# Nearfield noise source localisation with constant directivity arrays : a comparison - Application to tram noise

M.A. Pallas<sup>1</sup>, R. Perrier<sup>2</sup>

<sup>1</sup> INRETS-LTE, 25 avenue F. Mitterrand, case 24, 69675 Bron cedex, France, Email: pallas@inrets.fr <sup>2</sup> now at : INRIA, Saint-Ismier, France

# Introduction

Noise pollution due to transport, which affects residents along roads and railways, is of main concern within urban and suburban areas. The diagnostic and prediction of transportation noise requires knowledge of vehicle emission in real conditions. Simple point or linear representation of vehicles may sometimes be sufficient. But a more detailed description, involving a set of noise sources associated with the main noise emitting areas on the vehicle, is required when performing a fine analysis, studying noise source behaviour, or developing an emission model better adapted to a dynamic traffic description. This paper deals with the localisation of noise sources on moving road and rail vehicles using a nearfield microphone array; in these conditions sound waves differ from plane waves, and the vehicle motion induces Doppler effect on the received signals.

Beamforming is the basis method in array processing [1]; in its principle it is adapted to the localisation of stationary point sources. Regarded as a spatial filter, delay-and-sum beamforming gives out the source signal from a selected direction  $\theta_0$ . The output signal of a line array, with (2N + 1) microphones equispaced with  $\delta$  and steered in the direction  $\theta_0$ , is given for plane waves by :

$$S(t,\theta_0) = \sum_{n=-N}^{N} w_n x_n (t + n \frac{dsin\theta_0}{c})$$
(1)

 $x_n(t)$  is the signal received on sensor n,  $w_n$  is the shading coefficient (n = -N...N), and c is the sound celerity. The array length (2N + 1)d and the choice of the weights  $w_n$  determine the spatial performance (spatial resolution, sidelobe level), which depends on frequency. In particular spatial resolution, given by the mainlobe width of the array response, gradually deteriorates towards low frequencies.

In most common situations the sources are wideband: thus the spatial filter defined by (1) inappropriately presents variable performance over the signal spectrum. Several authors have proposed methods to correct this. We consider here only those deriving from classical beamforming: their main common point consists in using variable weighting, represented through a filter with impulse response  $w_n(t, \theta)$ :

$$s(t,\theta_0) = \sum_{n=-N}^{N} w_n(t,\theta_0) * x_n(t)$$
(2)

where \* stands for the convolution operator. In the

frequency domain:

$$S(f,\theta_0) = \sum_{n=-N}^{N} W_n(f,\theta_0) X_n(f)$$
(3)

where  $W_n(f, \theta_0)$  and  $X_n(f)$  are the Fourier transforms of  $w_n(t, \theta_0)$  and  $x_n(t)$  respectively. The various methods differ then by the way of determining these coefficients, in relation with the array geometry.

This paper presents three constant directivity methods deriving directly from beamforming, pointing out practical aspects and localisation performance. Each of them is assessed using three criteria: sidelobe level, mainlobe width and its uniformity over the frequency range, in a nearfield environment. In that step, the nested array is introduced as a basic reference for comparison. In a second part, the approach is implemented on a tram noise measurement configuration, which had been designed independently for standard 2D-array analysis. Several of the previous methods will prove ineffective in this broadband context; the results relying on the remaining one will be presented.

# 1 Comparison of constant directivity localisation methods

The different methods are first presented in a plane wave context, allowing easier mathematical notation and interpretation. The comparison will then be conducted with nearfield simulations.

#### 1.1 The nested array

The nested array is commonly used in pass-by vehicle source analysis. In this approach, the directivity problem is addressed geometrically, the array being composed of several subarrays, identical except for a homothetic factor r: the respective microphone spacing and subarray length are multiplied by r from a subarray to the next one [2][3]. Each subarray processes the signals within a restricted frequency bandwidth  $[f_0, rf_0]$ , with the classical beamforming (1):

$$W_n(f,\theta_0) = w_n \cdot e^{2j\pi f n d \sin \theta_0/c} \tag{4}$$

r is often chosen as an integer, such that some microphones are common to several subarrays, thus reducing the total number of sensors necessary. This method is not, strictly speaking, a constant directivity method and offers no variable sensor weighting; however it constitutes here the beamforming reference for comparison.

#### 1.2 Multi-beam method (MBM)

This method was proposed by M. Goodwin in 1993 [4]. Since array beamwidth widens towards low frequency, it selects the spatial performance at the lowest frequency, and consists in "damaging" performance at higher frequencies to make it similar to lower ones. For this purpose, at each frequency, the array is steered in parallel in (2M+1) directions neighbouring the main direction  $\theta_0$ , then the individual steered outputs are summed in order to form a wider lobe. These directions are fitted to each frequency:

$$W_n(f,\theta_0) = w_n \sum_{m=-M}^{M} e^{jng(f,\theta_0)\frac{m}{M}}$$
(5)

where  $g(f, \theta_0)$  is a function depending on the choice of the shading  $w_n$ . The tests show that this procedure should be preferably restricted to an octave. But the joint implementation of nested arrays with r = 2 (cf §1.1) allows the widening of the analysis bandwidth.

This approach chooses to favour a constant directivity at the expense of weakening the spatial resolution over the whole frequency range.

#### 1.3 Subarray combination (SAC)

This method consists in processing signals in a bandwidth  $[f_0, rf_0]$  (where r is an integer,  $r \in [2:4]$ ), through a linear combination of two subarrays with the following frequency equation [5][6][7]:

$$S(f,\theta_0) = Z_1(f)S_1(f,\theta_0) + Z_2(f)S_2(f,\theta_0)$$
(6)

where  $Z_1(f)$  and  $Z_1(f)$  are weighting functions of the subarray outputs  $S_1(f, \theta_0)$  and  $S_2(f, \theta_0)$ .  $S_1(f, \theta_0)$  and  $S_2(f,\theta_0)$  result from (3). The constant directivity requirement is met by constraining two fixed points on the frequency response for all frequencies : the -3dB direction and the array gain at the steered angle  $\theta_0$ . Determining the coefficients  $Z_1(f)$  and  $Z_2(f)$  for frequency f comes down to solving a system of two equations with two variables. The microphone spacing of both subarrays are selected to satisfy the Shannon sampling theorem respectively at frequencies  $f_0$  and  $rf_0$ . With the same number of sensors for each, the spatial responses of subarrays 1 and 2 at the respective frequencies  $f_0$  and  $rf_0$  are identical. In its principle, this method attempts to maintain a constant directivity over the frequency range by linearly combining an array with poorer spatial performance and another array with spatial undersampling.

### 1.4 Constant Directivity Beamforming (CDB)

This method proposes to design an array whose length automatically adapts to the frequency [8]. If the weight of microphone at abscissa  $z_n$  on the array axis is :

$$W_n(f,\theta_0) = fA(z_n f)e^{2j\pi f z_n \sin \theta_0/c}$$
(7)

then the spatial response is frequency invariant. A(.) is a function (*Beam Shaping*) defining the shape of the

frequency response of the array. In practice, the Beam Shaping functions may be defined by any standard low frequency FIR filter, with cutoff frequency  $f_c = \frac{Qc}{2|z|}$  where Q stands for the array aperture (in wavelengths), and c is the sound celerity.

The design of an array with logarithmic spacing allows the wise optimisation of the number of microphones necessary. However some classical shading such as Chebychev become unavailable for this geometry.

### 1.5 Nearfield array processing

The nearfield array processing implemented here derives from standard beamforming. The array is focused on point F by compensating the time delay differences  $\tau_{nF}$ from focus F to the n sensors ; attenuation differences, due to the various distances  $d_{nF}$  between focus and sensors are also corrected:

$$S(t,F) = \sum_{n=-N}^{N} \frac{d_{nF}}{d_{ref}} w_n x_n (t + \tau_{nF} - \tau_{ref})$$
(8)

 $w_n$  are the shading coefficients. The equation (8) achieves in a way inverse spatial filtering adapted to a point source located at the focus, relatively to some reference point. Nevertheless this nearfield processing is not optimal anymore in the maximum likelihood sense, contrary to standard plane wave beamforming.

This processing may be adapted to the constant directivity issue by substituting  $w_n$  for the various previous methods. A comparison has been conducted in this nearfield context (broadside, Shannon sensor spacing, source distance 2.5m) on the frequency range 300-2400Hz, thus covering three octaves. The microphone distribution may differ for one method or the other; the total sensor number is 17, except for the SAC array which uses 19 microphones for frequency range matching reasons. The nested, MBM, SAC subarrays are Chebychev shaded, for a theoretical plane wave sidelobe level of -25 dB. The CDB array involves Hanning shading.

Figures 1 to 4 and table 1 illustrate the performance of the different methods. Each figure presents the array geometry and the theoretical spatial response. Concerning the main lobe uniformity, which is the difference between the min and max 3dB width of the mainlobe over the frequency range, the SAC and CDB methods behave particularly well. The MBM method performs poorly for this requirement (mainlobe opening at each octave end); however its sidelobes are low: the summation of neighbouring steered responses creates destructive interferences outside the mainlobe. To sum up, the SAC method distinguishes by its uniform narrow mainlobe, despite a fair sidelobe level.

	nested	MBM	SAC	CDB
sidelobe max (dB)	-19.5	-20.1	-16.3	-21.8
-3dB width max (m)	1.19	1.3	0.84	1.26
-3dB width mean (m)	0.79	1.02	0.76	1.14
uniformity (m)	0.63	0.42	0.14	0.14

 Table 1: Nearfield performance of the methods



Figure 1: Nested array geometry and spatial response (in dB)



**Figure 2:** Multibeam array geometry and spatial response (in dB)

# 2 Application to a tram noise experiment

The developments of the previous section have been conducted using individual array configurations specifically designed to match each method. This section considers the case of a nested cross array, which had been designed for wideband analysis of tram noise sources [2], but outside the scope of constant directivity methods. The topic is here to investigate how these can behave and perform with that array.

The cross array was composed of two perpendicular line arrays. Each branch consisted of two nested subarrays, with respective microphone spacings 5 cm and 15 cm (homothety ratio r=3). In the cross array processing, each line array is beamformed individually, and the respective output signals are then cross-correlated, leading to 2D-directivity properties. The signals were analysed from low frequency up to the third-octave 4000Hz, with subarray switch at 1600Hz. The line arrays were shaded by a Chebychev spatial window, with 25dB sidelobe attenuation. Table 2 gives the performance of the array processing associated to one nested line branch of the cross array, for a measurement distance of 2.5m.

The constant beamwidth requirement is handled here on the range 300-3000Hz, i.e. one decade, even if the processing on the measured signals will be conducted



**Figure 3:** SAC array geometry and spatial response (in dB)



Figure 4: CDB array geometry and spatial response (in dB)

on a wider range. It is first tested on one line branch of the array. Among the three methods described in section 1, only one performs correctly. The MBM method yields poor results, since the subbands are too wide to produce uniform spatial properties and, as stated in  $\S1.2$ , it should be restricted to octave subbands. The SAC method is based on subarrays with homothety ratio r $(\S1.3)$ ; however it performs badly when used on a wider range than [f, rf]. Even attempts to proceed with several subbands, each with uniform behaviour, ended with unsatisfactory properties, particularly at low frequencies. The CDB method was presented in §1.4 with a logarithmic microphone distribution. A similar approach may however be applied to equidistant microphones : thus frequency beamforming is implemented, with frequency adapted shading, on either subarray. The array aperture Q = 4 has been selected, with subarray switch at 1500Hz. The performance associated to this CDB processing is given in Table 2 and figure 5 represents the nearfield array spatial response over the range 300-3000Hz. With

	nested	CDB
sidelobe max (dB)	-22.5	-19.5
-3dB width max (m)	1.4	1.4
-3dB width mean (m)	0.54	0.86
uniformity (m)	1.12	0.56

 Table 2: Nearfield performance comparison for the tram array design

Q = 4, a sufficiently narrow beamwidth has been chosen, to the detriment of uniformity at low frequency (where the array length limits the actual aperture available, with Q < 4).



**Figure 5:** CDB spatial response of the nested line array designed for previous tram noise study (in dB)

2D-array processing is then achieved using both CDBshaded line arrays : the signals are first filtered in thirdoctave bands, and next nearfield cross array processing is applied to each third octave. Figure 6 shows the acoustic cartographies resulting from simulations of a point source on a tram running at 30km/h, with a pure frequency component in each third octave. The focus point is swept with the source during pass-by, in order to increase integration time and improve the estimation of the corresponding noise source contribution. The constant directivity characteristics may be observed at medium and high frequencies: successive wheels of the same bogie can be separated identically on most of the frequency range suited for rolling noise. But care should be taken not to over widen the frequency range for one subarray in order to maintain sufficient microphone averaging. It should also be noted that with this third octave CDB processing, the weights remain implicitly constant in one frequency band, thus leading intrinsically to similar behaviour as for instance the nested MY13 [3] where subarrays are switched per third octave, but it involves here a far lower number of microphones.

## 3 Conclusion

This paper presents a comparison of several methods available in the literature for constant beamwidth array processing for noise source analysis. Two methods (SAC and CDB) behaved uniformly over three octaves when using specifically designed microphone distribution, in nearfield context. These were then tested on a cross array, first designed for tram noise source analysis with classical equispaced nested arrays and swept focussing. Only the CDB could perform correctly in this wideband context, with constant beamwidth properties over medium and high frequencies. This was confirmed through simulations with a moving point source. For still wider performance, a specific array should preferably be designed before processing.

### References

 D. H. Johnson, D. E. Dudgeon. Array signal processing. Prentice Hall, 1993



**Figure 6:** cross array processing with the CDB method, for a moving point source (in dB(A))

- [2] J. Lelong, M. A. Pallas, R. Chatagnon. Noise emitted by trams - Part 1 : investigations on emitted noise power, vertical directivity and localization of noise sources. Internoise 2007, Istanbul, Turkey, 2007
- [3] T. Kitagawa, An investigation into inconsistencies between theoretical predictions and microphone array measurements of railway noise. Institute of Sound and Vibration Research, University of Southampton, 2007
- [4] M. M. Goodwin, G. W. Elko. Constant beamwidth beamforming. ICASSP-93, Minneapolis, USA, 1993
- [5] R. P. Smith, Constant beamwidth receiving arrays for broadband sonar systems. Acustica 23: 21-26, 1970
- [6] J. Lardies, J. P. Guilhot. "A very wide bandwidth constant beamwidth acoustical end-fire line array without side lobes." Journal of Sound and Vibration 120(3): 557-566, 1988
- [7] F. Bongard, Microphone array for soloists. CFA/DAGA '04, Strasbourg, France, 2004
- [8] D. B. Ward, R. A. Kennedy, R. C. Williamson, Constant directivity beamforming. In: *Microphone* arrays - signal processing techniques and applications. M. Brandstein and D. B. Ward, Springer: 3-18, 2001.