Reciprocity measurements in acoustical and mechano-acoustical systems. Review of theory and applications

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1. Introduction

This paper presents a review on the validity and application of the reciprocity principle for measurements in acoustical and mechano-acoustical systems. Because of the fact that measurements in these systems are generally done with the aid of electro-acoustical or electro-mechanical transducers, electro-acoustical and electro-mechanical systems are also taken into account.

In 1860 Helmholtz published about reciprocity in acoustical systems [1]. Thirteen years later Lord Rayleigh showed that the principle is much wider and defined the General Reciprocity Theorem for linear dynamical systems [2, 3]. However, till 1970 the number of applications in practical acoustics remained very limited (see section 5 of this paper). In the period between 1970 and 1988 the author et al developed a number of straightforward applications, which are reviewed in section 6 of this paper. Around 1985 others joined the efforts and started to publish about their results. These are reviewed in sections 7 and 8.

The general reciprocity principle, its validity and the backgrounds for its application are discussed in sections 2, 3 and 4.

2. What is reciprocity?

A system is reciprocal when the transmission of vibration from an arbitrary position 1 to an arbitrary position 2 has a simple relation with the transmission from position 2 to position 1. In more precise terms, a system is reciprocal when a transfer function in terms of field quantities for transmission from position 1 to position 2 is equal to the corresponding transfer function for transmission from position 2 to position 1.

An acoustical system can be represented as a four-pole – see Figure 1.

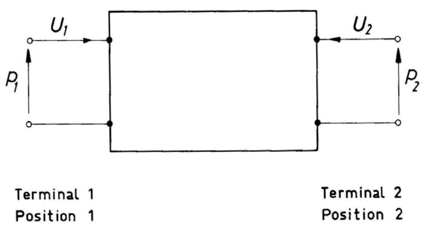

Figure 1: Four-pole representation of an acoustical system. The arrows define the positive directions for the sound pressure *p* and the volume velocity *U* at the positions 1 and 2.

When the sign convention for the sound pressure *p* and the volume velocity *U* in points 1 and 2 is chosen as shown in Figure 1 there are reciprocity relations for all four transfer functions for the transmission of sound from position 1 to position 2 [13, 14]:

$$\left(\frac{p_2'}{U_1'}\right)_{U_2'=0} = \left(\frac{p_1''}{U_2''}\right)_{U_1''=0} \qquad [\mathrm{Nsm^{-5}}] \qquad (1)$$

$$\left(\frac{p_2'}{p_1'}\right)_{U_2'=0} = -\left(\frac{U_1''}{U_2''}\right)_{p_1''=0} \qquad (2)$$

$$\left(\frac{U_2'}{U_1'}\right)_{p_2'=0} = -\left(\frac{p_1''}{p_2''}\right)_{U_1''=0} \qquad (3)$$

$$\left(\frac{U_2'}{p_1'}\right)_{p_2'=0} = \left(\frac{U_1''}{p_2''}\right)_{p_1''=0} \qquad [\mathrm{N^{-1}s^{-1}m^{5}}] \qquad (4)$$

Equation 1 is the classical reciprocity relation for acoustical systems as derived by Helmholtz [1]. However, for a reciprocal acoustical system, the three other relations are valid as well [4].

Sets of similar relations are valid for reciprocal electrical systems and for reciprocal electro-acoustical systems [13, 14].

It is obvious that the minus signs in the reciprocity relations disappear when squared values or root-mean-square values of the quantities are considered.

For systems with a mechanical terminal the situation is more complicated because the vibration at such a terminal has six "degrees of freedom". Each of these must be characterized by a conjugate pair of mechanical quantities (force and translational velocity or moment and angular velocity). In the case of a reciprocal mechano-acoustical system it can be shown that a set of 24 reciprocity relations of four different types is valid [12, 13, 14]. The first equations of types 1 and 2 are the following.

Type 1:

$$\left(\frac{p_2'}{v_{x1}'}\right) = \left(\frac{F_{x1}''}{U_2''}\right) \qquad [\mathrm{Nsm^{-3}}] \qquad (5)$$

All velocities at position 1 zero, except v_{x1}' ; $U_2' = 0$ — All velocities at position 1 zero.

Type 2:

$$\left(\frac{p_2^{'}}{F_{x1}^{'}}\right) = -\left(\frac{v_{x1}^{"}}{U_2^{"}}\right) \qquad [\mathrm{m}^{-2}] \qquad (6)$$

All forces and moments at position 1 zero, except F_{x1}' ; $U_2' = 0$ — All forces and moments at position 1 zero.

The left hand side of the equations represents the direct measurement of the particular transfer function, the right hand side the reciprocal measurement.

Note that all the point-to-point relations include requirements for the boundary conditions. Usually, the requirements at the acoustical terminal are easy to fulfil. The requirement $U_2' = 0$ when measuring p_2' for example, can be satisfied by using a small and rigid omni-directional microphone or hydrophone. The boundary conditions for the mechanical properties are often more difficult to fulfil.

In [70] Verheij provides an elegant overview of the various reciprocity relations. (Remark: because of the fact the equations (2) and (3) and the corresponding equations for electrical, mechanical and mechano-acoustical systems are basically identical, Verheij distinguishes three instead of four classes of reciprocity relations). Figure 2 shows his examples of reciprocity relations in reciprocal systems.

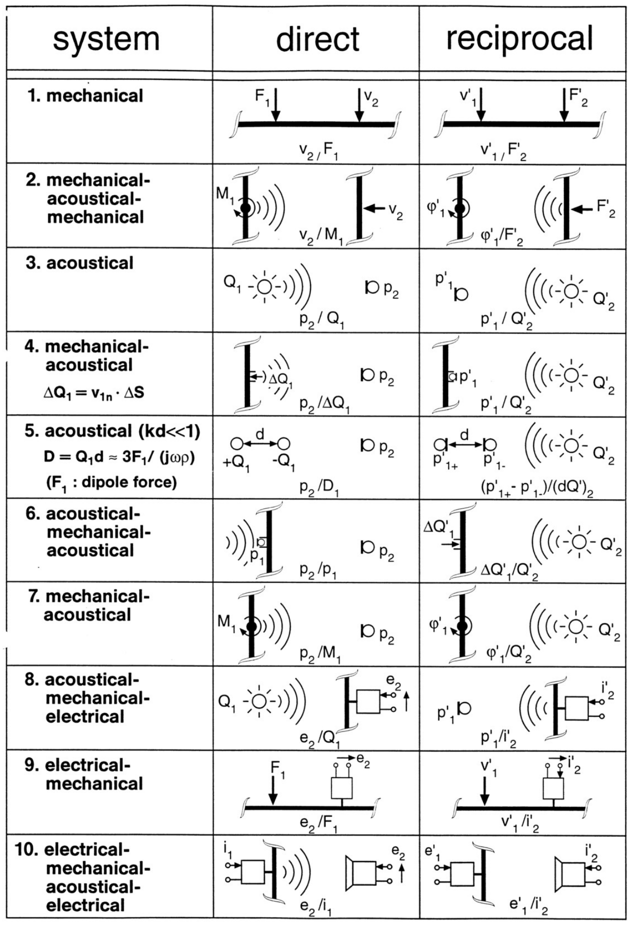

Figure 2: Examples of equal transfer functions (frequency response functions) in reciprocal systems. After Verheij [70]. In this figure the symbol Q is used for volume velocity.

3. Validity of reciprocity

Crucial for the application of reciprocity is of course its validity. Lord Rayleigh [1, 2] and also the electrical network theory combined with the theory of dynamical analogies, state that reciprocity is a general property of all stable, lumped, linear, passive, dynamical systems which only contain bilateral elements (like masses, stiffnesses, inductances, capacitors, transformers and resistors) [5, 6, 13, 70]. This covers a wide range of systems, but certainly not all. An important cause of non-reciprocity is flow. An example of a system with flow is the system between two points at a large distance (more than 100 m) in the open air. Due to wind and turbulence, elements of such a system are not bilateral, which makes the system non-reciprocal. There are, however, also situations with flow in which the disturbance of reciprocity is negligible [8, 14, 20, 46].

In spite of the clarification offered by the electrical network theory, doubts about the validity of the General Reciprocity Theorem for acoustical and mechano-acoustical systems remained. Formal proofs provided by Lyamshev in 1959 [7] and Chertock in 1962 [9] considerably improved the situation, but doubts about the role of porous materials and structural damping are even present today. In Rayleigh's time doubts were particularly expressed by Tyndall [38]. Tyndall arranged some experiments which showed, in his eyes that the principle did not hold. It is now clear however that Tyndall's experiments were too primitive to be conclusive. In more recent times (1958 and 1990), based on theoretical considerations, doubts were expressed by Janssen and by Fahy [6, 38]. Regarding Janssen's note on porous materials, Van Wulfften Palthe showed in 1967 that Janssen analysed a model in which certain details were neglected. When those details are included, the model becomes reciprocal [95]. Unfortunately, neither Van Wulfthen Palthe, nor Janssen have published this correction. Thus it could happen that in 1981 Pierce remarked on Janssen's note: "Whether this (i.e. the violation of reciprocity) is a necessary consequence of the material properties or an artefact of the model remains to be determined", and that in 1990 Fahy was still in doubt about the correctness of Janssen's note [38].

In 1990, Fahy assumed a violation of reciprocity due to non-velocity-dependent structural damping [38]. He based this conclusion on considerations about the hysteretic loss model. Verheij, referring to an article by Crandall, showed however that the hysteretic loss model does not perfectly represent the physical reality [97]. Also for this case a corrective note is missing in the literature. It can be concluded that linear dissipation is no obstacle for reciprocity.

Lyamshev demonstrated that the reciprocity principle applies whenever the differential equation of the motion is symmetric in the spatial variables [7, 8]. According to Heckl, the differential equations in acoustics are always symmetric, as long as the associated processes are linear; therefore, the reciprocity principle is valid for all acoustical problems [98]. In a footnote, Heckl remarked that the literature does contain nonsymmetric differential equations, for example, for vibrations of cylindrical shells and for sound propagation in porous materials (which refers to [6]), but that those equations are approximations, whose lack of symmetry results from the neglecting of small quantities. This observation (published in 1973) is in agreement with the above observations by Van Wulfften Palthe and Verheij.

It is clear that the principle may not (sufficiently) hold in the presence of flow. Attention is also required when there are poor connections in the mechanical (part of the) system (causing non-linearity and/or non-bilateral elements), when there are elements in the system which are non-linear (like certain resilient elements), and when there are rotating parts in the transmission path (which may cause gyration effects) [13, 14] . In such cases reciprocity may be violated to such an extent that it can not be applied. There are many cases however, in which the disturbing influences are limited and reciprocal measurements still provide data with an acceptably small uncertainty. For cases in which there is doubt, it is recommended to do one or more checks by performing experiments as well directly as reciprocally. Such checks can for example be done between the electrical terminals of two reciprocal electro-acoustical or electro-mechanical transducers at different positions in the system, as shown in example 10 of Figure 2.

Experimental proofs of reciprocity in particular systems are given in [14, 17, 18, 19, 38, 41, 46, 50, 52, 59, 60, 61, 65, 70, 73, 82, 90].

4. Why reciprocal measurements?

From a theoretical point of view there is no reason to replace the direct measurement of a transfer function by its reciprocal alternative. Considering practical aspects however, it appears that reciprocal experiments are often easier to perform and more accurate. There are three major reasons:

1. Sufficiently powerful electro-dynamical sources (acoustical or mechanical) are much larger than the corresponding, sufficiently sensitive, receivers (microphones, hydrophones, accelerometers). Consequently, in a direct set-up there is often too little space to accommodate a source, while there is sufficient place for receivers.
2. The independent excitation of three orthogonal forces and three orthogonal moments (couples) is very difficult and often even impossible. The independent measurement of six vibrational components is relatively easy and accurate, however [14, 18].
3. In a reciprocal set-up the measurement of mechanical or acoustical quantities can sometimes be replaced by the measurement of electrical quantities, which is considerably more accurate. This applies particularly to the reciprocity substitution method for the determination of acoustical and mechanical source strength and to the calibration of receivers (see the next sections).

Examples of specific cases in which reciprocal measurements are advantageous are given in the next sections.

5. Applications till 1970

The most straightforward application of reciprocity is of course the measurement of point-to-point transfer functions. However, till 1970 there were no publications on such applications for acoustical and mechano-acoustical systems.

A little bit more complicated is the use of reciprocity in substitution methods for the determination of source characteristics (acoustical or mechanical). In a direct substitution method, the unknown source is replaced by a known source and the relevant transfer function is measured directly. In the reciprocal alternative the same transfer function is determined by placing a known source at the receiving position and a suitable receiver (for example a microphone) at or near the position of the unknown source. Till 1970 there were no publications on such applications.

In 1970, the most straightforward application in acoustics was the reciprocity calibration of microphones and hydrophones, as proposed by Ballantine in 1929 [4]. This is a double substitution method with the aid of an auxiliary reciprocal transducer, applying the principle of electro-acoustical reciprocity. Figure 3 shows the three steps of this method. This method was, and is still, widely used, and can deliver very accurate data because the measurements are all electrical and the method requires no pre-measured sensitivity data of the employed transducers.

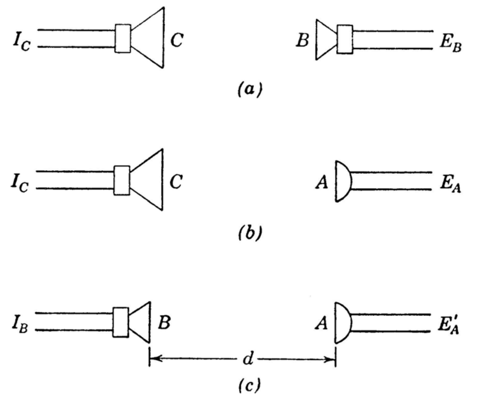

Figure 3: Schematic representation of the three steps used in the reciprocity calibration of microphone A. After Ballantine [4] and Kinsler and Frey [11].

In 1963 Heckl and Rathé published on the relationship between airborne and structureborne sound isolation in buildings, using the principle of reciprocity [10]. It concerns an application which is not directly focussed on measurements and it incorporates also other theoretical elements than reciprocity. Consequently it is somewhat outside the scope of this review. It shows however that in 1963, Heckl end Rathé had sufficient confidence in the validity of the principle. Later on, Heckl and his co-workers also used the principle in other applications, as for example presented in [26] and [32], but non of these is focussed on measurements.

6. Applications developed by Ten Wolde et al

6.1 Introduction

Between 1968 and 1988 the author et al developed a number of applications concerning the measurement of mechano-acoustical transfer functions, the measurement of acoustical source strength and the measurement of mechanical source strength [12, 13, 14, 18, 21, 22, 23]. Examples are summarized below. Most of the examples concern the transmission of sound from shipboard machinery to points underwater, but the same approaches are also applicable to similar systems, as for example the transmission of sound from the combustion engine to the driver's position in an automobile.

6.2 Mechano-acoustical transfer functions

The first example concerns the system sketched in Figure 4. It concerns the investigation of different sound paths from an engine aboard a ship to positions underwater [13, 14]. Investigation of these paths with the machine in operation is very difficult and costly – see the explanation in Figure 4. An alternative is to stop the machine and excite it by a force on the outside, assuming that such a force sufficiently represents the actual excitation by a running machine. Cooling water pipes, the exhaust pipe, resilient mounts and other elements can than be easily disconnected and/or replaced by other elements. The direct excitation with a force, however, requires extremely large mechanical exciters, so that such experiments are very difficult to perform. Reciprocity experiments with accelerometers attached to the machine surface and a strong underwater sound source offer a simple solution.

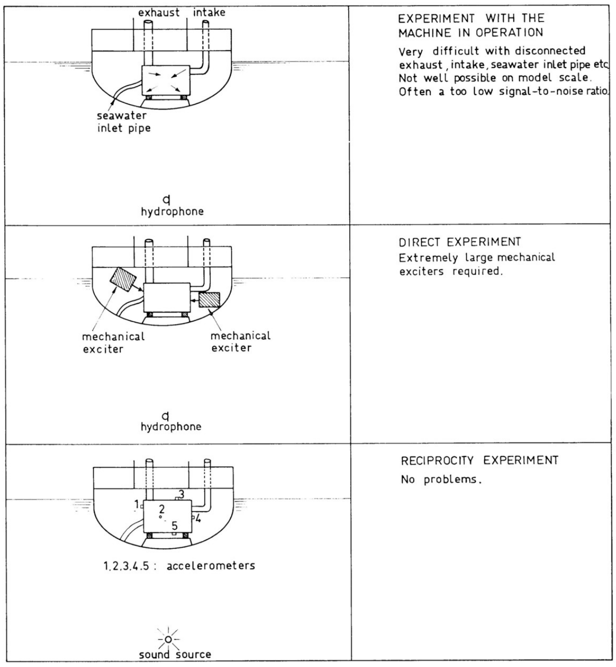

Figure 4: Investigation of various sound paths by reciprocity measurements [14].

The simulation of the actual excitation with one equivalent force at the outer surface of the machine can be acceptable for large machines and not too low frequencies [14].

The next example concerns a part of one of the most important transmission paths, which is the path through a resilient mounting, the machinery seating, the hull structure and the water. The starting point is the attachment position for a mounting on the seating. For the system between this position and a position underwater, the interest was in the transfer functions sound pressure/force or sound pressure/moment for all six degrees of freedom of the mechanical excitation. In practice, the independent and sufficiently strong excitation of three orthogonal forces and three orthogonal moments is nearly impossible. The necessary electro-mechanical exciters are too large for that purpose. In the reciprocal set-up (see Figure 5), the six components of the vibration of the machinery seating could be easily measured with pairs of accelerometers on a suitable block [14]. The volume velocity of the underwater sound source was determined in a separate experiment. The employed reciprocity relations are equation 6 and the five corresponding equations for the other degrees of freedom.

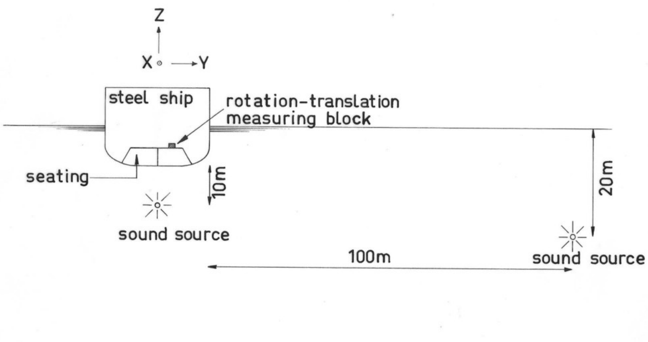

Figure 5: Reciprocal measurement of mechano-acoustical transfer functions for the system between a position on a machinery seating and positions underwater [14].

Results for the position 10 m below the keel are given in Figure 6. Shown are the six one-third octave band transfer functions for three different machinery seatings (A, B and C), which were mounted successively at the same position in the ship. The results cover the frequency range from 50 Hz to 3,15 kHz and a dynamic range of about 50 dB. The differences between the transmission through the three seatings are clearly shown.

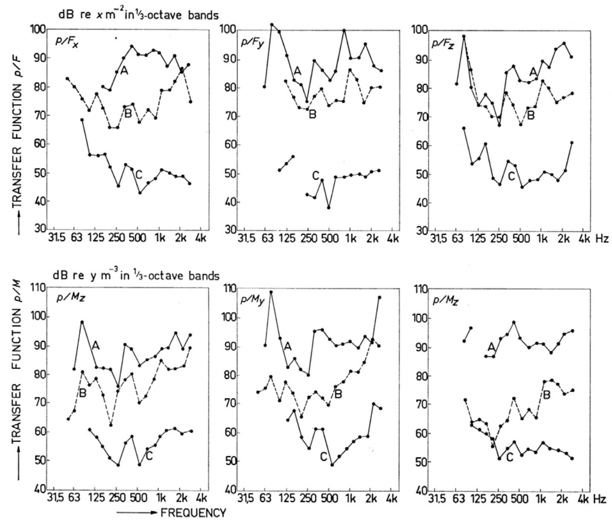

Figure 6: One-third octave transfer functions for the system between a position on the machinery seating and the position 10 m below the ship. Results for three different machinery seatings (A,B and C). After [14]. Note: *p/M*$_z$ in the low left figure must be *p/M*$_x$.

It is possible to extend the above transmission path with the resilient mounting. In that case reciprocity requires the measurement of blocked forces and moments (according to equation 5) on top of the mounting, which can be done with the aid of a stiff and heavy loading block of which the six components of the vibration are measured (see Figure 7). The forces and moments follow from the application of Newton's second law [14].

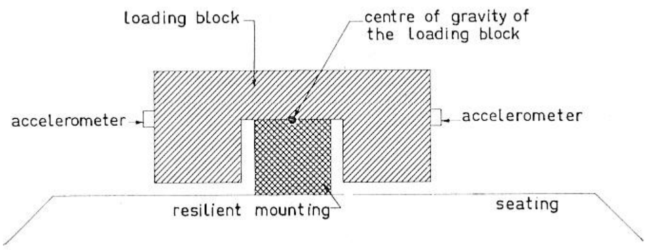

Figure 7: Loading block on resilient mounting [14].

The last example on transfer function measurements is the sound radiation from a railway rail – see Figure 8 [23]. Instead of exciting the rail with a mechanical exciter and measuring the sound pressure with a microphone, a loudspeaker with known volume velocity is placed in the far field and velocities (derived from accelerations) are measured on the rail (application of equation 6). By repeating the measurements for different positions of the loudspeaker, directivity patterns were derived as well. The measurements were performed for two degrees of freedom of the rail vibration i.e. the vertical and the transverse translation.

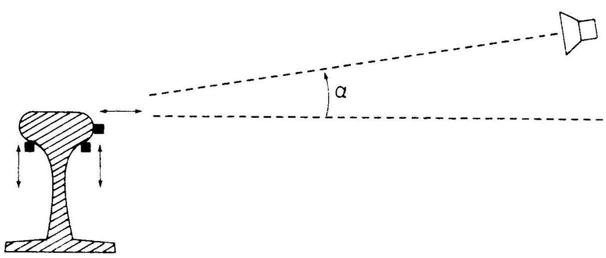

Figure 8: Rail radiation measured reciprocally [23].

The reasons for the choice of reciprocal instead of direct measurements were the following:

1. During train passages the accelerometers could remain on the rail, while force exciters could not.
2. The measurement of the velocity components in the reciprocal experiment is more accurate than the independent excitation and measurement of force components in the direct approach.

Other applications of the reciprocal measurement of mechano-acoustical transfer functions are:

- Sound transfer through cables, pipes and hoses [13, 14, 18].
- Sound radiation by a point excited ship's hull [13, 14].
- Shielding of propeller noise by a nozzle [13, 14].

6.3 Acoustical source strength.

Applying electro-acoustical reciprocity, the volume velocity of an unknown source can be determined in any reciprocal surrounding [14, 16, 19]. The principle is shown in Figure 9. The auxiliary transducer can be either located in the same space or in another space and may be either a reciprocal electro-acoustical transducer or a reciprocal electro-mechanical transducer.

Provided that the unknown source is a monopole, the volume velocity U of the unknown source follows from:

$$U = \left(\frac{e_2^{'}.i_2^{''}}{p_1^{''}} \right) \qquad [\mathrm{m^3s^{-1}}] \qquad (7)$$

The method provides the in-situ source strength and can be determined very accurately because it only requires the measurement of electrical quantities and the calibration data of the omni-directional microphone or hydrophone.

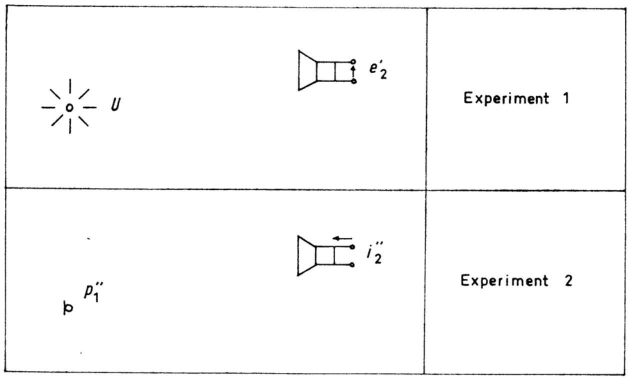

Figure 9: Principle of the electro-acoustical reciprocity substitution method for the determination of acoustical source strength [14, 16,19].

The method was tested for several sources by comparing the results with those of the reverberation room method or the free field method [16]. Some typical results are given in Figure 10.

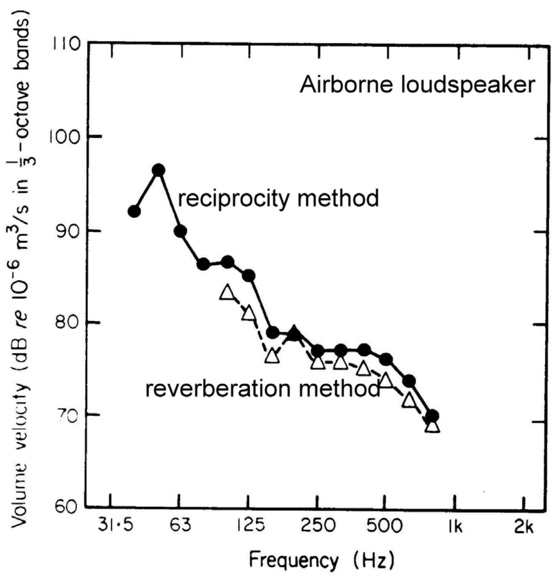

Figure 10: Volume velocity of a loudspeaker, determined with the reciprocal substitution method and with the reverberation room method [17, 18, 19].

In the frequency range where the reverberation method and the reciprocal substitution method were both applicable, the agreement is within the uncertainty margins of both methods. For low frequencies, only the reciprocal substitution method was able to provide valuable results.

The advantages of the electro-acoustical method are obvious:

1. Applicable in any reciprocal surrounding.
2. Determination of the in-situ source strength. When the free field source strength is of interest, the unknown source should of course be situated at a place where it can radiate freely.
3. The method is accurate because electrical quantities and sound pressure can be measured very accurately.

When the unknown source is not a monopole the results may become dependent on the position of the auxiliary transducer. In that case the strength of an equivalent monopole source is determined which causes the same sound pressure at the position of the auxiliary source as the unknown source does. An application in this sense is shown in Figure 11 [17, 19, 20]. It concerned the determination of the equivalent source strength of a cavitating ship propeller in relation to the noise aboard.

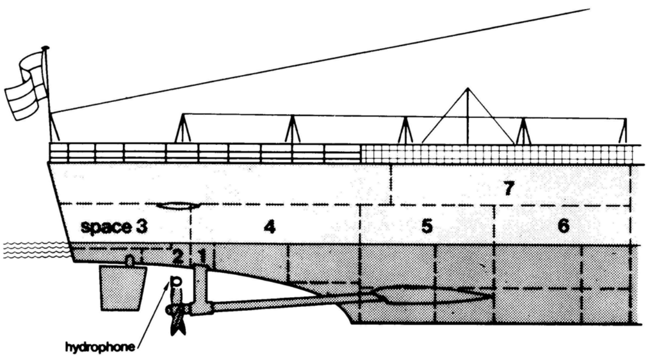

Figure 11: Determination of the acoustical source strength of a cavitating ship propeller [17].

The cavitating propeller was modelled by an equivalent monopole at the highest position of a propeller blade tip. This is reasonable because the heaviest cavitation occurs when the blade is in that position. Loudspeakers were placed aboard in the spaces 1 to 7 and reciprocal measurements were performed using the above method. Reciprocal instead of direct substitution was applied because a suitable underwater loudspeaker is too large to be placed at the position of the propeller tip.

The results for the spaces 3, 4, 5 and 6 agreed within a reasonably small margin, showing that the same source description can be used for the prediction of propeller noise in those spaces. The results for the spaces 1 and 2 were very different, showing that the data found from measurements in the spaces 3 to 6 can not be used for the prediction of noise in the spaces 1 and 2. The source description is obviously not sufficiently good for the prediction of noise in these spaces. Due to a too low signal-to-noise-ratio, no useful results were obtained for space 7.

6.4 Mechanical source strength

As indicated above, the author investigated the possibility to simulate the excitation by a combustion engine aboard a ship by one equivalent force at the outside of the engine [14] . The magnitude of this equivalent force can determined by reciprocal substitution, using loudspeakers at the reception position (aboard or underwater) and accelerometers on the engine's surface

A similar approach has been proposed for the structure-borne noise excitation by resiliently mounted machinery on heavy floors in buildings[30] – see Figure 12. This time the excitation is modelled by one equivalent force acting on the floor. Its magnitude is determined by reciprocal substitution, using mechano-acoustical or mechano-electrical reciprocity.

Gerretsen applied the approach successfully for washing machines [31].

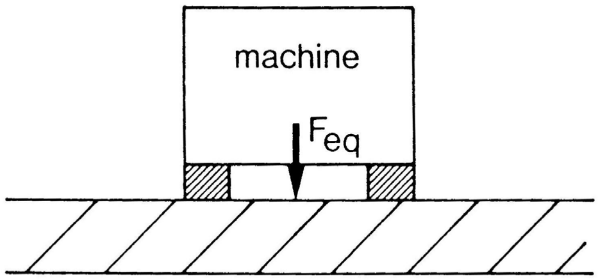

Figure 12: Simulation of the excitation by a resiliently mounted machine by an equivalent force [30].

7. Dissemination of the methods

From around 1985 other researchers tackled the subject and since than there has been a continuous stream of publications. The most productive authors are

- Verheij [21, 28, 42, 44, 45, 49, 55, 57, 58, 61, 69, 70, 74, 75, 76, 78, 79, 81],
- Fahy [24, 37, 38, 40, 41, 47, 48, 53, 56, 67, 68, 87] and
- Vér [25, 35, 46, 65, 66].

Today, the methods are generally accepted as valuable tools in the development of quiet cars and trucks. The most important contributions for this area were probably given by Verheij et al [42, 55, 61] and Van der Linden et al [52, 77]. There are however, various other authors [43, 50, 53, 60, 62, 63, 72, 88, 91, 92, 93]. The methods are used for:

1. Sound path analysis.
2. Data gathering for prediction methods.
3. Identification of the most important sources ("source identification").
4. Source characterisation.
5. Studies on the radiation from vibrating surfaces.

Many of the publications focus on the vehicle interior noise, but there are also applications related to the exterior noise.

The methods are also well known and widely applied in naval shipbuilding [21, 22, 28, 36, 39, 54, 73, 84]. Verheij and Musha are the most important contributors.

The methods are also applied in the aviation industry [33, 40, 41, 47, 48, 59, 64] and the rail transport industry [74, 75, 76, 79, 81], but the dissemination is probably somewhat less than in the automotive industry and in naval shipbuilding. Major contributions to the development of the methods for these areas are provided by Mason and Fahy for aviation [40, 41, 47, 48] and Geerlings and Verheij for rail transport [74, 75, 76, 79, 81].

Occasionally the methods are applied to noise problems in buildings [31, 89, 93]. There are also applications on the effect of sonic booms [51], the development of musical instruments [85], the development of quiet gearboxes in wind turbines [34] and the development of quiet fans [89].

Textbooks have a large influence on the dissemination of methods. So far, the author only knows one textbook in which the basic reciprocity measurement methods for acoustical and mechano-acoustical systems are properly presented, which is Beranek and Vér's book on Noise and Vibration Control [46].

8. Further development

Many of the applications are straightforward applications of Ten Wolde's work, applied to similar problems. There are however also applications which are rather different, there are applications to more complicated problems and there are further developments of the methodology. Sometimes there is no reference to Ten Wolde's work but to later authors or to the basic work by Rayleigh [2, 3], Helmholtz [1], Lyamshev [7, 8] or Chertock [9].

Most of the applications are based on the classical acoustical reciprocity relation (equation 1) and on the mechano-acoustical reciprocity of types 1 and 2 (equations 5 and 6). There are however also applications of acoustical reciprocity in the form of equation 2 [40, 41, 59]. The number of applications based on mechano-electrical or acousto-electrical reciprocity is small [28, 31, 65]. It seems that the possibilities of the latter forms of reciprocity are not yet fully recognised.

A very important extension of the application concerns source descriptions for machines and for vibrating surfaces with various mechanical or acoustical sub-sources. In those cases the reciprocity principle is often applied in combination with the superposition principle [29, 35, 37, 41, 44, 46, 53, 55, 58, 61, 69, 70, 75, 76, 79].

Below, some of the above developments are illustrated by examples.

Transmission of propeller noise into aircraft fuselages

Mason and Fahy [40, 41] developed an experimental reciprocity technique for calibrating an aircraft fuselage as a transmitter of sound pressure acting on the external surface to the interior. The technique is based upon the use of equation 2, which requires the measurement of volume velocity at the exterior of the fuselage – see Figure 13.

For the measurement of volume velocity a special capacitive transducer was developed [40, 48]. When the measured transmission data are combined with data of the impinging pressure field, sound pressure within the cabin can be computed. Mason demonstrated the validity of the approach on a scale model. A few years later Mac Martin et al described the successful application of the method for a full scale turboprop aircraft [59].

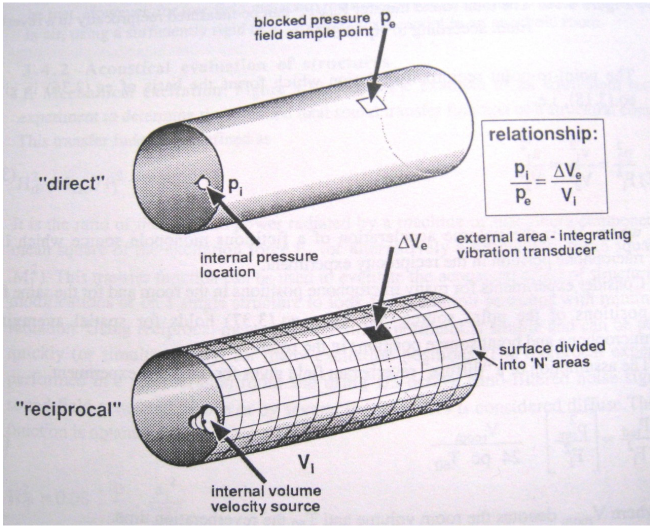

Figure 13: Sketch showing the principle of Mason and Fahy's approach [40].

Reciprocity measurement techniques to assess potential structural damage from sonic booms

The potential environmental impact of supersonic operations includes damage to structures by sonic boom overpressures. The assessment of such damage requires dedicated flyovers for each site at great expense. Garrelick and Martini showed that structural-acoustic techniques may be used to help provide such assessments in their absence [51]. In this procedure, transfer functions relating structural response to sonic boom overpressure are measured reciprocally by measuring the sound radiated by the structure when driven mechanically (application of equation 5).

Machinery noise source characterization for airborne sound and determination of related transfer functions

Mason and Fahy [37] proposed an approach for the characterization of a vibrating machine as a source of airborne sound and the reciprocal measurement of the related transfer functions. The principle is shown in Figure 14. It uses the velocity distribution over the sound radiating surface as a source strength descriptor. The machine's surface is divided into incremental areas ΔS_i with normal velocity v_i. Each sub-area is seen as an acoustical point source with volume velocity $Q_i = v_i \Delta S_i$. The source velocity v_i is measured when the machine is in operation. Measurement of the transfer functions related to each partial source with the aid of small loudspeakers is nearly impossible. Reciprocity experiments offer the solution (application of equation 1). The total sound pressure caused by the engine or by parts of it can be found by superposition of the partial contributions.

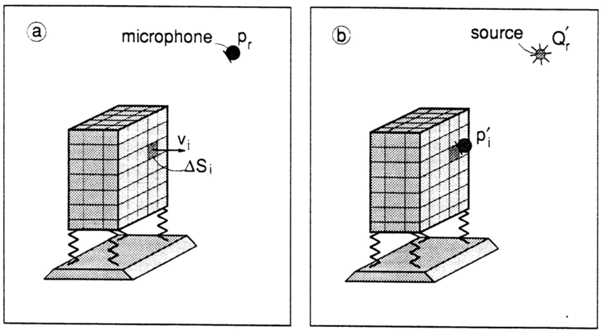

Figure 14: Source description with correlated monopoles and reciprocal measurement of the corresponding transfer functions, as proposed by Mason and Fahy [37].

The method can be useful for low frequencies and simple vibration patterns. For higher frequencies and complicated structural shapes and vibration fields, the large amount of data needed makes the method impractical. Also non-steady sources, like vehicle engines under running conditions, cannot be handled because the phase relations between the partial velocities cannot be determined. For those cases Verheij proposed and validated a method in which the engine is modelled by uncorrelated monopoles – see Figure 15 [44, 69]. The source strength of the uncorrelated monopoles is not determined from intensity measurements as for the stable case, but with the aid of a locally attached enclosure and application of acoustical reciprocity. The transfer functions from the monopoles to the reception position are also measured reciprocally.

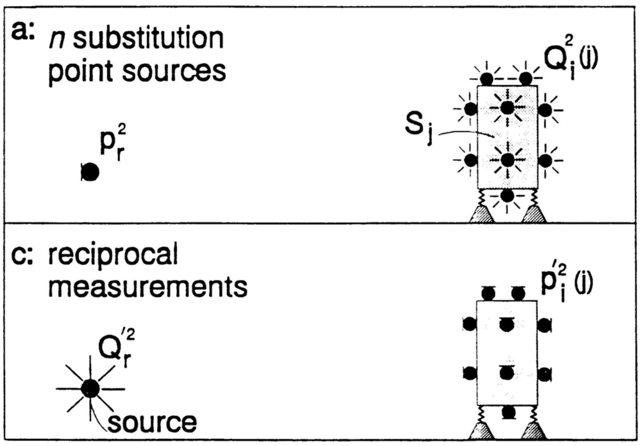

Figure 15: Source description with uncorrelated monopoles as proposed by Verheij [44, 69].

Verheij applied the method for the investigation of the transfer of airborne noise from a truck engine oil pump and from a diesel generator aboard a frigate [61, 62, 70]. In the latter case he separated the contributions from the bottom and the top part of the engine – see Figure 16.

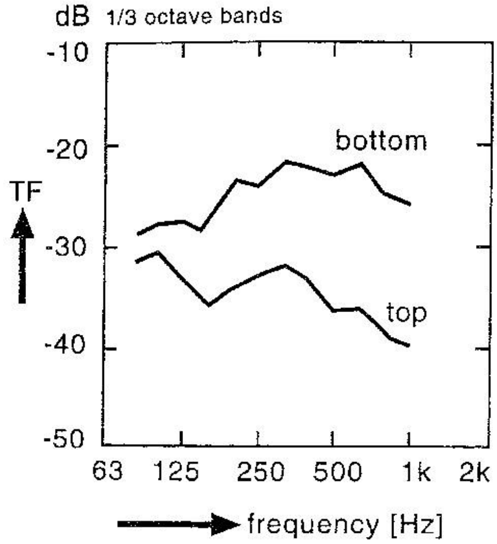

Figure 16: Transfer functions to underwater from different source areas on the diesel engine [70].

These results show that the method is very suitable for the investigation of noise reduction by partial enclosures of an engine.

Measurement of noise reduction of shielding measures for trains, using acoustical substitution sources

Basically the same approach was used by Geerlings et al [75, 81] for the investigation of different shielding measures for trains. In this case substitution sources in the form of monopoles and doublets were used for the characterisation of the radiation by train wheels – see Figure 17. For each of the partial sources, transfer functions with and without shielding were measured reciprocally. The results for the partial sources were weighted with the aid of the TWINS calculation model.

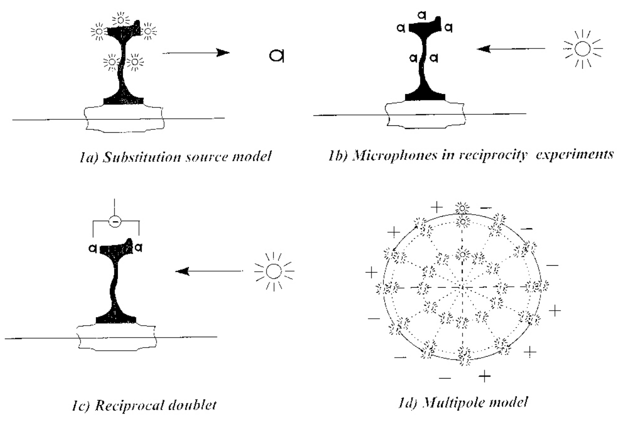

Figure 17: Model-based set of acoustical substitution sources for a train wheel and the principle of the reciprocal insertion loss measurements [81].

Geerlings et al developed a similar approach using mechanical substitution sources [74]. In that case the rail was also included.

Structureborne sound source description for small and compact machines

For small and compactly built machines, Verheij developed a source description in terms of a set of correlated pseudo forces which is able to reproduce all vibrations [69]. Figure 17 gives an overview of the approach. The accelerance matrix mentioned in part b of the figure, is measured reciprocally.

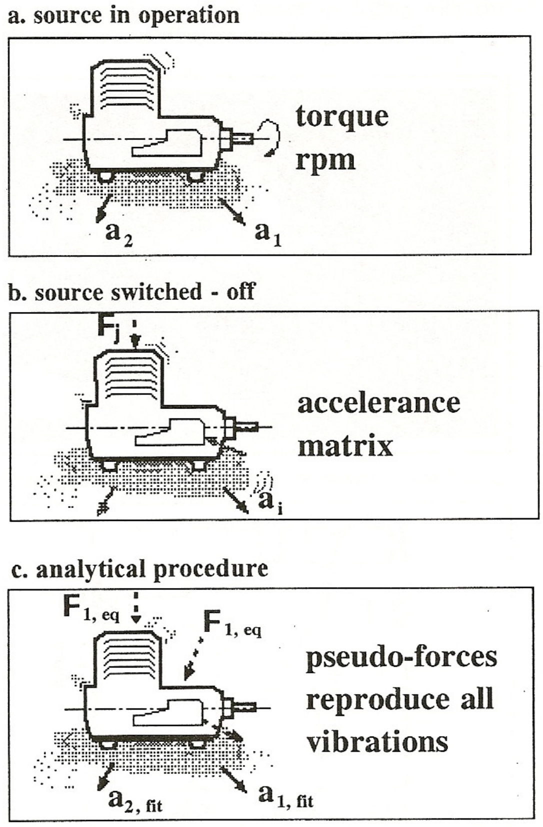

Figure 17: Sketch showing the procedure for the determination of the set of pseudo-forces [69].

9. Conclusions

Reciprocity is a property of all stable, linear, passive systems which contain only bilateral (reversible) elements. Flow may disturb reciprocity, but there are many cases in which the influence is so small that application of reciprocity is still acceptable.

With the aid of reciprocal measurements, it is often possible to do experiments which are very difficult to perform in another way. Application of reciprocity measurements may also be advantageous because it may provide more accurate results than direct measurements.

The reciprocal measurement techniques for the analysis of sound paths and for source characterization are well developed. The literature shows that these techniques are widely applied as tools in the development of quiet road vehicles and navy ships. There is also significant application related to the development of quiet aircraft and trains. For other areas of noise control the application is still rather small.

Records on the use of electro-acoustical and electro-mechanical reciprocity are scarce. It seems that the possibilities of these are not yet fully recognised.

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