On Solving the Tyre-Road Contact Problem at High Frequencies.

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Introduction

The typical tyre/road noise spectrum gradually raises up to 1 kHz, after which it decreases gradually. To study the effect of tyre vibrations on tyre/road noise, it is therefore essential to study vibrations at time scales less than a millisecond. The associated length scales are in the order of the treadblock size which poses a major problem in numerically simulating tyre/road noise; to correctly capture vibrations at these length scales, the numerical (finite element) models contain many degrees of freedom and computation times are large.

At the Structural Dynamics and Acoustics group of the University of Twente, we study tyre/road noise. Our goal is to perform a transient simulation of a rolling tyre. Based on the simulated vibrations, we then use boundary element software to calculate the radiated noise. In this two step approach, the transient simulation of the tyre in contact with the road is the most time consuming part of the algorithm.

In the field of elastohydrodynamic lubrication, similar contact problems have been solved by so-called multigrid techniques, see [1]. These numerical techniques can speed up calculation times tremendously. It is our intention to use these techniques for the tyre/road contact problem. However, the multigrid technique that we intend to use requires an iterative solution scheme that effectively reduces spatial, high frequency error components which was not available. We propose such an algorithm. It solves the dynamic equations and, while solving, satisfies exactly the contact condition (the condition that states that there is no penetration of the tyre into the road). As a result, there is no need for contact elements and no additional parameters are required to stabilize the solution process. In this paper the contact algorithm is explained for a finite element discretization.

Contact algorithm

A general finite element model is assumed to describe the tyre. We assume the road to be non-deformable hence only the equations of motion of the tyre are required. The discrete set of equations thus reads:

$$\mathbf{M}\ddot{u} + \mathbf{C}\dot{u} + \mathbf{K}u = \mathbf{f}, \tag{1}$$

where u are the displacements, $\dot{}$ denotes differentiation in time and f is the vector of external forces exerted on the tyre. Note that these equations are equations for the unknowns u but each row of the system is a simple relation for the external load f if u is given. When the tyre is in contact with the road surface, the displacement u is known and f is equal to the contact force required to put that specific node of the tyre on the road. We will exploit this notion in the contact algorithm.

The contact algorithm is actually an iterative solver for the equations of motion 1. We use Gauss Seidel relaxation but other relaxation schemes may be used as well. Then, while iteratively solving the equations of motion for each unknown u (we update u or any small number of unknowns) we correct u that would yield a displacement of the tyre below the road surface. Hence, if the new u (the iterate or update) does not satisfy the contact condition, we correct for it. The contact condition is a geometric relation, $g(u) \geq 0$. It states that the gap between the tyre and road surface (which may include roughness) should always be positive. Hence if the iterate would yield $g(u) < 0$ (a node of the tyre is below the road surface) we correct u such that $g(u) = 0$ (we put the node back on the road surface). After this correction, one needs to check whether the shear stresses are sufficiently low such that the node will not slip. If there is no slip, this corrected position is correct. Otherwise we solve for a new position of the node, based on solving the system of equations $g(u) = 0$ and, for a Coulombs friction model, $\tau = \mu p_n$, where τ is the shear stress, p_n is the normal stress and μ is the friction coefficient. Note that $\tau = \mu p_n$ should obviously be reformulated in terms of the unknowns u. The solution of this small set of equations may be obtained directly but can also be found iteratively. For nodes which were already in contact, we need to check if the updated position (the position for the node after the iteration) yields a negative normal force (or any small value to include stick-snap phenomena). In that case, the node will come out of contact and it is treated as a node which is not in contact.

Results

Results have been obtained by the contact algorithm for a simple bouncing ring. Snapshots of the solution at various time steps are shown in Figure 1. Similar simulations have been performed, which are not presented here, and compared to commercial software (e.g. ABAQUS). We have observed only very small differences.

Tyre/road noise

Once the vibrations of the tyre have been simulated, the vibrations are subsequently used as input for a boundary element simulation. This is done by transferring the rotating finite element mesh to a stationary boundary element mesh on which the calculated velocity spectrum can then be prescribed. Note that near the contact, there are nodes on the finite element mesh which come in and out of contact. From a sound radiation point

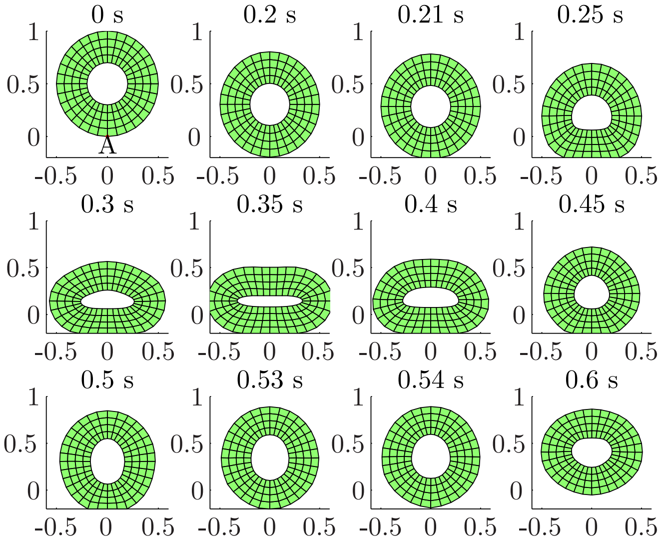

Figure 1: Bouncing ring on a rigid surface at different time steps

of view, this poses a problem, as these nodes will not emit any noise when in contact and only radiate noise if not in contact. In the boundary element simulation this is not incorporated as the velocity field is assumed to be harmonic. To mimic this behavior, we monitor in time a small volume just in front of the contact region. The Fourier transform of this volume changing in time is then used to calculate the velocity of the element on the boundary element mesh which closes off this region. In this way we create an equivalent volume source, see Figure 2.

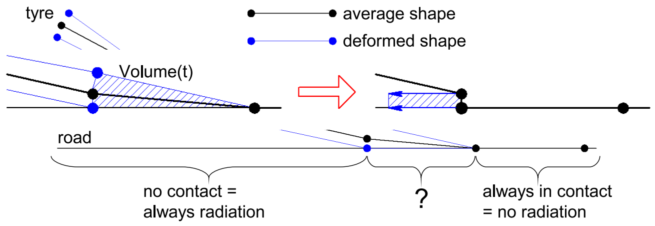

Figure 2: The equivalent volume source.

As an illustration of our goals for the 3D finite element model, we have performed a transient analysis using the same contact algorithm for a simplified 2D tyre model based on finite differences, see [2]. We have transferred the 2D results onto a 3D boundary element mesh and have calculated the emitted noise shown in Figure 3.

Conclusion

A contact algorithm has been developed (and validated with commercial finite element software) for general finite element models. The algorithm can be used to model the tyre/road contact problem. To reduce calculation times, we have started the implementation of the contact algorithm in a multigrid solver. The next step is to validate our results numerically and experimentally and to calculate the associated noise.

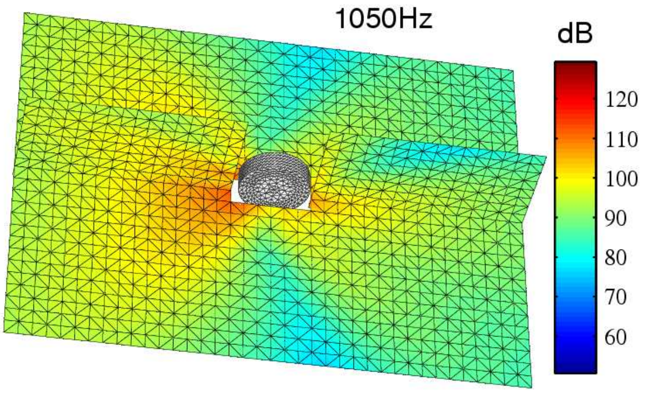

Figure 3: Pressure contours near the tyre at 1050 Hz.

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