Retarded-time source field analysis of vortex-pairing noise

F. Margnat[1]

[1] Arts et Métiers Paris Tech, Paris, France, Email: florent.margnat@paris.ensam.fr

Introduction

To date, the physical phenomenon that converts kinetic energy into acoustic waves escaping from the flow is not fully understood. Thanks to the increasing computational power, aeroacoustic prediction tools have become more and more fast and accurate. However, it is still challenging to link an acoustic emission pattern to the source flow, in terms of causal events. Lighthill's acoustic analogy [1] provides a way to extract the propagative motion from a flow through the expression of source term in an inhomogeneous wave equation. Unfortunately, when the flow is not compact, it is difficult to identify, from source field visualisations, flow locations where the acoustic energy could be produced.

In the present contribution, we analyse the source field as viewed by a given observer. Such point of view shows what is the source of an acoustic pressure probed at an observer point, through the formalism of the integral solution of Lighthill's equation. Thus, the source field is not represented at a fixed physical time for all source points, but at a time depending on how far is the source point from the observer point, in order that all contributions should reach the observer at the same time.

This methodology is applied to a 2D spatially evolving mixing-layer at Re=400 and 0.375 convective Mach number, using a direct numerical simulation database. Areas in the source domain are analysed through their net contribution to the aeroacoustic integral.

The Lighthill formalism

Consider the Lighthill equation written as follows with entropic and viscous terms neglected:

$$\frac{\partial^2 \rho}{\partial t^2} - c_0^2 \Delta \rho \approx \nabla \cdot \nabla \cdot (\rho \mathbf{u}\mathbf{u}) = T \qquad (1)$$

The integral solution in free-field gives the acoustic pressure at the observer position $\vec{X}$ and time t as:

$$p_a(\vec{X}, t) = \frac{1}{4\pi} \int_{\mathcal{V}} T\left(\vec{x}, t - \frac{|\vec{X} - \vec{x}|}{c_0}\right) \frac{d\vec{x}}{|\vec{X} - \vec{x}|} \qquad (2)$$

where $\vec{x}$ is the position in the source volume $\mathcal{V}$, and c_0 is the sound velocity.

For a compact source volume and a far-field observer, the expression of the aeroacoustic source term provide the true analogic source of sound. However, it is known that other effects than sound production are contained in T, such as flow effects on sound propagation. Moreover, in the near field and for non-compact source volumes, the identification of the source area or quantity which effectively do cause the acoustic signal is not so clear. For such a task, it is better to examine the source term T through the integral solution procedure, which acts like a filter. Thus, the quantity considered in the present study is the field:

$$S_{\vec{X},t}(\vec{x}) = \frac{T\left(\vec{x}, t - \frac{|\vec{X} - \vec{x}|}{c_0}\right)}{4\pi |\vec{X} - \vec{x}|} \qquad (3)$$

for a given observer $\vec{X}$ and some discrete times in its acoustic signal. It is exactly the integrant, and differs from the original T in two ways: the retarded time and the distance attenuation. An evaluation at retarded-time is usually denoted by bracketed expressions.

Application to the mixing-layer

Source flow

The present flow configuration is a two-dimensional spatially evolving mixing-layer, sketched in Figure 1, at Reynolds number $Re = \delta_\omega \Delta U / \nu = 400$, where δ_ω is the vorticty thickness at inflow and $\Delta U = (U_h - U_l)/c_0$, and Mach numbers $M_l = 0.25$ and $M_h = 0.5$. The subscripts l and h refer to low and high speed flow, respectively. The mixing layer is forced at its most unstable frequencies in order to control the roll-up and vortex pairing process. A sponge zone is added to dissipate aerodynamic fluctuations before they reach the outflow boundary and to avoid any spurious reflexion (see Moser *et al.* [2] for more details). The size of the computational domain is $Lx \times Ly = 800 \times 800$, and the grid resolution is $Nx = 2071 \times Ny = 785$.

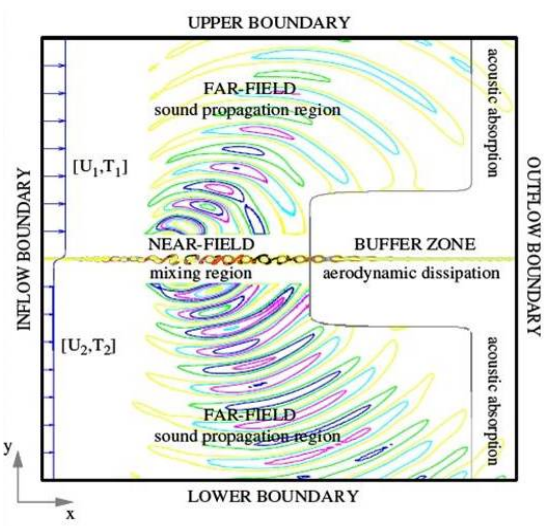

Figure 1: Flow configuration.

A snapshot of pressure fluctuation in the near-field, with vorticity contours, is plot in Figure 2. The vortex pairing process can be seen, as well as the first wavefronts escaping from the mixing region.

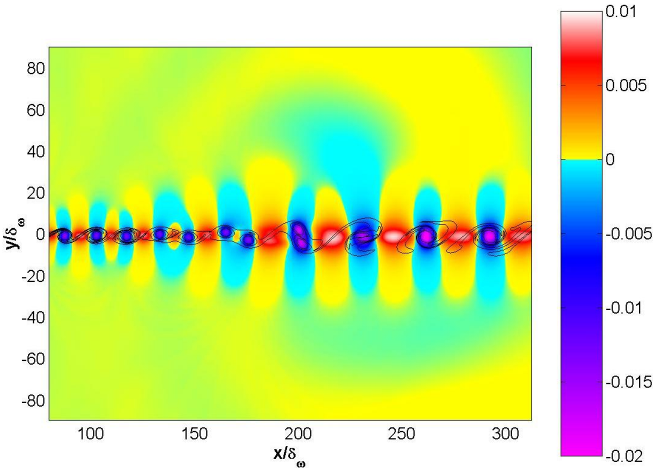

Figure 2: Mixing-layer flow visualisation. Colorscale: pressure fluctuation; Lines: vorticity contours.

Acoustic solution

The obtained database is the input for the computation of the acoustic pressure through the solution (2). The source domain $\mathcal{V}$ is the whole computational domain $Ly_1 \times Ly_2$ in order to take into account sound-flow interactions which are included in the Lighthill source term, as shown by Bogey *et al* [3]. Moreover, only the fluctuating part of the source quantity is used, the mean field being removed in order to compute centered acoustic signals. Finally, weighting functions applied at the boundaries of the source domain to deal with truncation errors. The obtained acoustic field is plotted in Figure 3. The emission is mainly directed at 35 degrees from the flow axis in the high-speed flow, and at 55 degrees in the low-speed flow.

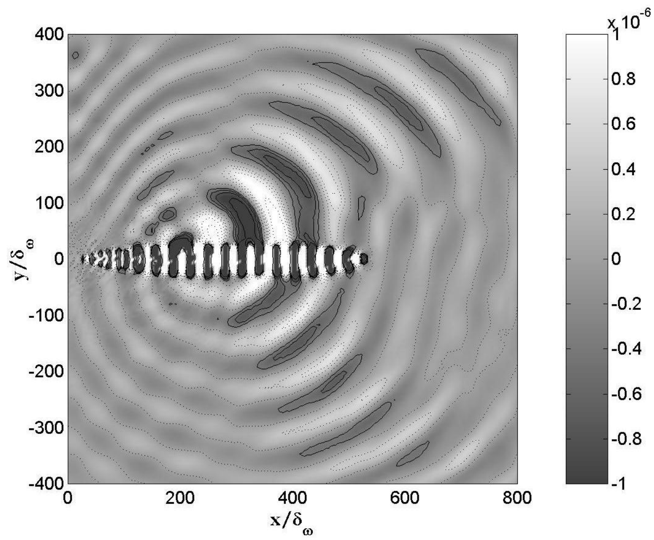

Figure 3: Acoustic pressure field computed with Lighthill's integral solution.

For the observer point located at $(X/\delta_\omega = 600, Y/\delta_\omega = 300)$, the acoustic signal is plotted in Figure 4. It exhibits a periodic shape at the pairing frequency. Observer times corresponding to maximum, minimum and zero value of this signal will be considered in the following for the source field analysis as defined by (3).

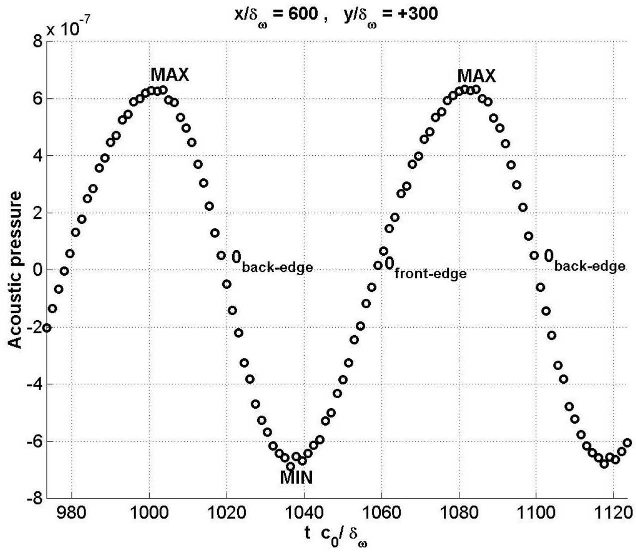

Figure 4: Acoustic signal at the position $(X/\delta_\omega = 600, Y/\delta_\omega = 300)$.

Source field analysis

Firstly, the retarded source field may be compare to the fixed time source field. This is done in Figure 5 along the line $y = 0$ for the time corresponding to the maximum in the acoustic signal. For this comparison, the distance attenuation is not taken into account. Thus the emphasis is put on the modification introduced by time delay difference between source points. This quantity depends on the Mach number and radiation direction, and is also plotted in Figure 5, with the source point $(x/\delta_\omega = 200, y = 0)$ taken for the time origin. Naturally, source points upstream from this position must emit before, while source points downstream from this position must emit after. Thus, because of the global convection movement in the flow, the visible wavelength of the vortex street is increased on the retarded time field.

Secondly, source fields at retarded time corresponding to the four main points in the acoustic signal are compared in Figure 6 in the mixing region close to the pairing phenomenon location. Indeed, acoustic waves seems to originate there. It is very interesting to note that no obvious trend can explain why one field will lead to a maximum of the acoustic signal and the other will lead to a minimum. In other words, noisy events can hardly be extracted from this sequence of vortex-pairing.

Moreover, from both Figures 5 and 6, it appears that a high source amplitude is associated with each vortex or vortex pair, even if no noise seems to come from all of them.

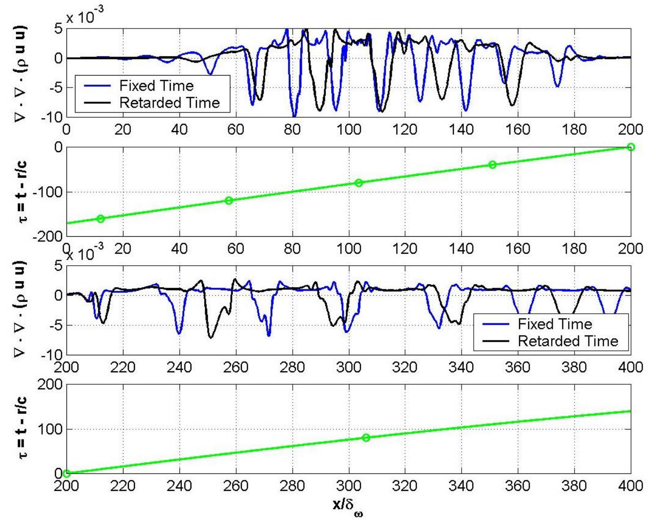

Figure 5: Evolution of Lighthill's source term and retarded time along x-axis. The observer position is $(X/\delta_\omega = 600, Y/\delta_\omega = 300)$ the observer time t corresponds to the maximum in the acoustic signal, and $r = |\vec{X} - \vec{x}|$.

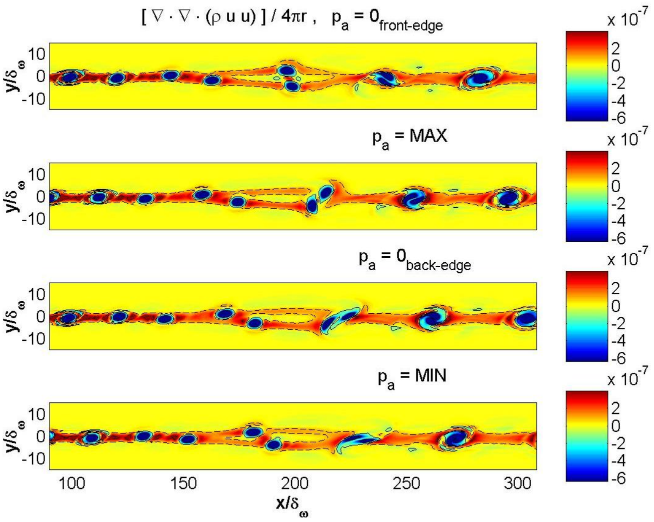

Figure 6: Source field at retarded times for main acoustic signal events. The observer position is $(X/\delta_\omega = 600, Y/\delta_\omega = 300)$.

Finally, the only missing step between the S-field and the computed acoustic signal is the integration. Now, considering the fields in Figure 6, in a vortex peak of radius $2\delta_\omega$ one have $S \approx 4.10^{-7}$. This will result in a contribution to the acoustic signal of 50.10^{-7}, which is 8 times the signal maximum. However, the mixing-layer exhibits a pattern of positive and negative zones that might balance one another. This is illustrated in Figure 7. The plotted field correponds to signal maximum $6,4.10^{-7}$. Integration of S over stripes starting inflow gives a contribution of $7,1.10^{-9}$ and $-2,7.10^{-9}$ for zones A and B respectively. The same conclusion is obtained outflow for well-chosen zones including the whole print of a paired vortex. Those observations confirm a known result, that a vortical structure subjected to convection without deformation does not radiate noise [4]. Also, as expected, the low-speed flow part, zone 2, and silent regions in the high-speed flow, zones 3 and 4, have negligible contributions to the signal. More surprising is the total contribution of the layer in the shear region, zone 1, whom integration gives $-1,6.10^{-7}$. This seems to be without intuitive relation with the value on the signal. A similar result is observed for the 3 other selected fields.

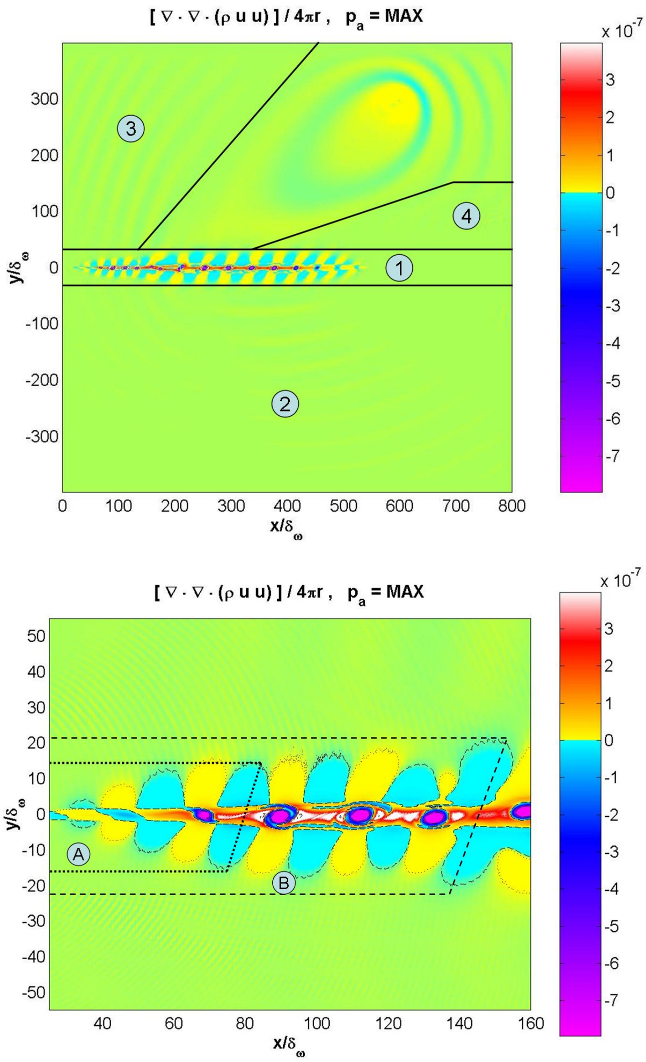

Figure 7: Source field at retarded time, with zones. Top: full source domain; bottom: closeup near inflow. The observer position is $(X/\delta_\omega = 600, Y/\delta_\omega = 300)$, and the observer time corresponds to the maximum in the acoustic signal.

However, the signal maximum, minimum and zeros are closely related to the corresponding full fields of S, plotted in figure 8. But the main cause of the signal seems to be located around the observer point. There, the intensity of T is 3 orders of magnitude less than in the pairing region, but the distance r is small, resulting in a significant intensity for S. Moreover, positive and negative zone pattern is visible there too, but spread over larger extents. These two trends may explain why this low-T region is able to contribute significantly to the integral value. So, the signal maximum correspond to the largest positive area of S around the observer point, and conversely for the signal minimum.

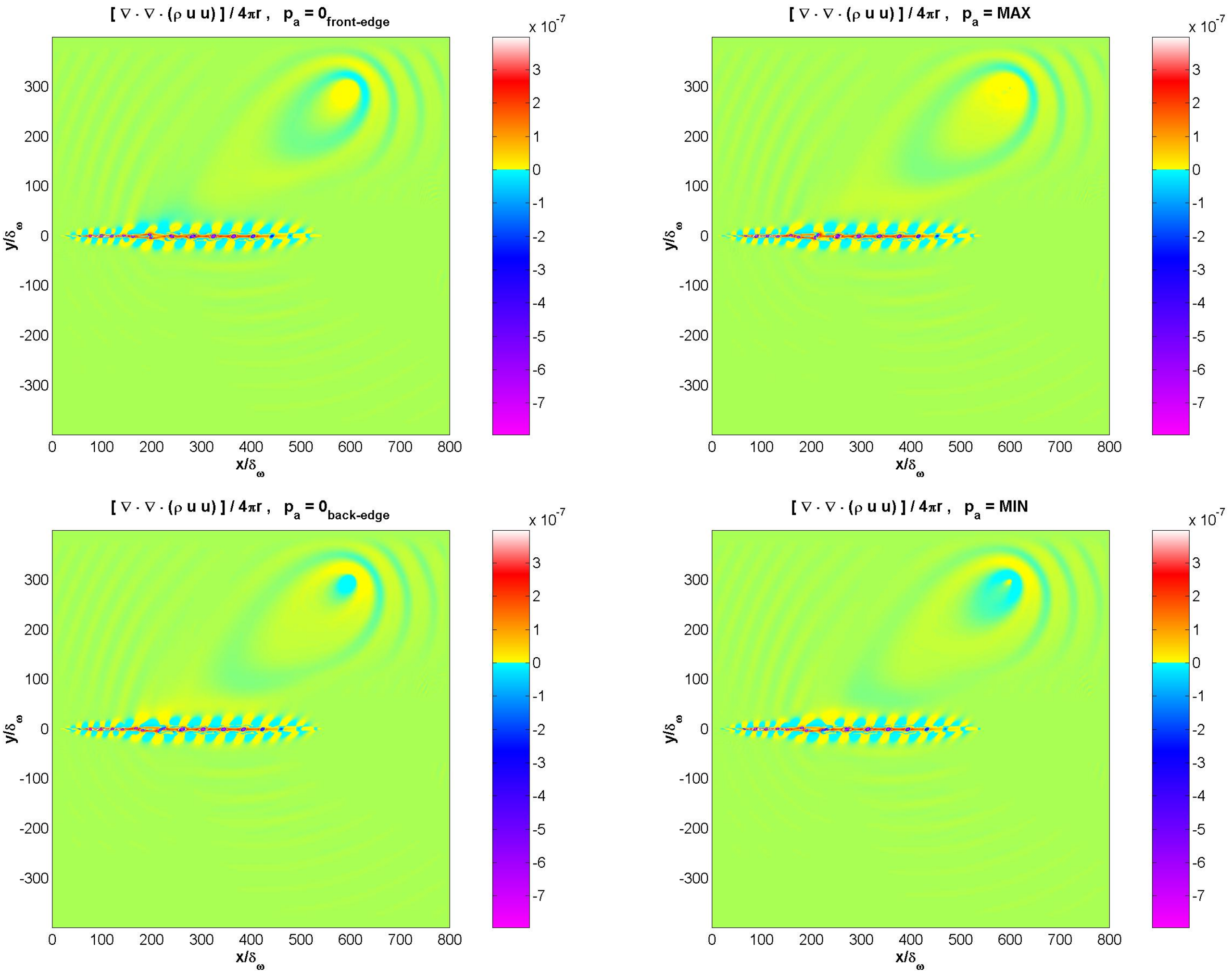

Figure 8: Source field at retarded times for main acoustic signal events. The observer position is $(X/\delta_\omega = 600, Y/\delta_\omega = 300)$.

Conclusions

The present observations showed that over the large source domain considered around a mixing layer, the acoustic pressure signal has not its source at a specific flow event or stress field in the vortex-pairing region, but is rather determined by the vicinity of the observer.

Is that way of thinking a fancy of the mind ? There are some intrinsic limitations of the present analysis: firstly, the observer is inside the source region, thus the analogy assumption of the source - observer separation is violated; but this separation is between the phenomena: the flow excites the wave equation as an externally applied source term. Thus the acoustic pressure can be computed at an observer located in the source region, provided the acoustic motion does not feedback the flow. Secondly, the integral solution (2) is written for a 3D-case, while our flow is in 2D, where the Green function is slightly different; but we expect few qualitative influence on the present observations. Finally, what makes the vicinity of the observer is acoustic waves coming from the pairing region, thus it could not be conclude that the pairing phenomenon is silent.

At least, there are two main lessons: on the one hand, a severe warning at interpreting non compact Lighthill's source field distributions as a map of noise sources. On the other hand, we are reminded that acoustics are a branch of fluid dynamics, and that a continous, material propagation medium is required.

Acknowledgements

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