The use of reciprocity in determining rainfall noise

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Introduction

Rainfall noise comes from the mechanical excitation of light structures by rain drops. In the past, lots of work has been done on this subject, ranging from empirical research to highly sophisticated theoretical prediction methods. A standardized laboratory measurement procedure also exists using artificial raindrops produced by a water tank. Because the infrastructure for this laboratory measurement is not always available, a ‘dry’ prediction method is developed in this paper. This method combines the realistic results of a simple ‘dry’ measurement based on reciprocity with the ease of a prediction method. Validation measurements were done for a single glass pane and a polycarbonate sheet.

1. Prediction of rainfall noise

Ballagh [1] developed one of the first prediction methods for the evaluation of rain noise problems:

$$L_p = F_R + F_C + F_A \tag{1}$$

According to (1) the sound pressure level in a room generated by rainfall excitation of the roof structure basically depends on three main factors: a first factor F_R comprising the characteristics of rain excitation, a second factor F_C depending on the vibrational response of the excited structure and a last factor F_A based on the properties of the receiving room.

According to [1] the predicted sound pressure level varies with 15.4 times the logarithm of the rainfall rate R:

$$F_R = 15.4 \log\left(\frac{R}{R_0}\right) \tag{2}$$

where $R_0 = 1\ mm/h$. The coefficient 15.4 was deduced by Ballagh from the Marshall-Palmer model [2] for the distribution of raindrops in natural rain. It can be compared to the empirical coefficient 17.3 measured by Dubout [3] for natural rainfall on steel roofs.

In case of a diffuse sound field in the receiving room, F_A defines the conversion of the radiated sound intensity level into a sound pressure level as function of the radiating surface area and the total absorption in the receiving room.

It is the factor F_C that is the most difficult to comprehend. Ballagh used a very simplified model for F_C based on the point impedance of an infinite plate in the frequency region below the coincidence frequency. Using this model, the only significant property of the plate is its surface mass. Validation measurements however showed that several other properties of the structure are in fact important, such as the damping of the plate and its mounting conditions and that these properties should also be included into the prediction method.

Akamatsu et al. [4] took the above recommendations into consideration and analyzed the radiated sound intensity from a baffled finite plate subjected to a stream of impulses random in space and time. He assumed the damping of the plate to be light enough and the impulsive excitation of ‘white noise’ nature. Because of the rainfall excitation, he also assumed the stream of impulses to be stationary and uncorrelated. Combining probability theory and modal analysis then resulted in the following equation for the sound intensity level radiated by a structure when excited by rainfall:

$$\begin{aligned} L_I = {} & 10\log\left(\frac{R}{R_0}\right) + 20\log\left(\frac{V_T}{V_{T0}}\right) \\ & + 30\log\left(\frac{D}{D_0}\right) + 10\log\left(\frac{K}{K_0}\right) - 8.5 \end{aligned} \tag{3}$$

where V_T is the terminal velocity of the raindrops, D the diameter, $V_{T0} = 1\ m/s$ and $D_0 = 1\ mm$.

The above formula is again easy to use, except for the complicated term $10\log\left(\frac{K}{K_0}\right)$. K describes a measure for the radiated sound intensity of the finite structure when excited by means of a unit point force. In [4] K is written as:

$$K = \sum_n \frac{\int_\Omega \phi_n^2(\xi)d\xi}{m_n^2 \eta_n} \int_{|\vec{k}|^2 \le k^2} \frac{\left|\Phi_n(\vec{k})\right|^2}{\sqrt{k^2 - |\vec{k}|^2}} d\vec{k} \tag{4}$$

with m_n, η_n and ϕ_n the modal mass, the internal damping and the mode function of the plate respectively, ξ is the space variable and $\vec{k}$ the wave vector on the plate. Equation (4) thus includes not only the mass properties of the structure, but also the damping and the modal characteristics, as recommended before. Theoretically the factor K can be computed numerically, but this is labor-intensive and cursed with inaccuracies.

2. Standardized laboratory measurement

The international standard ISO 140-18:2006 [5] describes a laboratory method for measurement of the sound intensity generated by rainfall on building elements such as roofs, roof/ceiling systems and skylights. The results obtained can be used for assessing the noise to be produced by rainfall on a given building element in the room or space below. But the results can also be used to relatively compare rainfall sound insulation capabilities of building elements and to design building elements with appropriate rainfall sound insulation properties.

ISO 140-18:2006 is based on measurements with artificial raindrops under controlled conditions using a water tank in a laboratory test facility in which flanking sound transmission is suppressed. The standard rainfall type used for comparison between products is denoted as 'heavy rain', of which the characteristic parameters are given in Table 1.

Rainfall type	Rainfall rate [mm/h]	Median drop diameter [mm]	Terminal velocity [m/s]
Heavy	40	5.0	7.0

Table 1: Characteristic parameters for artificial raindrop generation

The water tank should be built in order to generate raindrops with approximitively the above characteristics. The radiated sound intensity levels can be measured directly or can be obtained by converting measured sound pressure levels.

From the measurement results, the factor K can be calculated for a given construction and afterwards used for the prediction of noise from different rain types.

3. Building acoustic reciprocity

The theorem of reciprocity exists in many forms and is therefore applied in a wide range of scientific fields. In general, reciprocity is stated as

$$\left(\frac{v_1}{F_2}\right)_{F_1=0} = \left(\frac{v_2}{F_1}\right)_{F_2=0} \tag{5}$$

If a force F_1 that acts at a point P_1 produces a velocity v_2 at a point P_2, then this same force $F_2 = F_1$ acting at a point P_2 will produce a velocity $v_1 = v_2$ at the point P_1. In other words, the ratio of the exciting force to the observed velocity remains the same if the excitation and observation points are interchanged, provided that the direction in which the force acts in each case is the same as that in which the velocity is measured in the other case. As a second condition the dynamic system should be linear.

This principle of reciprocity can be extended towards building acoustics. When a vibrational point force F is applied to a certain point of a structure, this will cause the structure to vibrate. As a consequence the structure will radiate sound into the adjacent reverberant space. The radiated sound intensity I is proportional to the produced mean square velocity averaged over the radiating surface, v, and therefore also to the square of the applied force:

$$I = \alpha F^2 \tag{6}$$

α can be regarded as a measure for the radiated sound intensity of a structure excited by a unit point force (same as K). The response of a structure to a reverberant sound field, defined by the sound pressure p, can on the other hand be measured by β:

$$v^2 = \beta p^2 \tag{7}$$

Based on equation (5) and following [6] the very interesting reciprocity relation can be derived:

$$\frac{\beta}{\alpha} = \frac{4\pi}{\rho c k^2} \tag{8}$$

with ρ the density of air, c the speed of sound and k the wave number. The response of a structure to excitation by a diffuse sound field can thus be deduced from the sound radiation of this structure when excited by a unit point force and vice versa. The reciprocity relation (8) is only dependent on the frequency through k and independent of the size, shape, damping or any other property of the structure. It is also not limited by structural modes. This means that if the condition of a diffuse field is satisfied, the above formula can be used for the simplification of a great number of problems.

4. Reciprocal 'dry' measurement method

When looking at equation (6) it can be seen that the factor K, described in the first paragraph (equation (4)), restrains the same information as α. Both factors describe the radiation characteristics of a finite plate excited by a unit point force. The difference is that K is obtained through a difficult numerical procedure, while α can be measured in a very simple, reciprocal way, by exciting the structure with a diffuse sound field and measuring the resulting velocities by means of an accelerometer or a laser Doppler vibrometer. From this measurement the vibration response β can be deduced and thus also α (or K) through the reciprocity relation (8).

When a flexible structure is excited by a raindrop, the force pulse is lengthed. It is therefore necessary to also take into account the flow impedance of the raindrop. Using a linearized model for the flow phase, it was found in [7] that the flow impedance can be written as

$$Z_f = \frac{\rho_w \pi D^2 V_T}{4} \tag{9}$$

with ρ_w the density of water.

Following the above reasoning, equation (3) can be rewritten as

$$L_I = 10\log\left(\frac{R}{R_0}\right) + 20\log\left(\frac{V_T}{V_{T0}}\right) + 30\log\left(\frac{D}{D_0}\right) + 10\log\left(\alpha\cdot\frac{\alpha}{\alpha+Z_f}\right) - 8.5 \qquad (10)$$

Filling in the characteristic parameters given in Table 1 for the artificial raindrops used in the ISO 140-18 measurement method into equation (10) results in:

$$L_I = 10\log\left(\alpha\cdot\frac{\alpha}{\alpha+0.137}\right) + 45.4 \qquad (11)$$

When α is measured reciprocally, equation (11) should give a good approximation of the measurement result obtained with the ISO 140-18 method.

The only thing that was not accounted for in the above derivation was the rainfall excitation spectrum. In [8] the power spectrum was calculated for rain with the characteristic parameters as given in Table 1. This spectrum is shown in Figure 1.

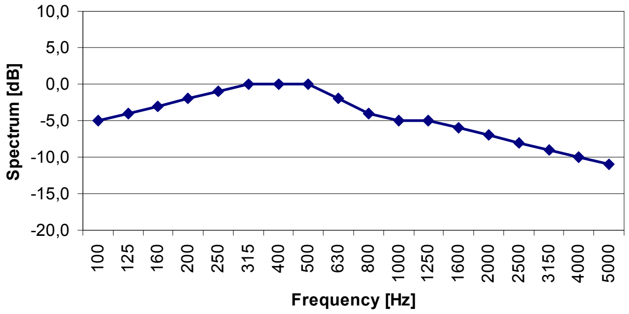

Figure 1: Rainfall excitation spectrum for selected rain

After calculating the values of L_I according to equation (11), the values should be weighted according the spectrum given in Figure 1.

5. Validation measurements

Validation is necessary to demonstrate the applicability of the method described above. For this purpose, two different test samples were measured in the Building Acoustic Laboratory of Cauberg-Huygen Raadgevende Ingenieurs B.V. in Zwolle.

The first test sample was a single glass pane with a thickness of 6 mm and an area of 1250 mm x 1500 mm. This sample equals the reference test sample described in the ISO 140-18 standard for use of a reproducibility check. Annex B of ISO 140-18 [5] lists the measured sound intensity levels radiated by this reference sample when excited with artificial ‘Heavy’ rain as explained in paragraph 2. The results of equation (11) after executing a ‘dry’ measurement of the sample should approach these values.

The test sample was installed in the measurement opening of the laboratory and excited with a diffuse sound field of which the sound pressure level was recorded. The response of the sample, in terms of velocity levels, was measured in a grid of 30 measurement points comprising one quarter of the plate.

For these measurements, the velocity levels were measured by means of a laser Doppler vibrometer. This way, there is no mass added to the sample. The measurements can also be done by using accelerometers, but these should be as light as possible.

From the measured velocity levels, the surface averaged mean square velocity was calculated and compared to the pressure in the exciting room. This resulted in the response factor β according to equation (7). From equation (8) α could then be calculated and the radiated sound intensity level from equation (11) taking into account the surface area of the sample and the spectral weighting according to the spectrum in Figure 1. The results for the A-weighted sound intensity levels $L_{I;A}$ are given in Table 2. In the last column of Table 2 the measurement results for the reference sample according to Annex B of ISO 140-18 are also given.

freq. [Hz]	$L_{I;A;meas}$ [dB(A)]	$L_{I;A;ISO}$ [dB(A)]
125	34	35
250	43	43
500	48	49
1000	45	49
2000	49	54
4000	50	53

Table 2: Validation results for a 6 mm single glass pane

The global result equals $L_{I;A} = 58\,dB(A)$ for the ISO 140-18 measurement and $L_{I;A} = 55\,dB(A)$ for the reciprocal measurement combined with the above described model following equation (11). In Figure 2 the measurement results are shown graphically.

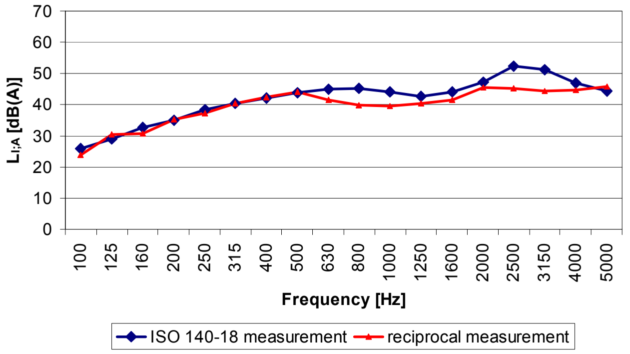

Figure 2: Measurement results for a 6 mm single glass pane

In november 2004 measurements were done, according to the ISO 140-18 standard, by the Building Research Establishment in Watford, UK, of the radiated rain noise of roof glazing, polycarbonate roofing and ETFE roofng. The second test sample used for this paper is the same polycarbonate sheet as in the report of the BRE [9]. It regards a 25 mm thick 5-layered polycarbonate sheet with an area of 1250 mm x 1500 mm.

The same validation procedure was followed as for the 6 mm single glass pane. In Table 3 the results are compared to the results as listed in [9].

freq. [Hz]	$L_{I;A;meas}$ [dB(A)]	$L_{I;A;ISO}$ [dB(A)]
125	44	44
250	53	52
500	57	57
1000	56	58
2000	59	59
4000	62	60

Table 3: Validation results for a 25 mm polycarbonate sheet

The global result equals $L_{I;A} = 65\, dB(A)$ for both the ISO 140-18 measurement results as well as for the reciprocal measurement combined with the above described model following equation (11). In Figure 3 the measurement results are shown graphically.

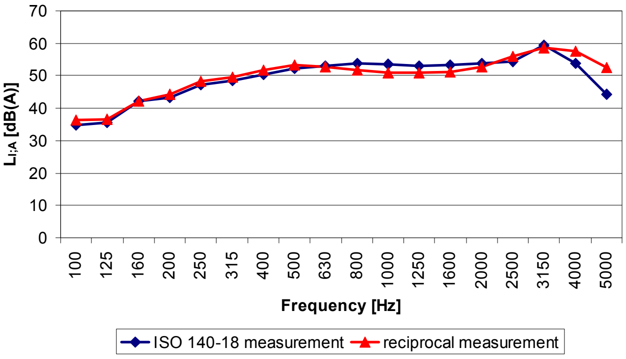

Figure 3: Measurement results for a 25 mm polycarbonate sheet

For both the single glass pane and the polycarbonate sheet, the validation measurements give reasonably good results. Around 1000 Hz however a discrepancy is shown, both in Figure 2 and Figure 3. The predicted results exhibit a slight dip compared to the ISO 140-18 measurements. This can probably be explained by looking at Figure 1. The rainfall excitation spectrum used also shows this non-smooth dip around 1000 Hz.

In Figure 4 the results are shown of the airborne sound insulation measurements for both the 6 mm glass pane and the 25 mm polycarbonate sheet. As expected the coincidence dips in Figure 4 appear as peaks in the predicted radiated sound intensity due to rainfall excitation in Figure 2 and Figure 3.

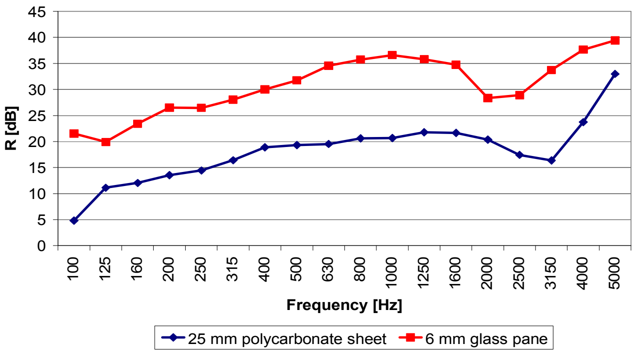

Figure 4: Airborne sound insulation of the glass pane and the polycarbonate sheet

7. Conclusion

A ‘dry’ prediction method is developed to determine the rainfall noise radiated by a structure. This method combines the realistic results of a simple ‘dry’ measurement based on reciprocity with the ease of a prediction method. Validation measurements are accomplished for a single glass pane and a polycarbonate sheet. The results are in reasonably good agreement with measurement results following the ISO 140-18 standard using artificial rain.

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