

# Estimation of primaries in seismic measurements by sparse inversion

G.J.A. van Groenestijn<sup>1</sup>, D.J. Verschuur<sup>2</sup>

<sup>1</sup> University of Technology Delft, The Netherlands, Email: G.J.A.vanGroenestijn@tudelft.nl

<sup>2</sup> University of Technology Delft, The Netherlands, Email: D.J.Verschuur@tudelft.nl

## Abstract

In exploration seismics the objective is to image the subsurface structures from acoustic reflection measurements, in order to localize and monitor oil and gas reserves. The reflection measurements are usually carried out with sources and receivers positioned at the surface of the earth. Especially in the marine case the water-air interface acts as an almost perfect acoustic mirror, reflecting all upgoing energy back into the medium. As a result, the measurements suffer from multiple reflections that mask the desired primary reflections from the inhomogeneities in the earth. However, these surface multiples have a physical relationship with the primaries: each primary event will be followed by a sequence of multiple reflections. This relationship can be exploited to estimate the primary reflection response, i.e. the multiple-free transfer function of the subsurface. This is done by a full waveform inversion process, in which the primary transfer functions are parameterized by spikes and a sparseness constraint is used during the optimization process. Examples will be shown for synthetic and field data.

## Outline

This paper begins with describing the seismic method. Primaries and multiples are introduced and it is made clear why it is relevant to separate both with a primary estimation method. A small literature overview of existing primary estimation methods is given before the Estimation of Primaries by Sparse Inversion (EPSI) method is introduced. EPSI is demonstrated on a synthetic and a field dataset.

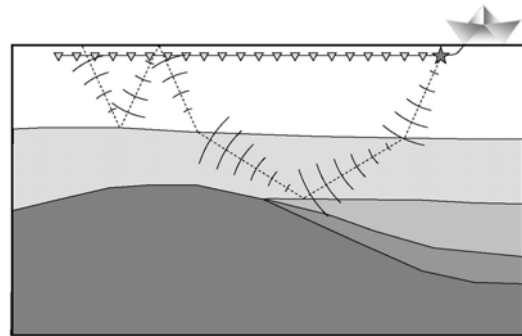
## An introduction to primaries and multiples

A message that is repeated multiple times is easier to understand. However, if the repetitions are started before the original message is finished, a method is needed to estimate the original message. In seismic recordings multiples are the repetitions that start before the original message, the primaries, is finished. In the past multiples were seen as noise that had to be removed to obtain the original message. This paper describes a method that uses the multiples to come to a better estimation of the primaries. Furthermore, the method can be used to reconstruct unrecorded parts of the primaries from the information in the multiples.

## The seismic method

In order to locate or monitor reservoirs in the subsurface that contain oil and gas or can store CO<sub>2</sub> an image of the subsurface is needed. Seismic exploration is the most common way to obtain these images. Seismic exploration can be divided into several stages. During the seismic acquisition stage (Figure 1) a source at the surface generates

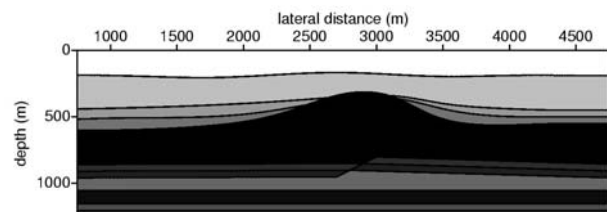
a wave field. This wave field propagates through the subsurface where it is partly reflected by the different layers in the earth. These reflections are measured by receivers at the surface. This experiment is repeated with different source positions. In the seismic processing stage the desired reflections in the recordings are enhanced, unwanted reflections are suppressed, and an image is constructed.



**Figure 1:** Seismic acquisition at sea. A boat pulls a source (air gun) and a cable with receivers (hydrophones) through the water. When fired, the source will send a wave field through the subsurface. The reflections that reach the surface are measured by the receivers.

## Primaries and multiples

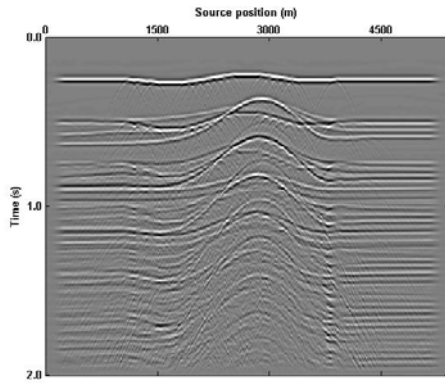
Figure 2 shows a synthetic subsurface model that is used to generate a seismic dataset. The dark coloured layer with the curved top represents a high-velocity salt layer. From each shot in this seismic dataset the recording of the receiver at zero offset is selected, and plotted in Figure 3. Looking at Figure 3, one might erroneously conclude that there are several salt domes on top of each other. However, the 'extra' salt domes are multiples; reflections that have travelled up and down at least twice (see Figure 1). Note that the multiples in Figure 3 not only give an erroneous image of the subsurface, but they also obscure the primaries.



**Figure 2:** Synthetic subsurface model.

Figure 4 shows the ray paths of possible events in a seismic measurement at the surface:

- Direct and surface waves (Figure 4a): waves that have not propagated downward but travel laterally, just below the surface. In the theory and all the examples in this paper it is assumed that in the data that is given as input to the primary estimation methods the direct wave is removed in preprocessing.

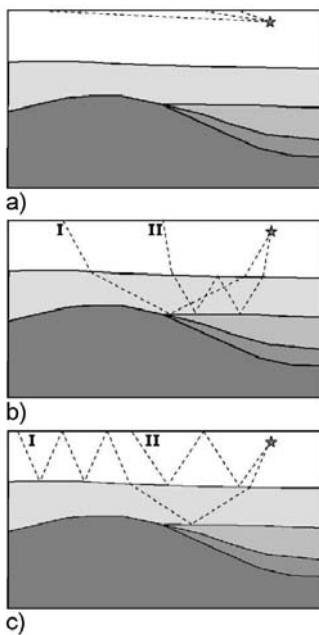


**Figure 3:** Zero offset section coming from the dataset generated from the subsurface model in Figure 2.

The distinction between primaries and multiples is made with respect to a boundary. If we take the surface as our reference boundary then we can make a distinction between surface-related primaries and surface-related multiples.

- Surface-related primaries (Figure 4b): waves that have propagated through the subsurface and have not bounced at the surface. If a surface-related primary has reflected more than once in the subsurface (like ray path II in Figure 4b) it is called an internal multiple, but still belongs to the surface-related primaries.
- Surface-related multiples (Figure 4c): waves that have propagated through the subsurface and have bounced at the surface at least once. The number of bounces at the surface determines the multiple order. Ray path II in Figure 4c describes a first order multiple and ray path I a second order multiple.

For the sake of convenience we will refer to surface-related primaries as primaries and surface-related multiples as multiples in this paper.



**Figure 4:** Ray paths of seismic events.

## Literature

This literature section deals with the literature that is related to the EPSI method. For general literature about the seismic method we refer to Jahn et al. (1998).

Although the correct elimination of surface-related multiples is possible in theory (Berkhout, 1982; Berkhout and Verschuur, 1997; Weglein et al., 1997), in practice many hurdles need to be taken. Often, the surface-related multiple elimination (SRME) method is implemented as a prediction and subtraction process (Verschuur and Berkhout, 1997), where the errors in the prediction are assumed to be compensated for in the subtraction process. However, in real life situations many factors limit the success of SRME, such as limited sampling and 3D effects (Dragoset and Jeričević, 1998) and distortion of primaries during the subtraction (Guitton and Verschuur, 2004). Therefore, it is proposed to avoid the prediction and subtraction process and consider the primaries as unknowns in an inversion process. A similar approach was described by van Borselen et al. (1996) and Amundsen (2001). Although there primaries were estimated under the assumption that the source wavelet or direct wave (including the near offsets) are known. Biersteker (2001) demonstrated the estimation of (missing) shallow primary reflections from the multiples, under a minimum energy constraint. van Groenestijn and Verschuur (2009) have introduced the estimation of primaries by sparse inversion (EPSI), like all of the above mentioned methods based on the same primary-multiple relationship. They propose a solution through an iterative inversion process, while introducing a sparseness constraint to the estimated primary impulse response. EPSI does not need to know the source wavelet or direct wave and does not use minimum energy.

## Estimation of primaries by sparse inversion

The theory of EPSI can be formulated with the aid of matrices described in the detail-hiding operator notation for 2D data (Berkhout, 1982). To obtain these matrices the measured samples are ordered into a cube;  $p(x_S, x_R, t)$ . The sample that was measured 3.1 seconds after the source at position 2.1 km was fired at position 6.4 km, is then stored at  $p(x_S=2.1km, x_R=6.4km, t=3.1s)$ . This cube is brought to the frequency domain by a Fourier transform along the time axis. A frequency slice is taken from this cube;  $\mathbf{P}(x_S, x_R)$ . This is a matrix which columns represent monochromatic shot records and rows represent monochromatic common receiver gathers. With the use of this notation we can express the upgoing data at the surface,  $\mathbf{P}$ , as:

$$\mathbf{P} = \mathbf{X}_0\mathbf{S} + \mathbf{X}_0\mathbf{R}\mathbf{P}, \quad (1)$$

where the primary impulse responses,  $\mathbf{X}_0$ , multiplied with the source properties,  $\mathbf{S}$ , equal the primaries,  $\mathbf{P}_0 = \mathbf{X}_0\mathbf{S}$ . The matrix multiplication of  $\mathbf{X}_0$  with the reflection operator at the surface,  $\mathbf{R}$ , and the total data results in the surface multiples,  $\mathbf{M} = \mathbf{X}_0\mathbf{R}\mathbf{P}$ . If we take  $\mathbf{S} = S(\omega)\mathbf{I}$  (meaning a constant source wavelet for all shots and neglecting directivity) and  $\mathbf{R} = -\mathbf{I}$ , equation 1 becomes:

$$\mathbf{P} = \mathbf{X}_0\mathbf{S} - \mathbf{X}_0\mathbf{P}. \quad (2)$$

This equation has more unknowns,  $\mathbf{X}_0$  and  $S_i$ , than knowns,  $\mathbf{P}$ , and, therefore, an extra constraint is required to solve it. van Groenestijn and Verschuur (2009) propose to use the constraint that  $\mathbf{X}_0$  is sparse in the time domain. The objective function  $J$  is introduced as:

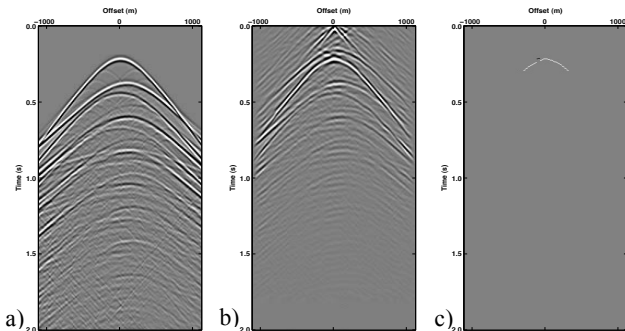
$$J_i = \sum_{\omega} \sum_{j,k} | \mathbf{P} - \mathbf{X}_{0,i} S_i + \mathbf{X}_{0,i} \mathbf{P} |_{j,k}^2, \quad (3)$$

where  $i$  denotes the iteration,  $\sum_{j,k}$  indicates a summation over all the elements of the matrix, and  $\sum_{\omega}$  indicates a summation over all the frequencies. In the first iteration we set the values of  $\mathbf{X}_{0,i}$  and  $S_i$  to zero. First,  $\mathbf{X}_{0,i}$  is updated. The update,  $\Delta \mathbf{X}_0$ , is a steepest descend-like step:

$$\Delta \mathbf{X}_0 = (\mathbf{P} - \mathbf{X}_{0,i} S_i + \mathbf{X}_{0,i} \mathbf{P})(S_i \mathbf{I} - \mathbf{P})^H, \quad (4)$$

where  $(S_i \mathbf{I} - \mathbf{P})^H$  is the complex adjoint of  $(S_i \mathbf{I} - \mathbf{P})$ .

A synthetic dataset based on a 2D subsurface model is used to illustrate this method. Figure 5a shows one shot gather of this dataset. Figure 5b shows the first update step. The term  $(\mathbf{P} - \mathbf{X}_{0,i} S_i + \mathbf{X}_{0,i} \mathbf{P})$  can be seen as the unexplained data or the residual. Since both  $\mathbf{X}_{0,i}$  and  $S_i$  are zero in the first iteration step, the first step equals a multi dimensional correlation of the data with itself,  $\mathbf{P} \mathbf{P}^H$ . A window is placed over the update of  $\mathbf{X}_{0,i}$  in the time domain and the biggest event(s) per trace are selected. By increasing the size of the window in each iteration convergence is improved.



**Figure 5:** Shot gather with all multiples (a) and the corresponding update of the primary impulse response before (b) and after (c) imposing sparseness.

Next, the sparse update,  $\Delta \bar{\mathbf{X}}_0$ , is added to the primary impulse response:

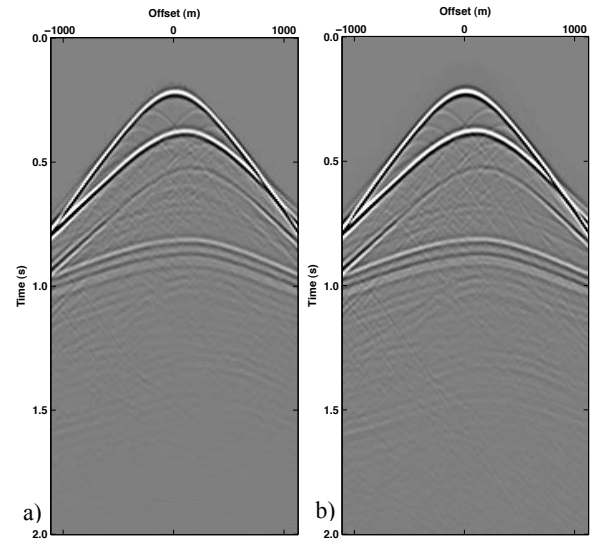
$$\mathbf{X}_{0,i+1} = \mathbf{X}_{0,i} + \alpha \Delta \bar{\mathbf{X}}_0, \quad (5)$$

where  $\alpha$  is a positive frequency independent factor that scales the update step, such that  $J$  decreases. Figure 5c shows  $\Delta \bar{\mathbf{X}}_0$ .

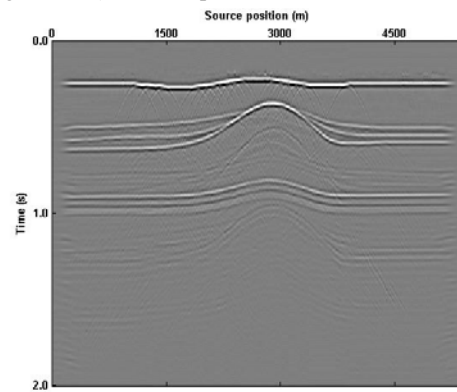
Next,  $S_{i+1}$  is estimated as a filter obtained from least-squares matching the impulse responses,  $\mathbf{X}_{0,i+1}$ , to  $(\mathbf{P} + \mathbf{X}_{0,i+1} \mathbf{P})$  in the time domain. These two update steps are repeatedly applied until the residual is small enough.

Finally, the estimates  $\mathbf{X}_{0,i}$  and  $S_i$  can be used to obtain a primary estimation by convolving the estimated (spiky) impulse responses with the estimated wavelets;  $\mathbf{P}_0 = \mathbf{X}_{0,i} S_i$ . The result is shown in Figure 6a. Figure 6b shows the true primaries. Figure 7 shows the estimated primaries in a zero offset section. This figure gives a much more accurate image

of the subsurface than Figure 3. Two very weak ‘extra’ salt dome events are still visible. However, these are no surface-related multiples but internal multiples. They will be removed further on in the seismic processing stage.



**Figure 6:** a) Estimation of the primaries of the shot gather in Figure 5a. b) The true primaries.



**Figure 7:** Zero offset section of the estimated primaries.

## Near offset reconstruction

During seismic acquisition it is not possible to place hydrophones close to the source, because the pressure pulse of the source would damage them. Therefore, the near offsets are not measured. These missing near offsets are a problem for data driven primary estimation methods, especially in shallow water. It is beyond the scope of this paper, but with some small modifications to the described EPSI algorithm the reconstruction of the missing near offsets can be included in the primary estimation algorithm (see van Groenestijn and Verschuur, 2009).

That this can have a significant impact on the primary estimation for shallow water marine datasets can be seen in Figure 8. Here we compare the primary estimation result of an existing method (SRME) on data with Radon interpolated near offsets (Kabir and Verschuur, 1995) in Figure 8b with the primary estimation and near offset reconstruction of EPSI in Figure 8c. The EPSI result looks much cleaner; for example the structure at  $t = 1.7s$ , CMP location = 14 km and the multiple in Figure 8b at  $t = 1.23s$ . The water bottom

reflection is stronger in Figure 8c than in Figures 8a and 8b. This is due to the near offset reconstruction. This dataset is treated in more detail in van Groenestijn and Verschuur (2009b).

## Discussion

We believe that there is large scope to apply this new combined primary and near offset estimation method for shallow water situations, as this is where SRME is known to have difficulties (see e.g. Verschuur, 2006). Although all tests were done on 2D data, we think that EPSI can be of value for the full 3D situation, since EPSI only needs to store the spikes of impulse responses. Furthermore, the missing data between the streamer lines might be treated in a similar way as the missing near offsets. We see possibilities to apply the EPSI algorithm to blended data (simultaneous source, Berkhout, 2008), passive seismics and deconvolution of up and down going wave fields (“Amundsen” deconvolution, Amundsen, 2001).

## Conclusions

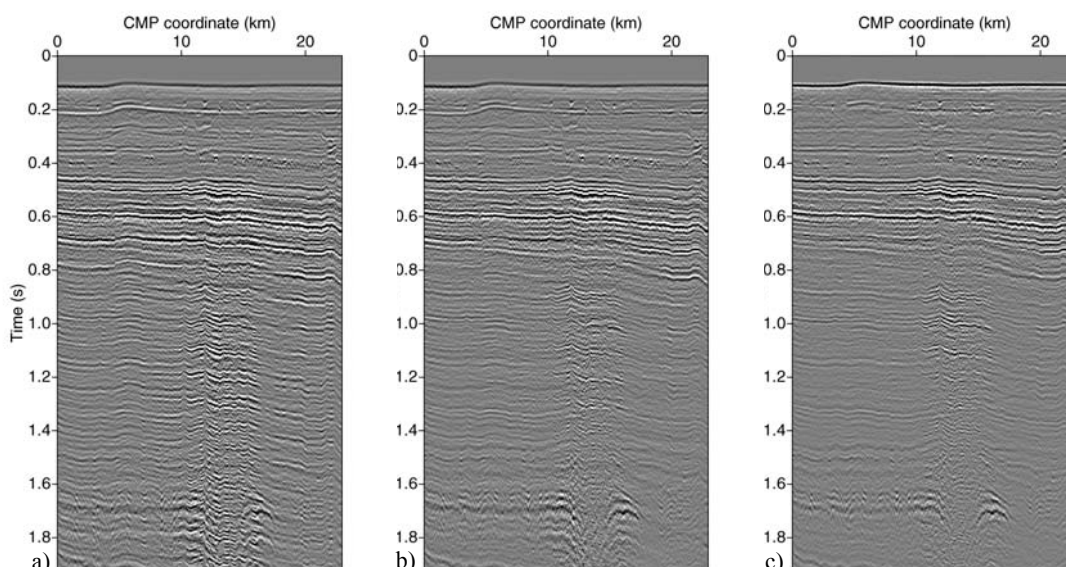
In this paper we have presented a new primary estimation method; estimation of primaries by sparse inversion (EPSI). The primaries are considered unknowns in a full waveform inversion process. EPSI does not need interpolated near offsets to estimate primaries which gave a better primary estimation result for a shallow water field dataset than using SRME.

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**Figure 8:** a) Stacked section of the shallow water marine data with multiples. b) The primaries obtained with iterative SRME using interpolated near offset measurements. c) The primary estimate of EPSI for which the near offset measurements have been estimated during the inversion process.