Introducing diffraction into beam tracing - some new results

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1. Introduction / The Basic Ideas

In room and city acoustics respectively noise immission prognosis, ray or beam tracing methods (RT/BT) are well approved (where a version of RT is the sound particle method with its detector technique and its statistical evaluation [1] and BT is an efficient straight forward implementation of the mirror image source method MISM). But these methods naturally neglect diffraction.
The aim is an efficient handling of arbitrary diffraction and reflection orders. A diffraction module is desired as an approximation for short, but not very short wavelengths.
As a high frequency approach for ray diffraction the UTD exists [2] and was recently utilized by Tsingos et. al. within BT [3]. Svensson developed a secondary edge source model valid even for low frequencies [4] (also only for hard wedges). But both methods work recursively for higher order diffraction, hence the computation time explodes.
So, basic hypotheses for an introduction of diffraction are:
- diffraction is mainly an edge effect,
- energetic superposition, hence RT can be used.

But there another problem arises: with RT, rays never hit edges exactly, they pass only near by.
Basic ideas for solving both problems are:
- not all combinations and paths of diffracted/ reflected rays or particles are important, only those where particles pass ***close to*** edges,
- the bending effect on a sound particle – the diffraction probability- should be the stronger the closer the by-pass-distance. This idea is inspired by Heisenbergs Uncertainty-Relation (UR): the by-pass-distance as an 'uncertainty'. Thereby, the diffraction pattern is the spatial Fourier transform of the transfer function of a slit. Already in 1986, the author made a successful approach for a sound particle diffraction based on the UR [5] (later affirmed by Freniere et al. who utilized the UR in another way successfully in optical RT [6]). In 2006 this approach has been generalized, embedded in a full 2D ray tracing program, now also for finite distances [7]. The results have been compared earlier with the Maekawa's 'classical' 'detour-model' [8], later with Svensson's model for the screen. (The impulse responses were Fourier transformed and the transfer functions octave band averaged.) Reference cases were the semi-infinite screen as a 'must' and the slit (two edges) as self-consistency-test.

The basic idea for solving the 'explosion problem' is a re-unification of 'similarly running' rays. This is only possible if rays are spatially extended, i.e. rather beams, in order to exploit their overlap, to interpolate and to re-unify them. For this purpose, Quantized Pyramidal Beam Tracing (QPBT) was developed in 1996 [9]. This chance is the reason, why now beam instead of ray diffraction is preferred.
A pre-condition for an effective pyramidal beam tracing is a subdivision of the room into convex sub-rooms. Diffraction events at 'inner edges' may be effectively detected on the transparent dividing 'walls'. Furthermore, RT is accelerated considerably. Fig.1 illustrates this vision.

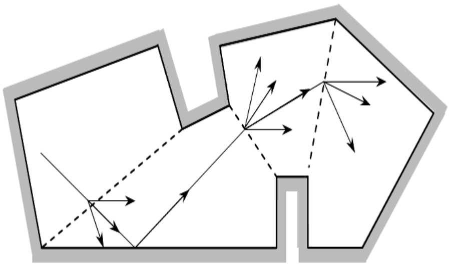


Fig. 1: Multiple diffractions in a (2D) room which is subdivided into convex sub-rooms: 'transparent' dividing walls are dashed; a ray is scattered/ diffracted several times on these 'walls' near edges (only one path is drawn)

This short paper reports about some recent results: 1) the checking of other by-pass-distance-dependent diffraction functions, 2) the fulfilling of the reciprocity principle, 3) an integral formulation is utilized and discussed. It assumes the reader is familiar with the mentioned models otherwise refers to the paper of last year [7] and the detailed paper in ACUSTICA [10]. Nevertheless, some basics and results shall be repeated.

2. The Sound Particle Diffraction Model

There are two basic concepts of implementation: the 'Diffraction angle probability density function' (DAPDF) and the 'Edge Diffraction strength' (EDS).
The idea of that DAPDF (with non-split-up particles) emerges from the UR. But it is more efficient (and physically equivalent) to split up the rays into new ones with partial energies according to the DAPDF. (fig 2).

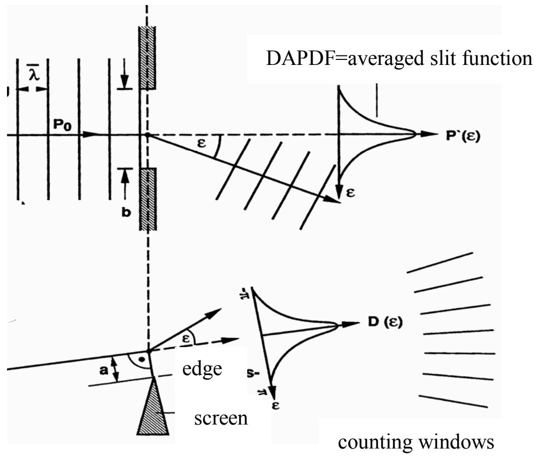


Fig.2: The sound particle diffraction model: Each moment a particle passes an edge of a screen at a distance *a* (below), it 'sees' a slit (above with the DAPDF on the right hand side). According to the uncertainty relation a certain EDS causes the particle to be diffracted according to the DAPDF= $D(\varepsilon)$. Below on the right some angle windows used to count the diffracted particles and to add up their energies to the transmission degrees (acc. eq.5). All the shifted DAPDFs of the different rays add up to the screen transmission function (as e.g. in fig. 5 [7]).

2.1. The DAPDF

The DAPDF (see fig.2.) is derived from the Fraunhofer diffraction at a slit $\propto \sin^2 v / v^2$, where $v = \pi \cdot b \cdot \varepsilon$, valid for parallel incident and diffracted rays. The DAPDF, averaged over a wide frequency band (similar as for 'white light') is roughly approximated by

$$D(v) = D_0 / \left(1 + 2v^2\right) \text{ with } v = 2 \cdot b \cdot \varepsilon \qquad (1)$$

where b is the apparent slit width in wavelengths, ε is the deflection angle and D_0 *is a* normalization factor such that the integral over all deflection angles is 1. The D_0-factor must be computed for each edge by-pass since its value depends on b and the angle limits of the wedge. In the following all distances are expressed in units of wavelengths λ.

2.2. The EDS

To develop a modular model which is applicable also to several edges that are passed near-by simultaneously, the 'Edge Diffraction Strength' (EDS(a)) is introduced such that the EDS of several edges may be added up to a total TEDS ,

$$TEDS = \sum EDS_i \qquad (2)$$

To be used as input for the DAPDF, an 'effective slit width' is then

$$b_{eff} = 1 / TEDS \; . \qquad (3)$$

By self-consistency-considerations (the RT experiment at a slit should re-produce the energy distribution of itself) it turns out that

$$EDS(a) = 1/(6 \cdot a) \qquad (4)$$

So, with only one edge, a by-passing particle would 'see' a relative slit-width of b_{eff}=6a.

2.3. Method of evaluation

For a systematic analysis, 2D -RT and -BT was evaluated for sources S and receivers R at finite distances r_s and r_r of 1,3,10,30,100 λ and 15 angles φ_r (and later also φ_s) -84…+84° in steps of 12°, in total 5*5*15=375 combinations at the screen (fig.3) as well as at the slit (of width b between two edges at –b/2 and +b/2 on the y-axis).

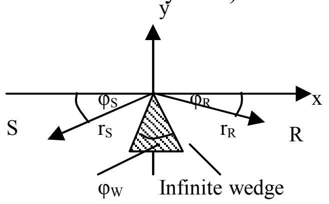


Fig.3: Geometrical definitions at the screen

For all these parameters, the transmission degree T was determined. T is defined as the intensity with the diffraction of an obstacle relative to the intensity in free field where 'intensity' in 2D is 'sound power/width' instead of 'power/ surface'. But the proportion of T is the same in 3D. For the slit, T equals directly the DAP, the energy portion for a certain angle range $\Delta\beta$ relative to the energy incident onto the slit. The results were compared with the known reference functions, evaluating the mean, max, min and the standard deviations over all. Curves as in fig. 5 may be plotted.

With RT, many – typically 10…100 – particles are shot over the edge. In the first approach [5], their energies were counted in 'angle windows' in infinity on the other side to compute from that the T of the semi-infinite screen. Now, in order to simulate also finite receiver distances, the particles are detected utilizing a grid of quadratic particle detectors [1,7]. For sound particles the immission formula [1] is valid

$$T_{BTSP} = \frac{2 \cdot \pi \cdot R}{M_0 \cdot S_d} \cdot \sum_M \sum_{n=1}^{n_0} w_{M,n} \cdot D'\left(\beta_{M,n}\right) \qquad (5)$$

where the $0 < D'\left(\beta_{M,n}\right) < 1$ are the energy fractions of diffracted rays (integrals of the DAPDF) in the angle range $\Delta\beta$ of the Mth incident ray and for each of the nth diffracted and received ray 'within an imagined beam', R is the direct distance source-receiver, S_d the detector surface for sound particles and w_{Mn} are the inner crossing distances of particles in detectors.

2.4. Results of ray diffraction experiments

At the first go (without any parameter fitting), the agreements with the reference function (Maekawa) were very good for almost all cases, now also for finite distances (standard deviation in most cases <1 dB, curves similar as in fig.5). In 1986, this happened even for many cases of the slit. Now, also the comparison with Svensson's result yielded good results (std.dev. 0.66dB).

3. From ray to beam diffraction

To prepare the later implementation of QPBT and to reduce the number of energy carriers, now beam diffraction was tested. For mirror image sources (as represented by beams), there is no stochastic variation and the 1/r² -distance law may be applied to compute the immitted intensities at the receiver points (in 2D a 1/r-law, r_{BM} is the distance bending point - receiver):

$$T_{BT}' = \frac{2 \cdot \pi \cdot R}{M_0 \cdot \Delta\beta} \cdot \sum_M \frac{D(\beta_M)}{r_{BM}} \qquad (6)$$

$D(\beta_M)$ is the same as $D'\left(\beta_{M,n}\right)$ in eq. 5 for the Mth incident beam which belongs to the Mth relevant incident and diffracted beam (elongated in fig.4). So, for one receiver, only one loop over all beams (M=1…M_0) is necessary, not a secondary loop over each time an additional number of secondary particles (n=1…n_0). So, RT can be equivalently be replaced by BT being much more effective. The valid by-pass distance of a beam is the middle ray's distance within the beam.

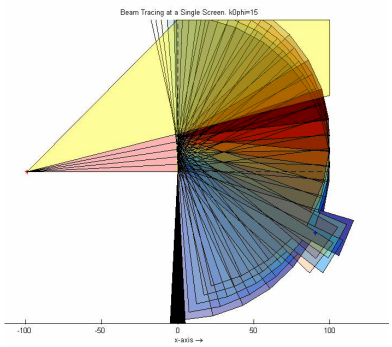


Fig. 4: 2D beam diffraction, specialized for the screen (black wedge in the middle): Typically 10…100 beams ('fans' in 2D) (left, pink) arrive within the by-pass distance range of 0…7 λ (here exaggerated). The direct sound passes above (yellow). To reach all receivers, beams are split up into each typically 10..100 secondary beams. To the right the diffracted beams: the darker the colour the higher the intensity –and this mainly in straight forward directions; bottom right: the beams relevant for one specific receiver are drawn elongated.

A mathematical analysis shows that, in order to reach a certain numerical accuracy, particles require a much, at least 10 times higher number of detector crossings and computation time of than beams do.

3.1.Results of beam diffraction at a screen

- The agreements RT /BT were very good (standard deviation of only 0.67dB);
- The direct comparison between BT and the Maekawa screen transmission functions yielded a std.dev. of 0.74dB,
- the comparison with Svenssons's exact coherent secondary edge source model as analytical reference model yielded only 0.39dB (see fig.5).

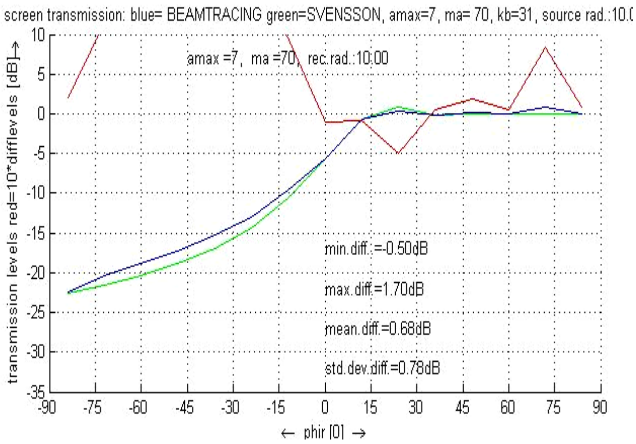


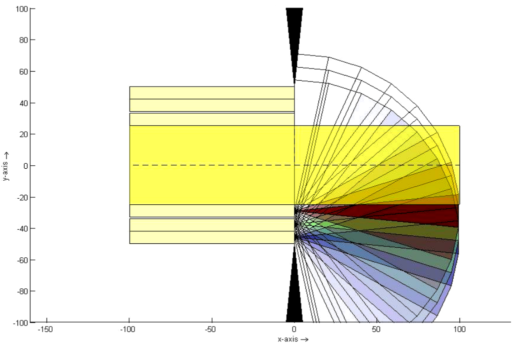
Fig. 5: Example of a comparison between beam tracing (green) and Svensson's reference method (blue, falling to the left). The transmission degree in dB is given as function of the receiver angle, to the left the 'shadow' region; red curve, rising to the left: deviation* 10 (70 incident * 31 diffracted beams within $a_{max}=7\lambda$, source and receiver distance: 10λ, source at y=0).

Also, **the influence of the inner wedge angle** φ_w (fig.3) was investigated: For smaller inner angles their influence is low, but for the case of 90°, compared with 0°, the differences in the transmission levels are up to 4dB (mean difference are typically 0.4dB). However, in Svensson's reference model, hard flanking walls are assumed whereas in the interaction model based on the UR only the position of the edge is relevant, not any flanking walls.

3.2.Parallel beam diffraction at a slit

For this self-consistency-test, both, source and receiver are in infinity, hence, the incident parallel beams carry a fraction of energy according the portion of the slit width, the diffracted beams carry energy according their angle width. Fig.6 shows the experiment similar as explained in Fig.4, drawn by the program with exaggerated beam widths.

Fig.6:



Only the beams near the lower edge are evaluated by reasons of symmetry, the yellow beam in the middle carry the undiffracted energy (outside certain max by-pass-distances.) Now the EDSs of the two edges were added (Eqs. 2-4).

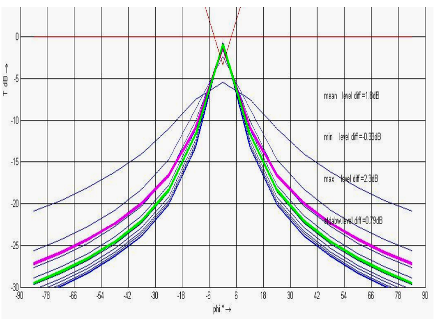


Fig.7. Transmission as a function of diffraction angle (upper violet curve) as the sum over all DAPDFs =array of lower blue curves. Green: reference function. Example for a 10λ wide slit.

The standard deviation for all cases is only 0.75 dB, but there are up to 3 dB too high levels at high angles ('deep in the shadow') compared with the slit function itself (green curve). (Without the a_{max}-limitation, even deviations up to 5dB, with the EDSE much better, see below). This result depends hardly on the number of beams.

4. From beam diffraction to integration

Now, to exclude any numerical error due to the finite number of beams (M_0), a comparison with an 'infinite number' of beams i.e. a (numerical) beam integration (BI) was also carried out. With $\Delta\alpha = 2\pi / M_0 \rightarrow d\alpha$ and $D(\beta_M)/\Delta\beta \rightarrow d(\beta(\alpha))$ (the DAPDF) eq. 6 converges to

$$T_{BI} = R \cdot \int_{\alpha \min}^{\alpha \max} \frac{d(\beta(\alpha))}{r_{BM}(\alpha)} d\alpha \qquad (7)$$

($\alpha_{\min/\max}$ are the min and max incident angles, $\alpha_{\min} = -\varphi_s$). The difference between BT and BI for the screen was only 0.38dB std.dev. The following was obtained also with BI.

5. Further investigations

5.1. Attempts of optimizations of the DAPDF

A somewhat improved approach for the DAPDF (instead of eq.1) was used in [5]:

$$D(v) = D_0 \cdot \begin{Bmatrix} 1 - v^2 & für & |v| \le v_0 \\ \dfrac{1/2}{\sqrt{2} - 1 + v^2} & für & |v| > v_0 \end{Bmatrix} \qquad (1b)$$

$$with \quad v_0 = \sqrt{1 - 1/\sqrt{2}} \approx 0.5412$$

This DAPDF2 has a wider top as the former, better approaching the averaged slit-diffraction function $\sin^2(u)/u^2$.

But, as it turned out astonishingly now: its use does not pay: The std.dev. at the screen became even slightly higher than with before (0.9dB). With the slit no improvement.

In [6] is proposed a gaussian distribution; but this is inconsequent, as the transfer function (and hence its Fourier transform) of a slit is not gaussian.

5.1. Optimization of the EDS

As it turned out, at least for the slit, the edge diffraction strength for wider by-pass-distances is to high. Therefore another EDS was tested again [5] with an exponentially decreasing strength and a limitation to 7λ:

$$EDSE(a) = \frac{1}{3 \cdot a + e^{a}} \quad \text{for } 0 < a < 7, \text{ else } 0 \qquad (4b)$$

With this (instead of the EDS of eq.4) at the slit the agreements become much better: max. deviation 1dB, std. dev. 0.5dB. (With the single screen, they become slightly worse, especially at short distances, std. dev. 0.8dB).

In [6] is also proposed to evaluate only distance to the nearest edge. Then the total EDS should (by self-consistency) be defined as $TEDS(a_1, a_2) = 1/(4 \cdot \min(a_1, a_2))$ (4c).

But, the result is much worse than with the EDSE: max. deviations were up to 5dB, std.dev. 1.4dB.

5.3. Test for reciprocity/oblique incidence

Do the same diffraction levels result with a permutation of source and receiver? This does not follow evidently from the application of the UR, resp. eqs. 1-4 or eq. 7. Hence, if the reciprocity were fulfilled, this would be an important indication of the correctness of the model. It turned now out: the reciprocity principle is fulfilled (max. dev. 0.49dB, std. dev. 0.21dB) at least if only r_s and r_r are interchanged, assuming only the ***total*** diffraction angle $\varphi_s + \varphi_r \approx \beta$ were relevant regardless of the position of the integration area (the former y-axis in fig.3. or a dotted line in fig.1.). So, restricted for $\varphi_s = 0$. If, however, also φ_s and φ_r are interchanged, severe deviations (mean dev. up to -10dB) occurred in cases of high negative values of φ_s. The reason is: Equ. 7 is ***not*** symmetric with respect to an interchange of source and receiver. Some geometrical transformations lead to the alternative integral over the by-pass-distance a:

$$T_{BIS} = R \cdot \int_0^{a\max} \frac{d(\beta(a)) \cdot \cos(\varphi_s)}{r_1(a) \cdot r_2(a) \cdot \cos(\Delta\alpha)} da \qquad (8)$$

where r_1 and r_2 are the radii to source and receiver from the bending point, $\Delta\alpha = \alpha_{\max} - \alpha_{\min}$ and $\alpha_{\max}$ corresponds to a_{max}.

It has to be checked weather an symmetrization of the formula into BT –an introduction of $\cos(\varphi_r)$?- could lead to an improvement. It has also to be acknowledged that the model is not made for 'backward scattering' ($|\beta| > 90°$).

Fortunately, the orientation of the 'diffracting surface' 'above' the screen (dashed lines in fig. 1) has only a weak influence (at +-45° less than 1dB). This is important for the practical implementation of the model in sub-divided rooms.

5.4. Optimum numerical parameters

Most important and useful is: a maximum by-pass distance of a_{max}= 7 λ (see fig.2) may be established; beyond that, direct transmission may be performed (figs. 4+6). In the case of the slit (or several edges), a_{max} even ***must*** be defined to reduce the effect of the EDS (if not the EDSE is used): the level deviations to the reference functions were without a_{max}: max 3.47dB, std.dev. 0.91dB ; with a_{max}=5 only max 0.94dB, std.dev. 0.4 dB. The maximum deviations increase with minimum by-pass distance, 1 λ is close enough. This cannot be improved with more particles. With RT, a decisive quantity is the number of incident particles within a close by-pass distance a_{min}. That should be maximum 0.1 λ. With BT, one incident beam onto the range near the edge is sufficient, a group of diffraction points within 0… a_{max} may then be established. The number of secondary beams should be in the order of the number of relevant targets or receivers on the other side.

6. Conclusions and Outlook

The agreements were in most cases very good – with some restrictions as now emerged. Consequently, it seems like Heisenberg's UR may be applied also to acoustics and sound may be handled as particles even with diffraction.

The cases of **beam diffraction at a slit, but with sources and receivers in finite distances,** and the **double diffraction at a cascade of edges behind each other** will still have to be evaluated. Experiments are going on.

In principle, it should not be a problem to extend the presented model to 3D and to multiple diffractions (if the edge = the *z*-axis is infinite, then $\Delta z \to \infty$ and there is no reason for any diffraction in the *z*-direction, one correctly gets $\Delta k_z = 0$). Edge diffraction happens only in the area perpendicular to the edge; it is basically a 2D effect.

The strong frequency dependence of diffraction (influencing the question what are 'near' edges /what is the best a_{max}) remains a problem. In final simulations, each beam should carry energies of several octave bands. This concerns also the question of the limiting distance between edges for 'independent' subsequent diffractions. This will to be investigated with the double-diffraction experiments.

A combination of beam diffraction procedures with QPBT seems now possible without explosion of computation time.

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