Sound Synthesis and Spatial Reproduction by Physical Modeling

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Introduction

Physical models of wave propagation or mechanical vibrations are becoming increasingly important for the design of algorithms for sound reproduction and sound synthesis. For sound reproduction, wave field synthesis has been established as a physically well-founded method for determining the loudspeaker driving signals of massive multichannel systems [1, 2]. On the other hand, algorithmic models for sound synthesis are derived from detailed physical descriptions of strings, membranes, etc. by physical modeling sound synthesis with e.g. digital waveguides [3, 4, 5] or the functional transformation method [6, 7].

The reproduction of audio material with wave field synthesis systems usually relies on recordings from musical events, where different sources are recorded on different tracks. Each source is then assigned to a certain location or trajectory for spatial reproduction. A recent wavefield synthesis demonstration at the 124th Convention of the Audio Engineering Society (Amsterdam, May 2008) showed a different trend with the reproduction of synthetic source tracks. Rather than relying on microphone recordings, source tracks were created from synthesized sounds, algorithmically separated sources from a vintage recording, or from a source track for a silent movie. However, creating individual tracks and reproducing them on a multichannel system still have been separate processes.

This contribution combines synthesis and reproduction to form a joint synthesis method of the sound production and sound propagation properties of real or virtual musical instruments. Three different building blocks are addressed: The simulation of string vibrations, the radiation pattern of the generated acoustical waves, and the determination of the driving signals for a multichannel loudspeaker array. From physical descriptions of string vibrations and sound waves by partial differential equations follows an algorithmic procedure for synthesis-driven wave field reproduction. Its processing steps are derived by mathematical analysis and well-known signal processing principles. The proposed method differs from previous wave field synthesis approaches since no prerecorded source tracks are required.

Fig. 1 shows the generic sound propagation model including the sound source, its radiation pattern and the reproduction by a loudspeaker array. The loudspeaker array in Fig. 1 is a circular arrangement with 48 loudspeakers, but also other geometries and different channel numbers are possible. An overview of the system is presented in the next section and then the individual building blocks are discussed.

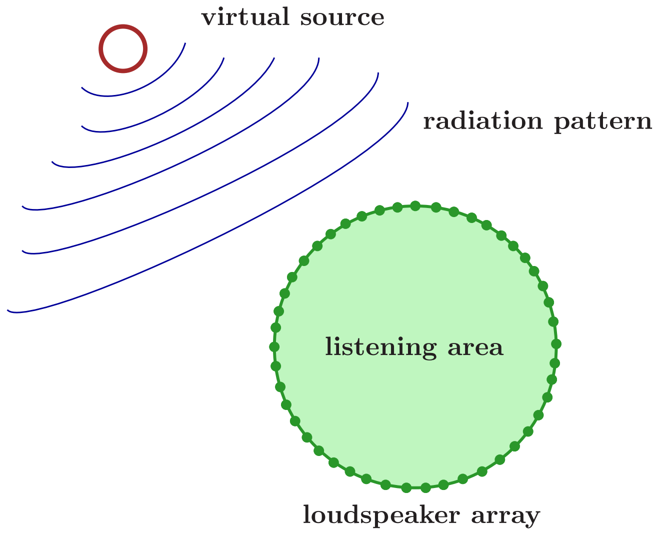

Figure 1: Generic sound propagation model from a virtual source to a loudspeaker array.

System Overview

The basic components of the generic sound radiation model from Fig. 1 are mapped to different building blocks of the joint synthesis and reproduction system as shown in Fig. 2.

The virtual source is realized here by physical modeling sound synthesis for a vibrating string according to the functional transformation method. The user input may consist of a stream of MIDI events which trigger the synthesis algorithm by a short sequence of digital samples as excitation signal. Also the physical properties of the string can be varied by the user at any time.

When the string is attached to a sound board, a part of its energy is radiated into the environment. A piston model is chosen as a simple model for a sound board. This model is closer to the properties of a real sound board than the point source or plane wave models usually employed in wave field synthesis. Furthermore, the piston model is well established in acoustics as a simple loudspeaker model. The position of the piston and its orientation in space are also under the control of the user.

For reproduction of the sound board radiation with an array of loudspeakers, the sound pressure and particle velocity at the location of each loudspeaker have to be known. They can be obtained from the known radiation properties of the piston model. Then the usual techniques for wavefield synthesis reproduction are applied to compute the driving signal for each loudspeaker [8].

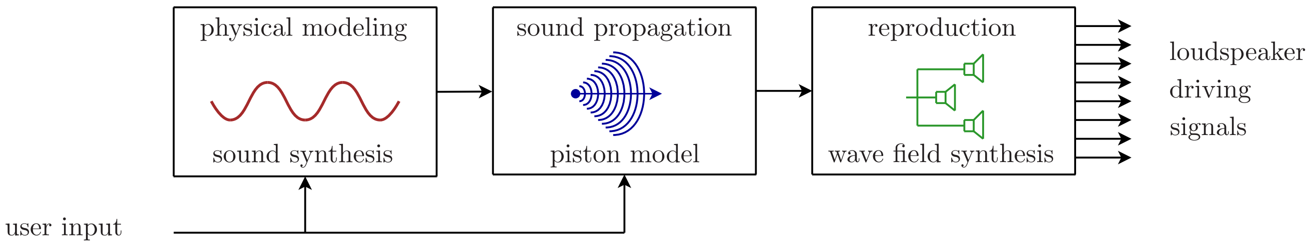

Figure 2: System overview of the combined sound synthesis and wavefield synthesis system.

Sound Synthesis

The sound synthesis method in the first block of Fig. 2 is based on a physical model in the form of a partial differential equation for the velocity of a vibrating string. Since the method has already been described in detail in [6, 7] only a short account is given here.

The underlying partial differential equation describes the propagation of transversal waves along a string and includes effects like dispersion and frequency dependent and frequency independent damping. The parameters of the model are the material parameters density, Young's modulus, and damping factors, and the geometrical parameters length, cross section area, and moment of inertia. Also the tension of the string is included.

The partial differential equation can be turned into a multidimensional transfer function by applying suitable functional transformations, i.e. Laplace transformation with respect to time and Sturm-Liouville transformation with respect to space. This way, the initial conditions resulting from striking or plucking a string and the boundary conditions induced by the fixing of the string ends are considered in a rigorous way.

The multidimensional transfer function for continuous time and space variables is converted to a discrete-time transfer function by a suitable analog-to-discrete transformation. It represents the synthesis algorithm as a parallel arrangement of second-order digital filters and a subsequent weighted summation (see Fig. 3). Thus the harmonics of the string are synthesized independently. This method differs from the well-known additive synthesis through the analytic computation of the filter coefficients from the physical parameters.

To be specific, the coefficients $b(\nu)$, $c_0(\nu)$, $c_1(\nu)$ for $\nu = 1, \ldots, n$ in Fig. 3 are calculated directly from the material and geometric parameters described above. Due to the simple realization, a variation of these parameters is possible during the operation of the synthesis algorithm. The weighting coefficients a_ν result from the boundary value problem which defines the Sturm-Liouville transformation. This digital filter structure is excited by a pulse of digital samples $f(k)$ which is triggered e.g. from each MIDI event "note on". The output $v(k)$ closely represents the velocity of the string within the limits of the discrete-time approximation of a continuous motion. The number n of second order sections should be chosen such that all relevant harmonics throughout the audio frequency range are synthesized.

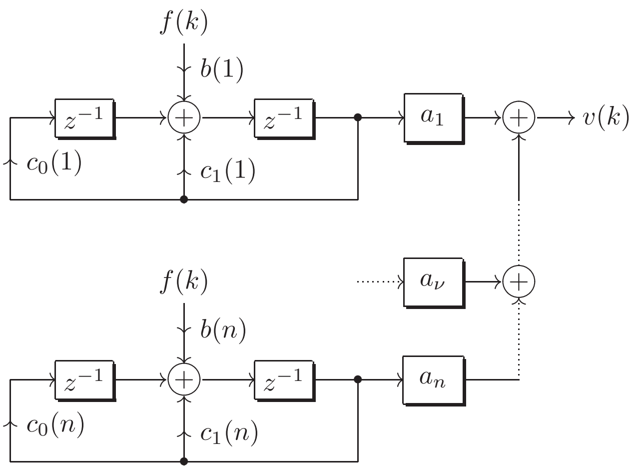

Figure 3: Synthesis algorithm as parallel arrangement of second order digital filters. The blocks labeled z^{-1} are delays by one sample.

Piston Model

Strings by themselves do not radiate enough sound energy for musical performances. In all acoustical instruments the strings are attached to some kind of sound board. To model this effect, the output of the synthesis algorithms, i.e. the velocity $v(k)$ in Fig. 3, is connected to a piston model. The piston is a classical model for a loudspeaker with a stiff membrane. Here, it is used as a first approximation to the sound radiation by the sound board of an acoustical instrument. No attempt has been made yet to adapt it to the specifics of a certain musical instrument. The piston model describes the sound propagation in Fig. 2.

A piston can be regarded as a circular disk which vibrates in the direction normal to its surface. The disk is stiff such that all points on the disk have the same phase. Then each point is modelled by a point source. The sound pressure at an arbitrary point in space results from the superposition of the effect of all point sources on the disk. The value of the sound pressure is obtained by integrating over the surface of the disk. The derivation of the resulting integral expression can be found e.g. in [9, 10] and is not repeated here.

Three different models for the piston have been investigated:

- The exact integral expression as described above. Since there is no analytic solution, a numerical evaluation has been implemented. This model serves as a reference for the following models.

- A classical approximation by a Bessel function.
  A geometrical approximation valid in a certain distance of the piston simplifies the integral expression such that the sound pressure at a distant point can be expressed in closed form by a Bessel function [10]. With this model the piston can be interpreted as a point source plus a certain directivity function.
- A transfer function model.
  A further simplification allows to express the relation between the velocity of the piston and the resulting sound pressure at a distant point by a transfer function, an amplitude factor, and a delay. In this simple model the frequency characteristic of the transfer function is constant for all directions. Directional differences appear only in the amplitude factor and in the phase caused by the delay. The simplicity of this model allows an efficient implementation as discussed below.

Wave Field Synthesis

The last stage in Fig. 2 is the reproduction by wave field synthesis. Most wave field synthesis implementations use point sources or plane waves as models for the sound propagation from a virtual source to the loudspeaker positions of the reproduction array. Here, the piston model is used instead. It predicts the sound pressure at each loudspeaker position such that the corresponding driving function can be calculated. According to [8], the driving function of a certain loudspeaker is obtained from the gradient of the sound pressure at the loudspeaker position by application of

- a spatial window which selects the active loudspeakers for a certain source direction,
- an amplitude factor $A(\mathbf{x}_\mu)$ which depends on the position $\mathbf{x}_\mu$ of the loudspeaker with number μ,
- a frequency selective filter $H_{\text{wfs}}(\omega)$ which is independent of the loudspeaker position.

The sound pressure respective its gradient at each loudspeaker position can be calculated from each one of the models of the piston radiation from above. The remaining determination of the loudspeaker driving signals is thus straightforward.

Signal Processing Structure

Combining the realizations of the three blocks from Fig. 2 as described in the previous sections gives the signal processing structure shown in Fig. 4. It describes ths signal flow from a MIDI input to each one of m loudspeakers. The realization of the physical model for the sound synthesis with the functional transformation method (FTM) is given by the structure from Fig. 3. The transfer functions $H_{\text{rad}}(\omega, \mathbf{x}_\mu)$ for the radiation pattern of the piston model to each loudspeaker position $\mathbf{x}_\mu$, $\mu = 1\ldots, m$ follow either from the exact integral relation, the Bessel model, or the simplified transfer function model. Finally the driving signals for the individual loudspeakers follow by multiplication with an amplitude factor $A(\mathbf{x}_\mu)$ and filtering with $H_{\text{wfs}}(\omega)$.

Fig. 4 shows the realization of the blocks from Fig. 2 without considering any possibilities for simplification. Especially for the simplest form of the piston radiation model, the transfer function model, a more efficient implementation can be achieved by shifting certain transfer functions and factors across the block boundaries. Both the simple piston radiation model and the loudspeaker driving functions contain transfer functions which are independent of the loudspeaker position. They can therefore be combined into one filter. It may be positioned at the front of the processing chain and combined with the excitation of the sound synthesis model (see $H(\omega)$ in the block filtering in Fig. 5). As long as the exitation impulse does not change, the output of this filter stays the same and needs to be calulated only once.

The radiation model and the driving functions contain also amplitude factors for each loudspeaker position which can be combined into one factor each ($B(\mathbf{x}_\mu)$ in Fig. 5). Finally there remain only the delays d_μ from the simple radiation model which are specific for each loudspeaker.

Comparing the structures in Fig. 4 and Fig. 5 shows that now the branches for each loudspeaker are free from any filtering operations. Since the number of loudspeakers is in the order of tens or hundreds, the resulting signal processing structure shown in Fig. 5 allows for a more efficient realization.

Conclusions

Wave field synthesis is versatile tool for multichannel audio reproduction. It is not restricted to the reproduction of recorded or pre-synthesized material only. It is also a natural building block for the synthesis of virtual musical instruments. So far physical modeling sound synthesis has been confined to the production of monophonic or two-channel stereo sound. Its capability for spatial reproduction is greatly enhanced by the combination with wave field synthesis. Both physical modeling sound synthesis and wave field synthesis rely on physical models in the form of partial differential equations. The missing link between a synthesized sound source and its reproduction by wave field synthesis is the spatial radiation pattern of the virtual instrument. The well-known and proven piston model has been used here as a proof of concept. For specific families of instruments like violins, brass instruments, or pianos, more elaborate radiation models are appropriate.

The signal processing structure from Fig. 5 has been implemented for reproduction with a 48-channel wave field synthesis systems at the Telecommunications Laboratory of the University Erlangen-Nürnberg. Musical examples with multiple strings at different locations and with the movement of sources along their individual trajectories demonstrate the feasibility of this joint synthesis and reproduction method.

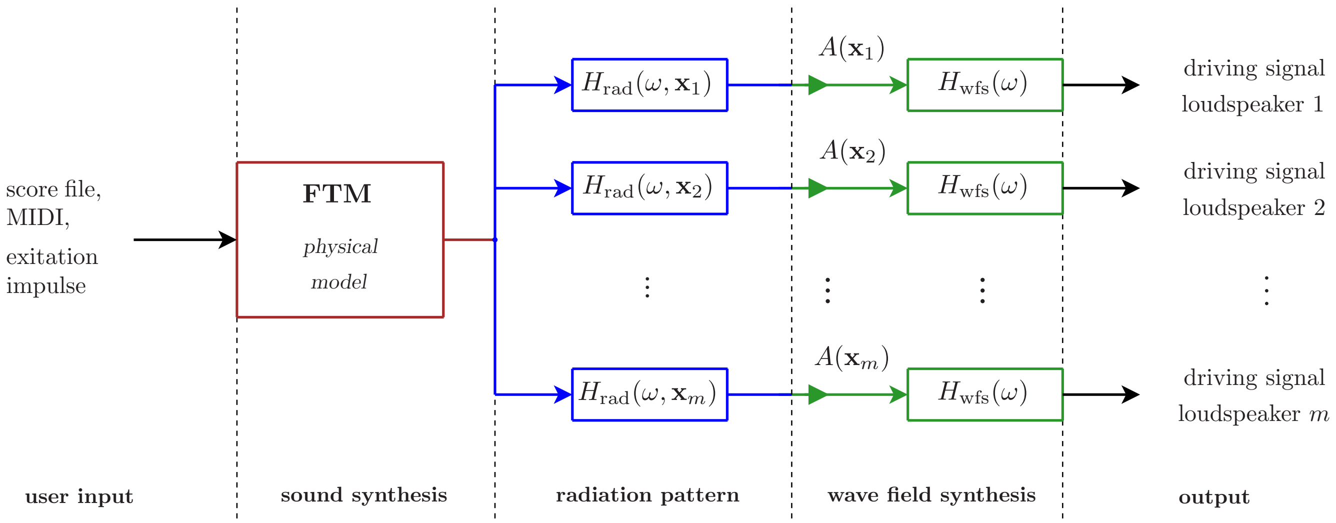

Figure 4: Signal processing structure from MIDI input to loudspeaker outputs.

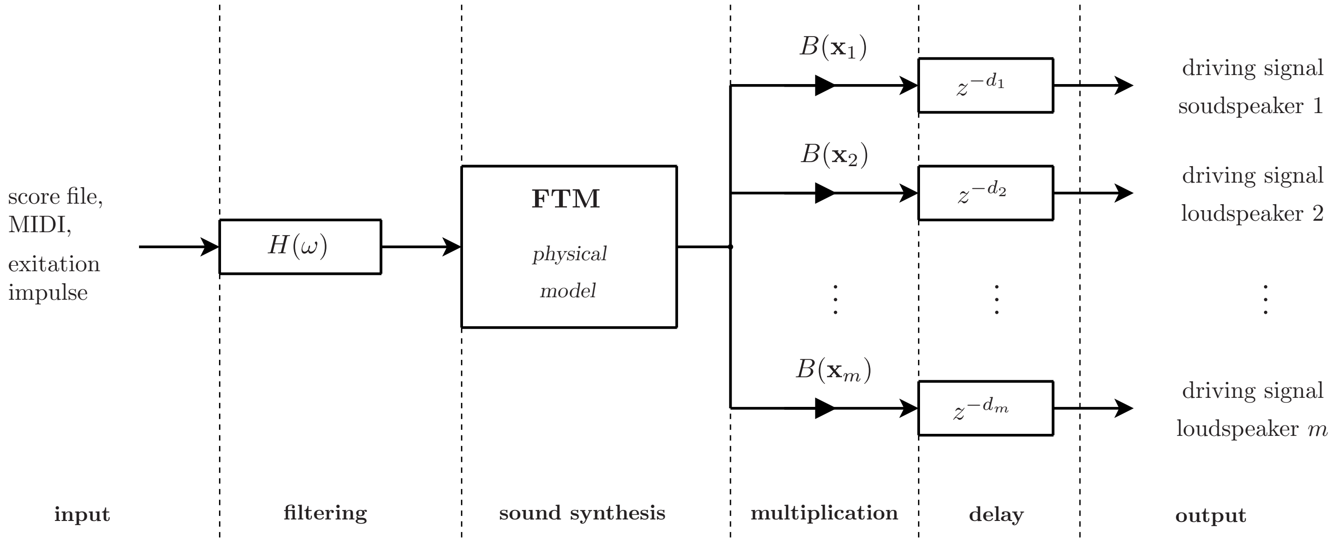

Figure 5: Efficient implementation of the signal processing structure by combination of transfer functions and amplitude factors across the block boundaries.

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