Reconstructing the Lituus: A Reassessment of Impedance, Harmonicity, and Playability

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Introduction

J. S. Bach's evocative funeral motet *'O Jesu Christ, meins Lebens Licht'* (BWV 118), appearing in two versions, was thought to have first been performed in 1736-7. On inspection the first version of Bach's score calls for four voices, cornett, three trombones, and two litui, and the second version for four voices, strings, bass continuo, and two litui. But what exactly are these mentioned litui? Are they a specific type of instrument or just an alternative name for an instrument already in existence at the time? To try and answer these questions a team of researchers from the Schola Cantorum Basiliensis in Basel, Switzerland, in collaboration with scientists and musicians from the University of Edinburgh, United Kingdom, have designed and built two functioning litui which may be similar to the instruments Bach could have had at his disposal during a performance of BWV 118. The project concluded with a public concert performance and formal presentation of the litui in Basel in January 2009.

These questions of exactly what the litui Bach refers to in his score looked and sounded like are not novel. Indeed, the musicologist, Ebenezer Prout, is known to have pondered these very same questions in an article to the Royal Musical Association back in 1885[1]. Work conducted by researchers at the Schola Cantorum Basiliensis indicated that the lituus may have been a long wooden natural instrument, sounding somewhere between a trumpet and an alphorn, and playing a series of natural harmonics. Scientists at the Musical Acoustics Laboratory at the University of Edinburgh were contacted to collaborate on the design process. Provided initially with a bore profile formed from interpolated measurements made for a Nagel trumpet (17th Century Nuremberg natural trumpet in D flat) and straight büchel (wooden alphorn-like instrument), theoretical models were then constructed and analysed using the Brass Instrument Evolution Software (BIES), recently developed at the University of Edinburgh[2, 5]. To acquire a better understanding of instruments similar in design to that proposed for the lituus, initial studies were undertaken on a 2.4m fiscorn (a type of Catalonian flugelhorn) in C, from the Edinburgh University Collection of Historic Musical Instruments. These initial studies included theoretically modelling the fiscorn from measurements of the bore profile, using BIES to calculate the theoretical input impedance curves, measuring the input impedance curves using the Brass Instrument Analysis System (BIAS) designed by scientists at the Institut für Wiener Klangstil, Vienna[3], and finally conducting a brief playing test using two experienced brass instrument players.

Following these initial studies an optimisation recipe consisting of an input impedance curve template, target input impedance peak frequencies, and a template for the instrument geometry with assigned upper and lower bounds were generated and input into BIES to find the optimal design for the lituus within specified parameters. Input impedance measurements were taken for one of the completed instruments and these measurements were then compared with theoretically calculated input impedance curves for the optimised lituus.

Measurement and Analysis Techniques

Input Impedance

The acoustic input impedance $Z(\omega)$ of any brasswind instrument is defined as the ratio of acoustic pressure $p(\omega)$ to volume flow $U(\omega)$, taken at the input plane of the instrument, where ω is the angular frequency for a sinusoidal input signal at the input plane. Acoustic input impedance is measured in units of acoustic ohms (Ω).

Impedance is an important measurement to have in the context of brasswind instruments, as it is this which determines the playing characteristics and linear acoustical behaviour for the instrument under investigation[4, 5]. The positions of the peaks on an input impedance spectrum also determine whether or not a note should be easy to play. On a trumpet, for instance, if a particular input impedance peak has a high magnitude at a frequency at which the player wishes to play, then this note will be easy to play. When the opposite happens and the player wishes to play a note at which the corresponding input impedance peak is of low magnitude, then the note is difficult to play. In this situation the player is required to lip away from the input impedance 'slots' in order for the note to sound from the instrument.

Equivalent Fundamental Pitch

Another important measurement in producing an optimal design for the lituus was the Equivalent Fundamental Pitch (EFP). Derived from the acoustic input impedance curves and closely related to the Equivalent Fundamental Frequency, an EFP plotted graphically provides a straightforward method for assessing how close individual input impedance peaks are to corresponding members of a single harmonic series. The EFP is calculated for each input impedance peak using the following equation

$$EFP(f_i) = \frac{1200}{\log(2)} \times \log\left(\frac{f_i}{iF}\right) \quad \text{[cents]} \quad (1)$$

where f_i is the frequency of the i^{th} input impedance peak and iF is the frequency of the i^{th} harmonic based on an arbitrary fundamental frequency F[2]. The EFP, expressed in the more musically relevant unit of cents, therefore indicates for our given value of i the deviation of the impedance peak frequency from the frequency of its corresponding component from the harmonic series. EFP was extremely useful throughout the optimisation routine, as it was possible to draw quick conclusions as to the suitability of theoretical instrument designs based on their harmonicity.

Brass Instrument Evolution Optimisation Software

The Brass Instrument Evolution Software (BIES), developed at the University of Edinburgh[2, 5], provides an integrated software package which allows for the design, testing, and optimisation of brass instruments theoretically. Specifically, BIES allows the musical instrument researcher to obtain calculated input impedance curves for theoretically described instrument geometries and an optimisation routine where a target input impedance is specified and, should suitable optimisation parameters have been chosen; an optimal input impedance curve and associated smooth bore profile is produced. Throughout the optimisation process spherical-wave propagation was assumed.

One of the features of BIES is that it uses the Rosenbrock direct-search algorithm to find an optimised bore profile with the required input impedance curve across a more constrained design space. This has the effect of improving the convergence to a realistic instrument design and the time within which it does this. A more detailed consideration of BIES and the optimisation process is available in other sources[2, 5]. Another feature of BIES is the ability to express the bell of an instrument as a series of end-on-end Bessel-horns[6], where the Bessel-horn is defined as being

$$r(x) = b(-x)^{-\gamma}, \quad \text{[m]} \quad (2)$$

where r is the bore radius a distance x along the axis of instrument, γ is a flare constant, and b is a constant usually defined in terms of specified length and input/output radii for the section expressed[2].

Initial Investigation using a Fiscorn

Considering it is accepted that discrepancies do exist between the input impedance of an instrument when measured and calculated theoretically[5] and to assess the input impedance results obtained theoretically with BIES, initial investigative work was undertaken on an instrument with notionally similar dimensions to the proposed lituus design. A 2.4m long fiscorn in C attached to a suitable mouthpiece had input impedance measurements taken using BIAS. Theoretical input impedance curves also were calculated using BIES from a model created from detailed measurements taken by hand and finally a brief playing test was conducted by two experienced brass players.

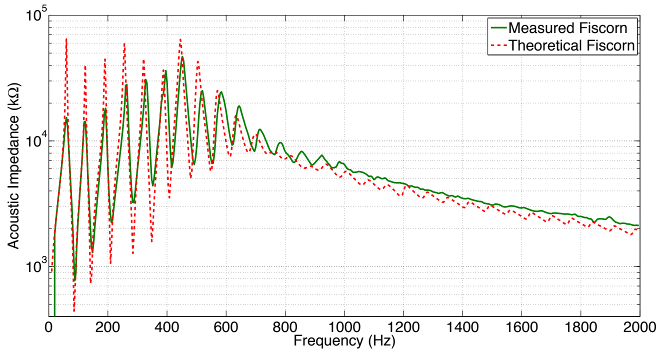

Figure 1: Input impedance curves measured for a 2.4m long fiscorn at playing conditions (*solid line*) and theoretically calculated (*dashed line*).

An interesting comparison to be considered was between the measured input impedance for the fiscorn, warm and in playing condition, and the input impedance calculated theoretically (see Figure 1). Although the measured and theoretical input impedance peaks are at almost the same frequency, their relative peak magnitudes are quite different. It is believed that this discrepancy is most likely to be a combination of acoustic energy loss due to dirt and sediment lining the internal tubing, as mentioned in Watkinson *et al*[7], and leaks due to the age of the instrument. The playing test results confirmed that the measured input impedance peak frequencies were close to the frequencies at which our two brass players played. Typically, the deviation was within ±5Hz of the measured impedance peak.

Designing the Lituus

With the initial investigation work complete, the original proposed lituus bore profile, provided by the Schola Cantorum Basiliensis, was then expressed theoretically as a series of cylinders, cones, bore jumps, and Bessel-horns. The rationale behind the interpolated bore profile is attributed to the fact that the researchers at the Schola Cantorum Basilensis believed that a combination of the tone quality produced by a trumpet-/büchel-like instrument would provide the sort of sound they would expect to blend well with the cornett and trombones also playing in BWV 118. To provide an impedance template for BIES, a theoretical input impedance was calculated for the unoptimised lituus bore design. It was requested that the optimised lituus play as close to a harmonic series based on C2 (with A4 at 415Hz) as possible. With this in mind, and considering the range of notes Bach writes for the two litui to play, which extend up to the 15th resonant mode (see Figure 2), the optimisation target was specified such that the lituus would play in tune from the 4th to the 18th resonant mode.

Figure 2: All of the notes to be played by the two litui in BWV 118 and their respective ranges.

Having assigned a template input impedance for the optimisation process, target input impedance peak frequencies were specified. As the BIES optimises within assigned design space constraints, the unoptimised lituus design was then expressed as a series of cones, cylinders and Bessel-horns with reasonable upper and lower limits. In an attempt to keep the results of the optimisation process realistic, care was taken to express each constituent element of the lituus model such that no discontinuities could become apparent in the final optimised design (beyond those that might be formed by the join between the mouthpiece and receiving pipe). Finally, tuning parameters to control how strictly the optimiser tries to find solutions with impedance peaks that lie closest to the target values were specified and tested. Many different combinations of tuning parameter values were tried and tested in BIES to find an optimal set that provided a realistic bore profile, good EFP results, and for which the optimiser would converge.

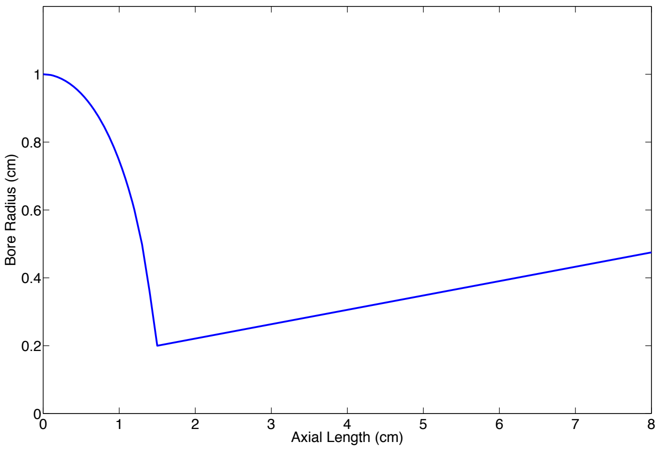

Figure 3: A plot of the internal bore profile for the lituus mouthpiece.

Throughout the optimisation routine the design of the mouthpiece, expressed as a list of conical and cylindrical tubing, was kept constant. This was due primarily to the players expressing a preference for a particular design. The design of the mouthpiece agreed for the lituus was elliptical in shape with a diameter measuring 20.0mm at the opening, extending to a depth of 15.0mm from the opening to a throat measuring 4.0mm in diameter, then finally extending conically over a distance of 65.0mm to a diameter of 9.5mm (see Figure 3).

Results

In the final design, the body of the litui were carved out of pine wood, split into three sections (for reasons of portability) and the mouthpieces were hand-carved from cow horn by Matteus Wetter, the instrument maker in Basel.

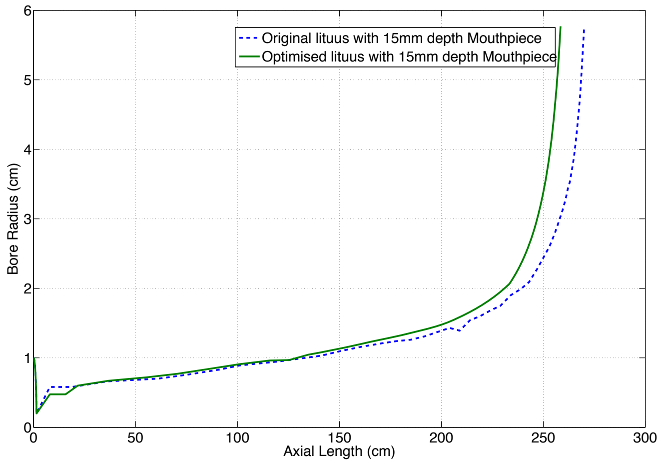

Figure 4: A plot of the internal bore profile for the unoptimised (*dashed line*) and optimised (*solid line*) lituus design.

Figure 4, showing the internal bore profile for the unoptimised and optimised lituus design, demonstrates the extent to which the original bore profile supplied by the Schola Cantorum Basiliensis has been altered by the optimisation process.

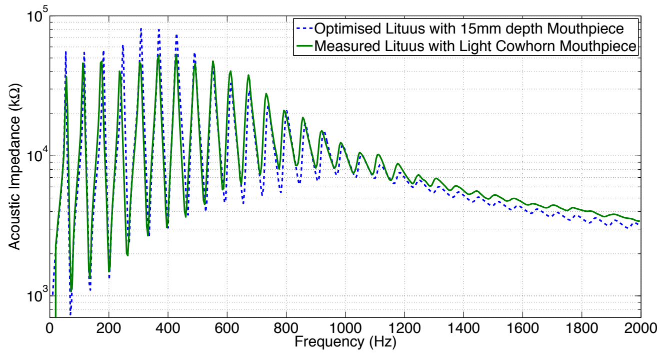

Figure 5: Input impedance curves measured for the constructed lituus (*solid line*) and theoretically calculated from the optimised design (*dashed line*).

Figure 5, shows input impedance curves measured for the constructed lituus and theoretically calculated from the optimised design. With the exception of one or two points, there appears to be good agreement between the two input impedance curves, despite the known discrepancy between theoretically calculated and measured input impedances. Furthermore, it is shown that the difference in peak magnitude between the two input impedance curves is significantly less than the differences observed for the fiscorn.

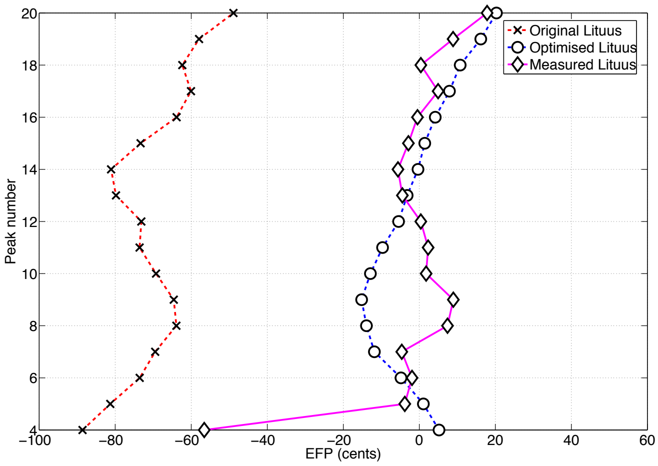

Figure 6: Plot of mode number versus Equivalent Fundamental Pitch (in cents) for the original lituus design, theoretical optimised lituus design, and the constructed lituus.

Figure 6 shows an EFP comparison plot between the original lituus design, theoretical optimised lituus design, and the constructed lituus. An immediate observation which can be made from this figure is that both measured and predicted optimised designs have improved harmonicity when compared to the unoptimised design. This difference is most likely due to the unoptimised design being tuned to a different pitch centre although it is noted that this design is still relatively in tune in itself. The discrepancy between the harmonicity of the two optimised litui is thought to be caused by slight differences in the dimensions of the two instruments, most likely caused unavoidably by the manufacturing process.

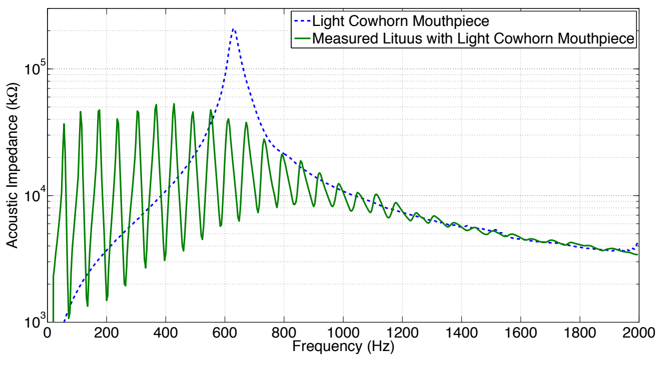

Figure 7: Input impedance curves measured for the constructed lituus (*solid line*) and the constructed lituus mouthpiece (*dashed line*).

Finally Figure 7, showing input impedance curves measured for the constructed lituus and its respective mouthpiece, makes it possible to see how the input impedance of the instrument relates to the input impedance of the mouthpiece and what effect this has. In considering the work of Caussé *et al*[8], it is believed that the lituus behaves in much the same way as the French horn. This is due to the fact that the cup frequency of the mouthpiece (the accentuated 'spike' in the mouthpiece input impedance) appears to govern the point at which the input impedance peak magnitudes for the instrument begin to reduce.

Conclusions

Following initial investigative work a real, playable instrument was built based on a purely theoretical design. In addition, the sound of the reconstructed lituus blended well with that of the other instruments, as was originally hoped. The results presented here illustrate that, given an initial design for a brasswind instrument and the desired frequencies at which the instrument is to be tuned, it is possible to theoretically create a realistic optimised design. Furthermore, it has been shown that, when constructed, the input impedance curve for the instrument shows good agreement with the input impedance curve calculated theoretically.

Further work on the lituus will most likely consist of detailed measurements taken of the bore profile of the instrument, further comparison between theoretical and measured input impedances from these measurements, and investigative work into the instrument's playability.

References

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