6 shows the test results for 2 immission points. Both points are located in 7.5m distance, P1 with hight h=1.2m and P2 with h=3m as can be seen in Figure 7. The results show a very good agreement with the analytical solution. Also the directivity which can be observed in point P2 is reproduced quite well. The difference in the low frequency region is expected. Short circuit effects of the numerical model without wall are not present in the analytical model with an infinite wall.

A second test case examined the sound radiation of a railway wheel. The impulse response function in the time domain was calculated by a FEM calculation. Based on these data the sound radiation of the wheel could be calculated. A comparison to a FD-BEM calculation performed with Virtual Lab, shows a good agreement (Figure 8). There is no measurement data available, so no conclusion about the simulation accuracy can be made.

Conclusion

The TD-BEM is a promising method for the computation of the transient sound radiation. Numerical test cases show the reliability and stability of this method. The accurate evaluation of the boundary integrals plays a fundamental role, because errors in the integration process seem to stimulate the influence of the eigenfrequencies of the structure. In order to ensure accurate simulation results, it was shown that the CHIEF method can be used to regularize the eigenfrequencies. More research regarding the efficiency and the limits of the CHIEF

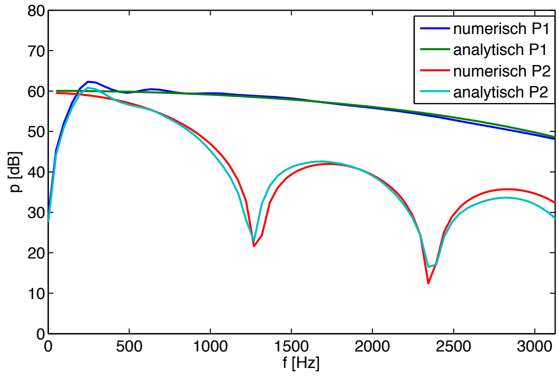


Figure 6: Sound radiated by a circular piston (in free field), compared to far field approximation (in infinite wall).

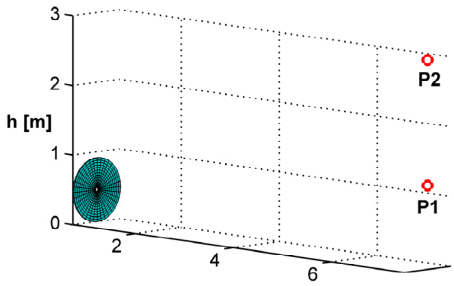


Figure 7: Setup of circular piston test case with 2 immission points located in 7.5m distance, P1 with hight h=1.2m and P2 with h=3m.

method in TD-BEM calculations has to be done. The results will be presented in one of the next publications.

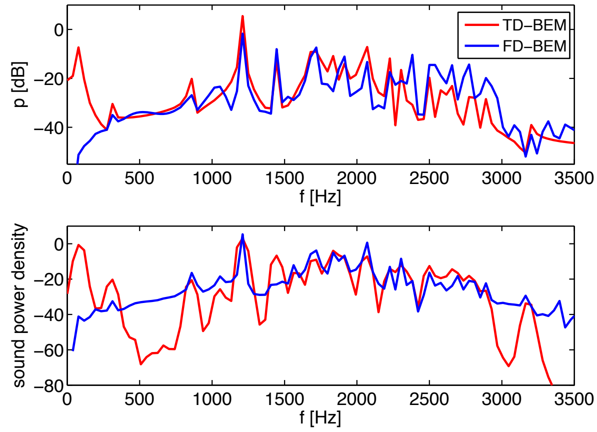


Figure 8: Sound radiation of an railway wheel computed with TD-BEM and V-Lab (FD-BEM).

References

[1] M. Friedmann, R. Shaw: Diffraction of pulses by cylindical Obstacles of Arbitrary cross Section, J. Appl. Mech.,Vol. 29 (1962), S.40-46

[2] T.A. Cruse, F.J. Rizzo: A direct formulation and numerical solution of the general transient elastodynamic problem I, J. Math. Analysis and Applic., Vol.22 (1968), S.244-259

[3] Mansur, W. J.: A Time-Stepping technique to solve wave propagation problems using the boundary Element Method, PhD thesis, University of Southampton, 1983

[4] Meise T.: BEM calculation of scalar wave propagation in 3-D frequency and time domain (in German). PhD Thesis, Technical Reports Nr.90-6, Fakultät für Bauingenieurwesen, Ruhr-Universität, Bochum, Germany; 1990

[5] H. Antes, A Boundary Element Procedure for Transient Wave Propagations in Two-dimensional Isotropic Elastic Media, Finite Elements in Analysis and Design 1 (1985) 313-322

[6] Jäger M. Entwicklung eines effizienten Randelementverfahrens für bewegte Schallquellen. PhD Thesis. Fachbereich für Bauingenieur- und Vermessungswesen der TU Carolo-Wilhelmina zu Braunschweig, Germany; 1993

[7] Antes H, Baaran J. Noise radiation from moving surfaces. Engineering Analysis with Boundary Elements; Volume 25, Issue 9, October 2001, Pages 725-740 Combustion

[8] A.A.Ergin, B. Shanker, E. Michielssen: Analysis of transient wave scattering from rigid bodies using a Burton-Miller approach, JASA, 106, 1999, pp. 2396-2404

[9] D.J.Chappell, PJ. Harris, D. Henwood, R. Chakrabarti: A stable boundary element method for modeling transient acoustic radiation,JASA,120, 2006, pp. 74-80

[10] M.Stütz, M.Ochmann, Stability behaviour and results of a transient boundary element method for exterior radiation problems, Acoustics'08-Paris, 2008

[11] A. J. Burton, G. F. Miller: The Application of Integral Equation Methods to the Numerical Solution of Some Exterior Boundary-Value Problems, Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences, Vol. 323, No. 1553, A Discussion on Numerical Analysis of Partial Differential Equations (Jun. 8, 1971), pp. 201-210

[12] Harry A. Schenck: Improved Integral Formulation for Acoustic Radiation Problems, JASA , July 1968, Vol 44, Issue 1, pp. 41-58

[13] M.Stütz, M.Ochmann : Calculation of the acoustic radiation of an open turbulent flame with a transient boundary element method, International Congress on Acoustics in Madrid (ICA), 2007