Mode coupling in the sound generation in wind instruments

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Abstract

The Fourier components generated by the non-linear excitation process are coupled to the acoustic modes of the pipe (and to those of the vocal tract). At low amplitudes the fundamental determines the frequency of the played note; at higher amplitudes higher modes are active, sometimes leading to unexpected occurrences. For example, for certain changes in fingerings on a woodwind the playing frequency may go up while the first peak of the input impedance goes down. This can be observed for fork fingerings. Similar effects can be expected in flaring horns and bends, where resonances in transverse direction can occur. In this paper some effects on woodwinds are studied; blowing observations are verified by time domain simulations.

Introduction

In a wind instrument a (nearly constant) pressure source supplies air through a channel (fixed, or variable between lips or reeds). The difference between the pressure in the mouth and the resonating air in the top of the instrument controls the channel dimensions or the direction of the flow, defining the magnitude of the flow entering the pipe. When in resonance, the air column controls the flow it needs for maintaining its oscillations. Since the process is non-linear, except at very low amplitudes, the components of the input spectrum of the pipe need to be close to harmonic for a stable oscillation. Only at very low amplitudes, the lowest resonance peak in the spectrum defines the playing frequency. At higher amplitudes, the higher modes have an important role in stabilizing the playing ("mode locking" [1, 2]). If the modes are harmonically related, the playing frequency remains the same when the amplitude increases. However, if they are not, they can acquire a dominating influence and shift the playing frequency: the system seeks for an oscillation pattern where the energy output is maximized. In the case of the so-called fork fingerings, pipe pieces are formed which cause resonances at higher modes not harmonic with the fundamental. In extreme cases shifts of a semitone are possible. Understanding the phenomenon quantitatively is not easy. Even if the input impedance can be obtained accurately by measurements or by calculations, influences of mouth resonances, reed impedance, flow displaced by the moving reed and transverse flow in bends and quickly flaring horns are badly defined.

In the present study, a continuation of [3], some cases on woodwind instruments amenable to experiments as well as calculations are investigated and serve to illustrate the magnitude of the effects. Experiments and calculations appear to correspond reasonably well.

Table 1. Effects of fingering changes on the frequency on some instruments

Instrument	Fingering no.	Left hand 1 2 3 4	Right hand 2 3 4 5	Possible cents change
Recorder	1	● ○ ○ ○	○ ○ ○ ○	
	2	● ○ ● ○	○ ○ ○ ○	– 80
	3	● ○ ● ○	● ● ○ ○	– 100
Clarinet	1	○ ○ ○ ○	○ ○ ○ □ □	
	2	○ ○ ● ○	● ● ○ □ □	– 10 to + 5
Baroque oboe	1	○ ● ○	○ ○ ○ □	
	2	○ ● ○	● ● ○ □	– 20 to + 40

Observations

In Table 1 some examples of fork fingerings on the woodwinds investigated are shown. An open hole is indicated by ○, a closed hole by ● and an open hole with a key hanging above it by □. The instruments were blown; the frequency of the stationary state was determined using a recording program with a Fourier transform facility. On a recorder, successively closing holes (leaving one open, forming a fork fingering) effectively lengthens the resonating air column, which lowers the playing frequency; see Table 1, fingerings 1 to 3 (100 cents = 1 semitone = 6%).

Performing this action on a clarinet, the frequency lowers only at low levels, for higher levels the frequency goes up. On a baroque oboe, the effect is stronger; the note may go down 20 cent or go up 40 cent, depending on embouchure and blowing pressure. Similar effects are found on other instruments; they are especially strong on a bassoon. Musicians are acquainted with the effects and use them for humouring and stabilizing weak notes.

The behaviour of the system can be studied using the harmonic balance method, but this is not always stable [4]. We have chosen for direct time domain simulation (TDS) a procedure pioneered by Schumacher [5], see also [6] and [7].

Time domain simulation

Procedure

The present study concerns both single and double reed instruments. Blowing experiments with single-reed mouthpieces build for bassoon and oboe have shown that there is hardly a difference between the two types of blowing, provided the single reed mouthpiece equals the bore of the double reed configuration as closely as possible. So it was assumed that single and double reed excitation can be described by the same mathematical expressions [8]. The well-documented equation of reed motion in the time domain [9, 10, 11, 12] gives a relationship between its displacement from equilibrium, y, and the pressure difference across the reed, $p_d = p - p_m$, where p is the pressure in the top of the tube and p_m in the mouth. The reed has a stiffness k_r, a mass m_r, and a damping G_r. When no forces are acting on the reed, H is the rest opening of the slit.

We extend this equation with a term T_{NL}, allowing for the increase in reed stiffness when the reed nears the lay

$$T_{NL}\left(m_r y'' + G_r y' + k_r y\right) = p_d \tag{1}$$

where the dash denotes differentiation with respect to time, and the term

$$T_{NL} = 1 + H_S/(y+H) \tag{2}$$

is a fit formula taking into account the increasing stiffness when the reed nears the lay. This formula is inspired by measurements of Ollivier at al [13], plotted in Figure 1. In the same figure, the fit formula for the non-linear stiffness function is plotted.

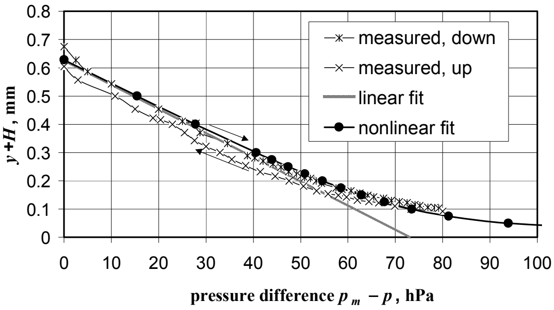

Figure 1. Reed opening versus pressure difference

The flow U entering the instrument is the sum of the well-known Bernoulli-flow and the flow induced by the moving reed (its area diminishing when nearing the lay and assumed to be equal to S_r/T_{NL})

$$U = -b(y+H)\,\mathrm{sgn}(p_d)\sqrt{2\,|\,p_d\,|\,/\,\rho} - y' S_r / T_{NL} \tag{3}$$

where b = the width of the slit (or reed) and ρ = air density. The pressure in the pipe is obtained from a convolution of Green's function $G(t)$ and the volume flow $U(t)$, where $G(t)$ is the impulse response function, the inverse Fourier transform of the input impedance. For a faster convergence we use the reflection function, the inverse Fourier transform of the reflection coefficient, $R=(Z–Z_c)/(Z–Z_c)$, where Z=input impedance and Z_c=characteristic impedance [5, 6, 7, 14]. The time step $\Delta t = 1/2f_{\max}$, where $f_{\max}$ is the maximum frequency of the impedance sampling, where this maximum is chosen such that the imaginary part is zero. For the time-derivatives

$$\begin{aligned} y' &= \left[y - y(t-\Delta t)\right]/\Delta t \\ y'' &= \left[y - 2y(t-\Delta t) + y(t-2\Delta t)\right]/\Delta t^2 \end{aligned} \tag{4}$$

The impedance was calculated with the transmission line method using the dimensions of the instrument, obtained from accurate measurements – if necessary after taking it apart. An example is given in Figure 2 for the two fingerings of the baroque oboe. It is obvious that it is difficult to predict the behaviour for these two fingerings from this spectrum.

Results

Clarinet

The clarinet investigated was a Selmer Centered Tone no. P8182. The impedance spectrum was calculated with the transmission line method. The maximum frequency was chosen just below 11 kHz, and it was sampled in 4096 steps. Chosen parameters: $k_r = 6\times10^6$ Pa/m, $m_r = 0.018$ Pa.s^2/m, $G_r = 51$ Pa.s/m, $H = 1$ mm [15]. The parameter chosen for best fit from the plot in Figure 1 was $H_s = 2$ μm. For the calculation with reed motion flow $S_r = 60$ mm^2, without flow $S_r = 0$. In the latter case the reed motion flow is equal to zero.

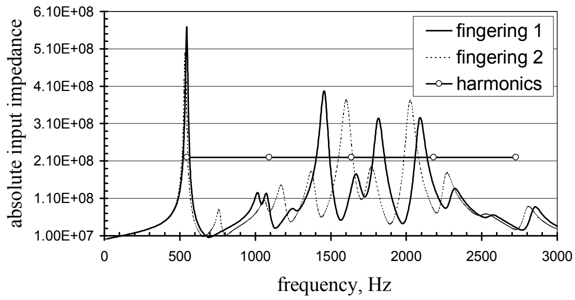

Figure 2. Input impedance for two fingerings of the baroque oboe

Figure 3 shows the results for the stationary frequency as a function of the mouth (blowing) pressure. The positions of the first peaks of the input impedances are also indicated. The frequency of the first impedance peak is lower for fingering 2 than for fingering 1, and this is also the case for the frequencies of the playing frequencies at low amplitudes. At these low levels the pressure in the pipe is close to sinusoidal and the resonance is based on the first resonance peak. Including the reed motion lowers the frequencies, but the difference between the two fingerings remains the same. At increasing sound level, the pressure shape becomes close to square. Then higher modes lower the note some 5 to 10 cents, which nicely corresponds to the blowing observations (Table 1). As can be seen from Figure 3, the played target note, F4 = 349 Hz, is predicted satisfactorily by the simulations. The influence of the reed motion is substantial: it amounts to about half of a semitone. It explains the flexibility the player has of adjusting the frequency to his needs by changing the embouchure.

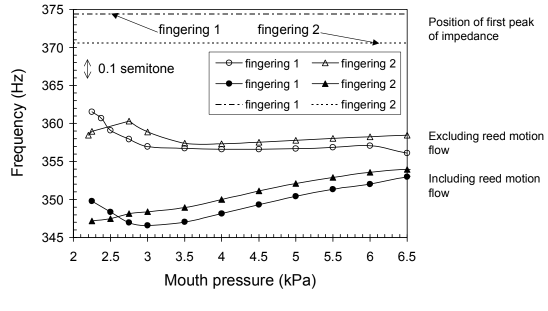

Figure 3. Playing frequency as a function of mouth pressure for two fingerings of the clarinet for the note (sounding) F4=349 Hz

Baroque oboe

A similar investigation was done for a Schermer baroque oboe. The impedance calculated was sampled up to the first peak below 10 kHz, in 5000 samples. Parameters were: $k_r = 12\times10^6$ Pa/m, $m_r = 0.036$ Pa.s^2/m, $G_r = 100$ Pa.s/m, $H = 0.4$ mm, $H_s = 2$ μm, $S_r = 20$ mm^2. The calculated stationary frequencies as a function of blowing pressure for the two fingerings are shown in Figure 4. Above a certain maximum pressure the oscillations jumped into an overtone, at a pressure different for the two fingerings. The results were verified by actual blowing the instrument. The oboe is conventionally blown with a double reed; in the present investigations it was blown by a specially constructed single-reed mouthpiece, in which a small microphone could be inserted to study the pressures in the top of the instrument. Pressures were found to correspond with those calculated. Listening tests did not reveal major differences between single and the double reed blowing. In the actual blowing tests, overblowing occurred at about the same pressures as predicted by the calculations. The differences between the two fingerings were the same. It was found that the oscillations stopped approximately at the same pressures as the calculations predicted, fingering 2 acting up to higher pressures than fingering 1. As can be seen from Figure 4, the blowing frequency is 494 Hz (B4) and not far from the calculated values. Note that musicians prefer fingering 2 for playing B4 (written C5).

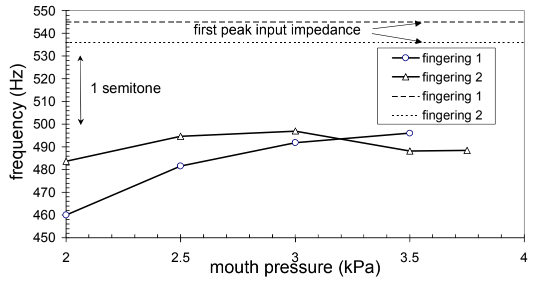

Figure 4. Playing frequency (from simulation) for two fingerings of the baroque oboe. Target: 494 Hz

Discussion

The excitation of wind instruments is a complex interaction between the valve (for example the reed) and the air pressures around it, in particular those in the upper part of the pipe. Exactly describing all interactions is a complicated task. In this paper only the interaction with the air in the tube is considered. Its input impedance was obtained from calculations. Theoretical predictions and experiments correspond satisfactorily.

Note that beside the input impedance other phenomena have to be considered. For example, resonances in the mouth can serve as support for higher notes, as has been shown for playing high notes on the saxophone [16]. Another influence may be a bend in an air column, which causes a local change in the inertance. For low frequencies the inertance decreases, which can be compensated by a local diameter reduction [11]. However, in a recent study it was found that for bends larger than a quarter wavelength the inertance change may diminish and even change sign [17]. In real instruments this situation will be rare. It also corresponds to experiences of a flute builder who constructed a flute with a bend head (Figure 5), to ease the somewhat awkward handling of the instrument; he did not experience changes in the blowing properties of the flute (the bend was not too long) [18]. It is also worthwhile to look into the impedance of quickly flaring horns, since transverse flow can modify the local inertance similar to that of the bend; changes may also be frequency-dependent [19].

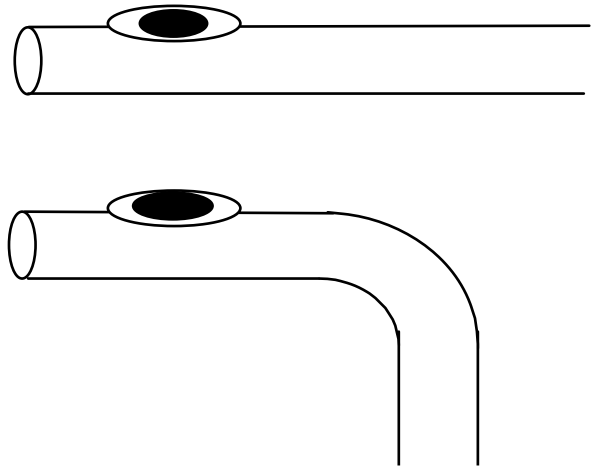

Figure 5. Normal flute (above) and flute with bend head (below)

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