Sound Generation and Propagation with the Nonlinear EIF–Approach

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Introduction

Aerodynamic sound generation can generally be analysed by solving numerically the Navier–Stokes–Equations. However, large disparities in the length scales and power efficiency of the hydrodynamic and acoustical processes make this direct approach impracticable due to high calculation costs [1]. An interesting alternative is a so called “Expansion about Incompressible Flows” (EIF) approach, which is a hybrid methodology avoiding the disparities by splitting the solution domain into an incompressible viscous flow and a non viscous compressible acoustics. The details of the method can be found in [3], [4], and [5].

In the present work the EIF approach is implemented in the Finite Volume Method. The basics of the formulation are given, and some results for a validated numerical example from the ALLESIA Report [2] are presented.

Governing Equations

Assuming that an acoustic wave can be described as a fluctuation about the mean incompressible viscous flow one can split the governing equations into the incompressible nonlinear part yielding the mean pressure P and velocity U fields and the nonlinear equations for small acoustic quantities ρ', p' and u_i'

$$u_i = U_i + u_i', \quad p = P + p', \quad \rho = \rho_0 + \rho'. \tag{1}$$

With the assumption, that only the first part involves losses, whereas the second part is assumed to be isentropic, the derivative of the pressure with respect to time is given by

$$\frac{\partial p}{\partial t} = \left(\frac{\partial p}{\partial \rho}\right)_s \frac{\partial \rho}{\partial t} = c^2 \frac{\partial \rho}{\partial t}. \tag{2}$$

Inserting the equation (1) into equation (2) and using the splitting procedure described above, one obtains the nonlinear system of equations governing the propagation of the aerodynamically induced acoustic waves

$$\frac{\partial(\rho')}{\partial t} + \frac{\partial(f_i)}{\partial x_i} = 0, \tag{3}$$

$$\frac{\partial(f_i)}{\partial t} + \frac{\partial(f_i \cdot u_j + \rho_0 \cdot U_i \cdot u_j + p' \cdot \delta_{ij})}{\partial x_j} = 0 \tag{4}$$

$$\frac{\partial(p')}{\partial t} + c^2 \cdot \frac{\partial(f_i)}{\partial x_i} = -\frac{\partial(P)}{\partial t} \tag{5}$$

with

$$f_i = (\rho_o + \rho') \cdot u_i' + \rho' \cdot U_i \ ; \ c^2 = \frac{\gamma p}{\rho}. \tag{6}$$

Here c denotes the speed of sound and γ is the ratio of the specific heats. It should be noted, that the only source term is present in the pressure equation (5). The methodology is similar to the perturbation method with the only difference, that one obtains a nonlinear system of equations and not a sequence of linear boundary value problems. Although the description of the acoustic problem is nonlinear, it is still confined to “small” nonlinearities, since the acoustic wave does not influence the basic flow.

Finite Volume Formulation

In order to implement the EIF into the Finite Volume Methodology, the equations (3) to (5) have to be transferred into a set of integral equations, such that

$$\frac{\partial}{\partial t}\int_\Omega \rho' \, d\Omega + \int_S (\rho_0 \mathrm{v}' + \rho' \mathrm{V}) \, n \, \mathrm{d}S = 0 \tag{7}$$

$$\frac{\partial}{\partial t}\int_\Omega f_i d\Omega + \int_S f_i (U_j + u_j') \cdot \vec{n} dS + \ldots \\ \ldots + \int_S (\rho_0 U_i) \vec{u}' \cdot \vec{n} dS + \int_S p' \cdot \vec{e} \cdot \vec{n} dS = 0 \tag{8}$$

$$\frac{\partial}{\partial t}\int_\Omega p' d\Omega + \int_S c^2 \vec{f} \cdot \vec{n} dS + \frac{\partial}{\partial t}\int_\Omega P d\Omega = 0, \tag{9}$$

where the integration is to be carried out for each cell of the discretized fluid domain. The solution of the discretized boundary value problem can then be computed using a time-marching scheme in which various time and spatial integration schemes can be applied. In the present implementation the boundary integrals are solved by a linear interpolation and the volume integrals are computed using the mid values of each cell. For the time integration the Runge Kutta scheme of second and fourth order as well as explicit and implicit Euler formulations were applied.

Implementation

The implemented code is based on a computer program named NS3D developed by the Technical University of Munich. The CFD code provides the fluctuating pressure and velocities of the viscous and incompressible flow which excites the acoustic waves.

First, the code was implemented and tested for various 2D examples. Then it was extended to 3D-problems. Basic test cases were confined to simulations of the noise induced by the flow passing through elementary sources (monopole, dipole or quadruple). To validate the accuracy and efficiency of the developed computer code, the computations were performed for some validated computational examples (see ALLESIA Report [2]). One of them refers to a cross flow around of a circular cylinder with the following flow specifications:

Strouhal frequency:	265 Hz
Strouhal number:	0.201
Dynamic viscosity:	1.778e-5 Ns/m²
Density of the fluid:	1.18 kg/m³
Sound velocity:	163.5 m/s

Some results are presented in the Figures 1 and 2. Figure 1 shows the acoustic pressure induced by the stable vortex street at the time of 1.5 periods.

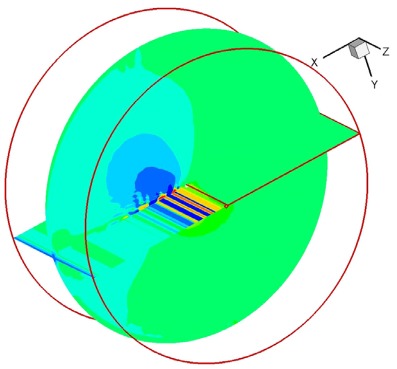

Figure 1: 3D-Cylinder in cross flow: acoustic pressure after 1.5 periods

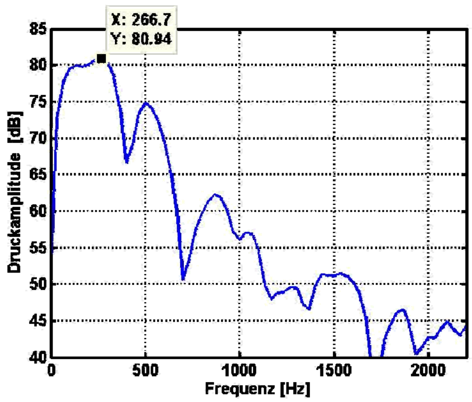

Figure 2: Spectrum of the acoustic pressure at a point located 1 m downward the cylinder at the crossing line of the vertical and horizontal symmetry planes

The spectrum of the acoustic pressure at a point located 1 m downward the cylinder at a crossing line of the vertical and horizontal symmetry planes is presented in Figure 2. The maximum acoustic pressure level of 81 dB occurs at the frequency of 267Hz. The result of the ALESSIA report (see Figure 3) compare very well with the EIF-results and confirm the accuracy of the computations.

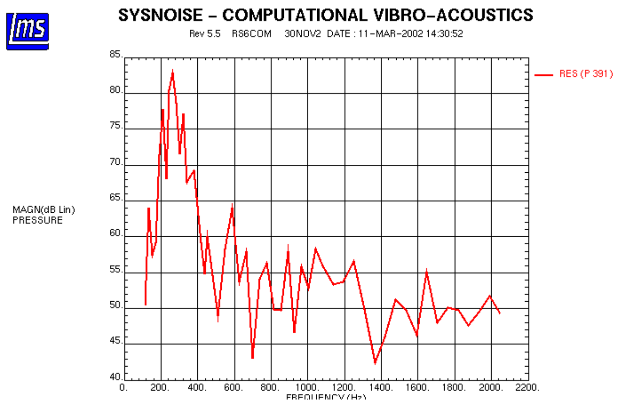

Figure 3: ALESSIA Validation Report frequency spectrum

Conclusions

The EIF Method proved to be an interesting compromise between the (yet) impracticable direct numerical solution of the aero-acoustic problems and the methods based on acoustic analogies. It allows for the simulation of the propagation of nonlinear acoustic waves. However, the limits of the validity of the underlying expansion are still to be investigated.

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