HBEM Simulation of Noise Radiation from Rolling Tires

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Introduction

The sound radiation from rolling tires is studied numerically by using a sequential finite element (FEM) – hybrid boundary element (HBEM) approach. The equations of motion for the rolling wheel are developed in the frame of an arbitrary Eulerian-Lagrangian description with a time independent formulation for steady state rolling. The strongly nonlinear problem is treated by an incremental-iterative approach including the computation of contact shear and normal tractions. After the configuration of steady state rolling is known, a modal analysis is performed in this deformed state. For the eigen-analysis, small linear vibratory displacements are superimposed on the large deformations of the fixed control state. For further details on the FEM formulation, the reader is referred to Nackenhorst [6] and Gaul et al. [2].
The noise radiation caused by the vibration modes is computed by the symmetric hybrid boundary element method [4, 7] . The relative normal velocities at the wheel surface are the Neumann data of the acoustic domain. The sound field in the domain surrounding the wheel and road is determined by an efficient field-point algorithm inherent in the HBEM [3]. The road surface is modeled as hard–walled by a mirroring technique of the fundamental solution.

Hybrid Boundary Element Method

A time-harmonic formulation of the Hybrid Stress BEM for acoustics is derived from the Hellinger-Reissner potential. As field variable, the velocity potential $\phi(x,t) = \mathcal{R}\{\hat{\phi}(x)\mathrm{e}^{\mathrm{j}\omega t}\}$ is chosen. The complementary energy formulation of the Hellinger-Reissner potential implies an independent Neumann variable which is the gradient of the velocity potential $\phi_{,i}$ in the domain and the flux ψ on the boundary. The resulting potential field $\phi^{\nabla} = \phi^{\nabla}(\phi_{,i})$ is an exclusive function of the gradient.
The Hellinger-Reissner potential for acoustics in frequency domain reads

$$\hat{\mathcal{H}}^{\mathrm{R}} = \int_{\Omega} \frac{1}{2}\rho\left(\hat{\phi}_{,i}\hat{\phi}^{*}_{,i} + \kappa^2\hat{\phi}^{\nabla}\hat{\phi}^{\nabla^*} + 2((\hat{\phi}_{,i})_{,i})\hat{\lambda}^{*}\right)\mathrm{d}\Omega$$

$$- \int_{\Gamma_\psi}\rho(\hat{\psi}-\hat{\bar{\psi}})\hat{\phi}^{*}\mathrm{d}\Gamma - \int_{\Gamma_\phi}\rho\hat{\bar{\phi}}\hat{\psi}^{*}\mathrm{d}\Gamma \Rightarrow \text{stat.}, \quad (1)$$

with Dirichlet and Neumann boundary data $\hat{\bar{\phi}}$ and $\hat{\bar{\psi}}$, respectively. Compatibility between the potential field $\hat{\phi}$ on the boundary and $\hat{\phi}^{\nabla}$ in the domain is enforced in a weak sense by weighting with Langrangian multipliers, which can be interpreted as $\hat{\lambda} \equiv \hat{\phi}^{\nabla}$. Vanishing of the first variation of $\hat{\mathcal{H}}^{\mathrm{R}}$ leads to the stationary condition

$$\delta\hat{\mathcal{H}}^{\mathrm{R}}(\hat{\phi}_{,i}, \hat{\phi}^{\nabla}, \hat{\phi}) = 0\,. \quad (2)$$

For a numerical solution, proper approximation functions have to be applied. The fields on the boundary are discretized by the product of shape functions in the matrices $\boldsymbol{N}_\phi$ and $\boldsymbol{N}_\psi$ with nodal values $\check{\boldsymbol{\phi}}$ and $\check{\boldsymbol{\psi}}$. The key idea of the derivation of the HBEM is the approximation of the velocity potential and gradient fields in the domain by the fundamental solutions $\boldsymbol{\Phi}$ and $\boldsymbol{\Psi}$ weighted with generalized loads $\boldsymbol{\gamma}$

$$\begin{aligned} \hat{\boldsymbol{\phi}}(x) &= \boldsymbol{\Phi}^{\mathrm{T}}(x,\xi)\,\boldsymbol{\gamma}(\xi) \\ \text{and} \quad \hat{\boldsymbol{\psi}}(x) &= \boldsymbol{\Psi}^{\mathrm{T}}(x,\xi)\,\boldsymbol{\gamma}(\xi)\,. \end{aligned} \quad (3)$$

Modification of the domain such that small spheres with radii ε – centered at the load points collocated with the nodes where the fundamental solutions are singular – are subtracted, the modified domain Ω' with boundary Γ' is introduced. The properties of the Dirac loads acting at points located outside of the considered domain lead to a vanishing domain integral in the limit $\Omega' \to \Omega$. Inserting the approximations in the first variation of the reduced variational principle and applying the fundamental lemma yields

$$\begin{bmatrix} -\boldsymbol{F} & \boldsymbol{H} \\ \boldsymbol{H}^{\mathrm{H}} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{\gamma} \\ \check{\boldsymbol{\phi}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{f} \end{bmatrix}, \quad (4)$$

with equivalent nodal forces $\boldsymbol{f}$.

Application: A Simple Wheel

The pressure field radiated from a wheel representing a simple tire model is depicted in Fig. 1 for the second eigenfrequency at 1279 Hz. The results of the FEM contact simulation and subsequent eigenanalysis are used as Neumann boundary data for the HBEM computation of the acoustic field. In the pure Neumann problem the equivalent nodal forces $\boldsymbol{f} = -\int_\Gamma \boldsymbol{N}_\phi \mathbf{v}_n \, \mathrm{d}\Gamma$ are given completely by the normal velocities $\mathbf{v}_n$ on the tire surface. Thus, by eq. (4) the weighting parameters $\boldsymbol{\gamma}$ can be obtained and then inserted in the domain approximation eq. (3) yielding the complete field solution. For this, only the complex conjugate transpose $(\,)^H$ of the matrix $\boldsymbol{H} = \int_\Gamma \boldsymbol{\Psi}^* \boldsymbol{N}_\phi^T \, \mathrm{d}\Gamma$ needs to be calculated and no further boundary integrations are necessary.

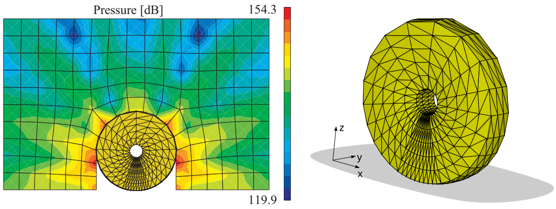

Figure 1: Noise radiation from rolling tire

Mirroring Technique

In a first approach the road surface influence can be modeled as hard–walled, which is efficiently implemented by a mirroring technique of the fundamental solution as discussed in the following. The fundamental solution for the Helmholtz equation in full space [1] is given by

$$\Phi = \frac{1}{4\pi r} e^{-ikr} \, . \tag{5}$$

Herein, r is the Euclidean distance between the field point $\mathbf{x}$ and the so called load point $\boldsymbol{\xi}$. By the use of the modified test function

$$\Phi_h(r, r') = \frac{1}{4\pi} \left(\frac{1}{r} e^{-ikr} + \frac{1}{r'} e^{-ikr'} \right) \, , \tag{6}$$

a hard–walled surface of the road is implemented without discretization of the road surface. r' is the distance between the field point and the mirror image of the field point on the hard–walled surface. This can be visualized as a superposition of two sound fields, the actual sound field generated by the radiator and a reflected wave from the road surface. This is called the mirror technique. In conclusion only the surface of the tire has to be discretized.
In Fig. 2 the radiated sound field calculated with the modified test function (6) is shown. One notices an increase of sound pressure level by approximately 3 dB compared to the full space solution in Fig. 1. The radiated sound waves do not hit the surface perpendicular and thus, the increase of sound pressure is not a doubling as one would expect from theory.

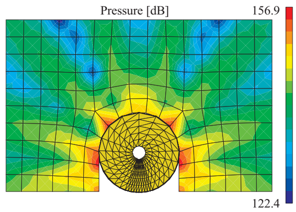

Figure 2: Noise radiation on hard–walled surface

Conclusions

Based on eigenmodes, which are determined in the deformed state of a stationary rolling wheel by finite element techniques, sound radiation analysis is carried out by the hybrid boundary element method. This allows the simulation of the radiation characteristics of single mode shapes.
For the computation of noise radiation from the rolling tire, the HBEM offers the following advantages: a) The problem dimension is reduced by one and the Sommerfeld radiation condition is fulfilled inherently, b) for a Neumann problem, field point evaluation is numerically very efficient and c) a hard–walled road surface can be modeled by mirroring technique without discretization.

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