Analysis of Enclosed Sound Fields using Multichannel Impulse Response Measurement

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Introduction

Audio reproduction inside small enclosures is strongly influenced by the acoustics of the surroundings. Thus, for evaluating the performance of spatial audio systems, the knowledge about the sound propagation process inside the listening area gives additional information to assess the influence of room acoustics on the reproduction quality. The room impulse response taken on a certain point in a room contains information about the temporal structure of the direct sound, the reflections and the reverberant field at this point in space. Wave field analysis based on multichannel impulse response measurement delivers insight to the temporal and spatial properties of a sound field. Closely spaced impulse responses are usually recorded along a line, a circle or a grid resulting in a coherent acoustic data set. This enables the possibility for additionally analyzing the spatial characteristics of the sound field in detail. The result of an array measurement can be decomposed into its spatial components. For example a so called plane wave decomposition (PWD) can be achieved by applying a spatial Fourier transform [1].

In this paper the application of a multi-dimensional Fourier transform is described. First a theoretical overview on spatial transformation and the possibilities for visualization of the results is given in brief. After the theoretical chapter a mirror image source model (MISM) is used to demonstrate the analysis concept within a reflective enclosure. Recordings from a virtual microphone array are used for these examples. The virtual array responses were calculated at a two-dimensional grid along a thought listening plane inside a small enclosure. It was expected to identify the strongest reflections and to obtain directional information from the data set.

Multichannel IR measurement and multi-dimensional Fourier Transform

Acoustic measurement at a single position will provide information about the wave field properties at that location. To get access to the spatial characteristic of the wave field it is necessary to carry out measurements with a higher number of closely spaced sensors. In case of a digital recording the wave field now is sampled in space and time. The array topologies used for such a multichannel measurement may mainly depend on the following factors:

- desired grade of dimensionality (2D, 3D, …)
- desired/achievable amount of microphones
- geometrical aspects of the subject under investigation

Depending on the geometry of the sensor setup, different methods for wave field analysis are favorable or become applicable. A helpful overview about the potential features of linear, cross and circular array topologies can be found in [2].

The microphone array used in this paper forms a horizontal two-dimensional regular grid along the Cartesian x- and y-axis and covers the region of interest inside an enclosure (see fig. 6). In contrast to well known array geometries like a circular or a spherical array the acoustic data now is collected covering a plane and not only at the border of a plane. This means that the measurement results in a three-dimensional data set, containing two spatial dimensions and one temporal dimension (fig. 1). The data set represents a sampled version of the original wave propagation process in the measurement plane. A simple space-time domain investigation of the propagation process can be established by just playing back the impulse responses sample by sample. Even if this representation delivers a good insight into the wave field, a more advanced approach for analysis of the sound field can be established. Based on the grid data set it is e.g. possible to apply a multi-dimensional Fourier Transform. For example a 3D Fourier Transform will decompose the measurement data into plane waves along the two spatial dimensions of the horizontal plane.

Figure 1: View of the resulting three-dimensional data set of a two-dimensional grid array measurement: A direct interpretation of data slices taken in the x-t or y-t plane provides information about temporal details while using the x-y view can be used to investigate the spatial sound field propagation. The application of a 3D Fourier transform makes use of all three domains together.
To describe the multi-dimensional Fourier transform a linear grid array is assumed as depicted in figure 2. For an incoming plane wave which is incident to the array under an angle $\theta$ there is a distinct relation between the wavenumber $k$ and its component $k_x$ in direction of the array (eq. 2), which is in this case the x-axis. The wavenumber $k$ is the length of the wavenumber vector $\mathbf{k}$, which is always pointing perpendicular to the wave fronts.

For an array along the y-axis, the formulation of analogue relations between $k$ and $k_y$ is straightforward.

$$\lambda_x = \frac{\lambda}{\sin \theta} \quad [\text{m}] \quad (1)$$

$$k_x = \frac{2\pi}{\lambda_x} = k \sin \theta \quad [\text{m}^{-1}] \quad (2)$$

Now a plane wave decomposition based on a 3D Fourier transform is applied to the three-dimensional data set for further investigation of the spatial properties of the sound field (eq. 3).

$$P(k_x, k_y, \omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} s(x, t) e^{i(k_x x + k_y y + \omega t)} \, dx \, dy \, dt \quad (3)$$

After this wave field analysis step the resulting plane wave components $P(k_x, k_y, \omega)$ can be depicted as a three-dimensional object in the so called wavenumber-frequency domain (fig. 5). Regarding the dispersion relation as

$$k_x^2 + k_y^2 = k^2 \quad ; \quad \text{with} \quad k = \frac{\omega}{c} \quad (4)$$

where $\omega$ denotes the angular frequency and $c$ the speed of sound, it becomes clear that the contributions of an incoming wave field appear as points on a cone surface. Only for points on this surface the pressure $P(k_x, k_y, \omega)$ is non-zero. Some examples are given in the following.

**Example 1: Plane Wave**

A plane wave is considered to propagate towards a two-dimensional array under a certain incidence angle. If the signal of the plane wave only contains one single frequency component, then this entity is transformed into a single point in wavenumber-frequency domain. Assuming a broadband signal the transformation result forms a line on a cone surface (fig. 5, left side) because the wave field of a plane wave contains components from one single spatial direction only. The appropriate projection into the $k_x$-$k_y$-plane can be seen in figure 3 (right side). From this point of view also the directional components $k_x$ and $k_y$ of the wavenumber vector $\mathbf{k}$ can be indicated. According to equation 4, all components with the same angular frequency $\omega$ lie on a circle in this plane.

**Example 2: Point Source**

In this case the wave field of a point source, emitting a broadband signal is measured by the array. Now the incident wave field contains of multiple spatial and temporal frequency components. This is explainable due to the fact that the wave field of a point source can be interpreted as summation of plane waves arriving under various incidence angles. In the wavenumber-frequency domain this results in a fully shaded sector on the cone surface. Again the appropriate projection into the $k_x$-$k_y$-plane shows the inherent spatial wave field components from a top view (fig. 4).

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**Figure 2:** The wave fronts of a plane wave are spatially sampled by the microphones of a line array. The wave length $\lambda_x$, which is “seen” by the array, depends on the angle of incidence $\theta$. If $\lambda_x$ becomes smaller than twice the sensor distance $\Delta x$, then spatial aliasing occurs.

**Figure 3:** Broadband plane wave travelling towards the array (left side). On the right side the result of the PWD is shown from a top view.

**Figure 4:** Broadband point source: The wave field of a point source can be interpreted as summation of plane waves arriving under various incidence angles.
Application of the Analysis Methods to a MISM

The analysis method described in the previous section is now applied to a simulated wave field. The underlying mirror image source model is depicted in figure 6. The dimensions are chosen in accordance to an automotive use case. In a first step the model is used to produce a two-dimensional sound field, so no reflections from above or the ground attend to the measurement result. A two-dimensional virtual microphone array is placed inside the enclosure. The position of the virtual microphones is following a real measurement setup which was placed into a car cabin [3].

The impulse response for each microphone is calculated by using this model and as a result a three-dimensional data set, as the one shown in figure 2, is gained. In a future step the analysis method will be used to investigate the sound field of this real measurement.

The spatial Fourier transform is now calculated for two situations. As shown in figure 7, the free field case (left) is compared to the one with fully reflecting side walls (right). In the wavenumber-frequency domain the contribution of the direct sound equals the expected result for a point source. When including the reflections two additional plane wave contributions occur. Due to the length of the travelling path the side wall reflections tend to be recognized as plane waves.

Conclusions

The application of a 3D Fourier transform gives insight to the spatial structure of a wave field. Based on a two-dimensional measurement it is possible to spatially decompose the sound field into both the x- and y-components. The results for a PWD of a mirror image source model follow the theoretical expectations.

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References

