DIMENSIONAL ANALYSIS AND CONCERT HALL ACOUSTICS – AN APPLICATION INVOLVING THE REVERBERATION TIME

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1. Introduction:

Dimensional Analysis (DA) is a powerful tool that helps physical modelling in many fields of sciences. Its methodology can be also useful in Acoustics. This work uses the approach of DA in the investigation of room acoustics. For this purpose, dimensionless numbers involving an important acoustic parameter – the reverberation time (RT) – are obtained and used to plot data from various concert halls. The conditioning of the data in this way evidences a common behaviour of these halls in regard to this acoustical parameter. Dimensionally homogeneous formulas, similar to Wallace C. Sabine's expression for the reverberation time, are obtained for each frequency band. These formulas have the advantage of being based only in geometrical characteristics of the hall, which can be obtained directly from their architectural drawings. Posterior comparison with measurements made in a Brazilian hall shows reasonable agreement.

2. Methodology and results:

The correct application of the methodology of DA starts with the choice of a complete set of physical quantities relevant to the phenomenon under study. This can be physical constants, geometric and physical parameters and one unknown quantity of interest. The choice of quantities is critical and will depend on the problem and mostly on the experience of the researcher.

For the problem in question, the following quantities should be considered (SI system): the unknown quantity *reverberation time* RT (s); the *volume* of the hall V (m^3); the *area* of the exposed absorbing surfaces S (m^2); the *frequency* f (s^{-1}); the *sound velocity* c_0 (m/s); the *acoustic pressure* p (N/m^2) and the dimensionless ratios, shape factors and absorption coefficients.

Apart from the unknown RT, one geometric characteristic of the hall was chosen, its volume V, and quantities that are related to the phenomenon itself, i.e. S and f. One characteristic of the medium, c_0, and one inherent to the sound decay, p, should also be included in this set of physical quantities.

These quantities are then organized in a dimensional matrix and later into linear equations whose solutions leads to the dimensionless numbers.

Following the procedure indicated by CARNEIRO (1996), 3 dimensionless numbers are obtained and shown in Table 1 below:

$\Pi_1 = \dfrac{\sqrt[3]{V^2}}{S}$	$\Pi_2 = \dfrac{f\sqrt[3]{V}}{c_0}$	$\Pi_3 = \dfrac{c_0 TR}{\sqrt[3]{V}}$

Table 1. Dimensionless numbers as a solution of the dimensional matrix.

The numbers generated are general and could represent data for different types of enclosures. The reason for choosing concert halls is that data is compiled for this kind of hall in literature, once measuring a large amount of data around the world would be impracticable. Also because the kind of music these halls are made for, their purpose, the building techniques, materials and shapes are somewhat the same, which suggests that these halls should keep a certain degree of similarity among them.

The area of the audience (S_t) is responsible for the majority of absorption in a concert hall and therefore is a representative area of the hall and though can be chosen to figure in the number Π_1. Furthermore, it is assumed that $\Pi_3 :: \Pi_1$ or, in other words, Π_3 / Π_1 = numerical constant.

The existence of similarity can now be investigated with the help of DA. Figure 1 shows a plot of Π_3 x Π_1 for the data for concert halls taken from BERANEK (1996) in the band of 1kHz:

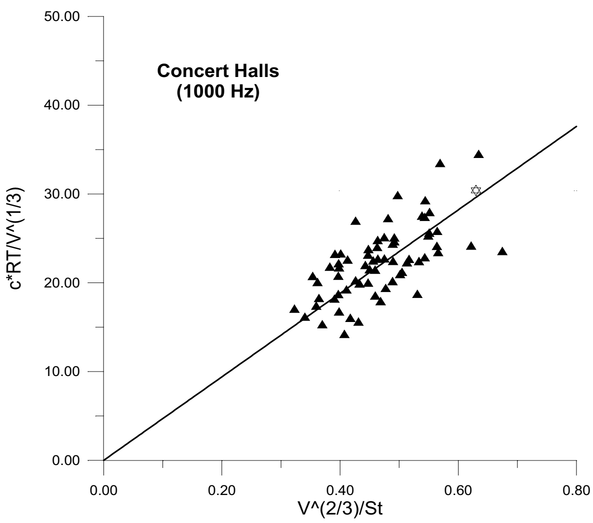


Figure 1. Data for concert halls in the band of 1kHz. The star indicates Leopoldo Miguez Hall.

A best fit for the data on Figure 1 generates:

$$\frac{\frac{c_0 RT}{\sqrt[3]{V}}}{\frac{\sqrt[3]{V^2}}{S_t}} = \frac{c_0 RT}{\frac{V}{S_t}} = 47 \quad \therefore \quad RT = \frac{47}{c_0}\frac{V}{S_t}$$

The expression above is very similar to Sabine's formula for the reverberation time. However, it has some particularities. This dimensionally homogeneous expression is obtained only on dimensional basis with no prior assumption about the phenomenon or restriction over the sound field or the kind of room that can be represented in this manner. Of course, it is important to compare rooms of the same kind if useful information is to be obtained. One advantage of conditioning the expression in this way is that it is based only in geometrical parameters of the hall that can be obtained directly from the arquitectural drawings with no necessity of the difficult task of assigning absorption coefficients to exposed surfaces.
This formula is based on data from various halls and, as stated before, it is not an estimator of the actual value of *RT* in a room. It concentrates information in one graph and serves as a guide in early stages of the design of a new hall where a first estimative of *RT* or the size of the hall or audience is required. This guidance is also useful in stablishing a range of variation of this parameter based on data from successful halls in regards to their acoustical behaviour.

Similar plots can be derived for the other frequency bands and are shown in Figure 2 and summarized in Table 2. The coefficients for each frequency are dimensionless.

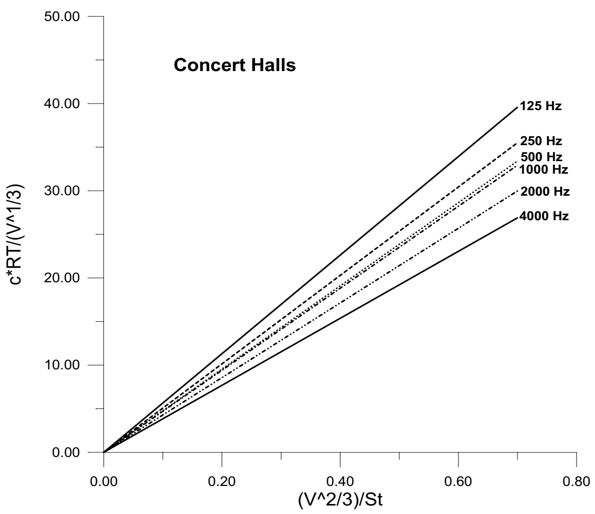
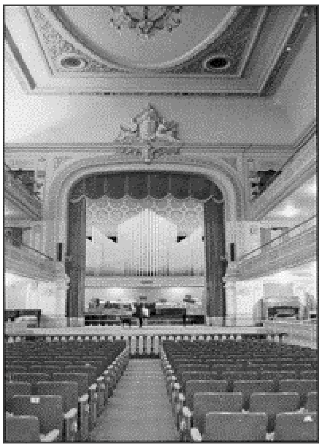


Figure 2. Results for various halls and different frequency bands. Points are ommitted.

Figure 2 reproduces an important feature in concert halls: the mean behaviour of these halls indicates that the absorption of sound increases with frequency. Table 2 shows progressively lower coefficients toward higher frequencies. It is well known that sound absorption provided by the audience increases with frequency. The individual behaviour of absorbing materials can be quite different from each other and their absorption coefficient not necessarily increases with frequency. However, in general, the mean behaviour of these halls indicates that lower *RT* is expected in higher frequencies.

Table 2. Dimensionless coeficients for concert halls.

$\mathbf{TR} = (\ldots)\frac{\mathbf{V}}{\mathbf{c_0\, S_t}}$						
***f*(Hz)**	**125**	**250**	**500**	**1000**	**2000**	**4000**
Coeff. (...)	56,5	50,7	47,7	47	42,8	38,4



The Leopoldo Miguez Hall (1100 seats, 2 balconies) was built in 1922 and inspired in Gaveau Hall in Paris. With the help of its architectural drawings, its volume (5680 m^3) and audience absorbing area (504 m^2) can be calculated.
The measurements undertaken there show agreement with the mean behaviour of the concert halls for the mid-bands (differences less than 3,4% in RT) and reasonable for low- and high-bands (around 15%). However, the difference between the measurements in the low- and high-frequency bands is also bigger than in the mid range.

3. Conclusions:

The non-dimensional numbers obtained through the application of the methodology of DA represent an alternative way of analysis. It furnishes a straightforward tool in approaching physical problems difficult to be modelled and can be useful in other problems in Acoustics also. The data for the concert halls reveals a certain degree of similarity in the acoustical behaviour of these halls in regard to the reverberation time. The measurements in a Brazilian hall show reasonable agreement.

4. Bibliography:

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