A waveguide using the geometrical properties of conicoids:

**Principles, design and applications.**
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In the last ten years, coverage improvement of sound reinforcement systems has become a major issue for loudspeaker manufacturers.

Good results have been obtained with distributed sound systems, installed as “light projectors” providing constant “lighting” over the zone to be covered. However, intelligibility tends to a limit related to the induced reverberant field.

In other words, there are acoustical situations which do not allow satisfactory speech reproduction with any distributed sound system. Moreover, these designs require many sources, and installation costs often exceed the equipment cost.

To reduce these costs, loudspeakers are often assembled in clusters, which are aesthetically good looking, but do not provide satisfactory results because of incoherent and uncontrolled sound dispersion.

Professional loudspeaker manufacturer, Nexo, has been working over the last three years to develop a new generation of loudspeaker system. This design process is (partly) described in the presented paper.

**Some theory…**

**The point source**

Point source radiation properties are well known. Assuming that a source can be considered as a “point in space” as long as its dimensions are small in relation to the emitted wavelength, the radiated pressure is given by:

\[ p_s(r) = \frac{jk \rho C q_s \exp(-jkr)}{4\pi r} \]

where:
- \( q_s \) is the volume velocity (m³/s)
- \( r \) the distance from the source to the observation point (m).

From the above formula, it clearly appears that:

- Sound pressure level decreases at –6 dB per doubling of distance: a point source will provide uneven sound distribution if the distance \( r \) varies too much (figure 1);
- A point source is omnidirectional: a small source in relation to wavelength cannot achieve good directivity control (typically, loudspeakers are omnidirectional at low frequencies, and achieve “nominal coverage” above 2 kHz).

**Arrays of point sources**

When \( n \) point sources are line-coupled with equal spacing \( d \) and equal volume velocity \( q_s \) \[1\], assuming that distance is large in relation to the array dimension, the resulting pressure is given by:

\[ p_n(r) = \frac{jk \rho C q_s \exp(-jkr)}{4\pi} \frac{\sin(\frac{nd\sin \theta}{2})}{\sin(\frac{kds\sin \theta}{2})} \]

(1)
Directivity function, which expresses the ratio between off-axis and on-axis pressures in the far field ($r \gg nd$) is consequently:

$$h(\theta) = \frac{\sin\left(\frac{nkdsin\theta}{2}\right)}{n\sin\left(\frac{kdsin\theta}{2}\right)}$$

(2)

These formulae show that:

- directivity function varies with frequency x spacing product;
- at low frequencies, where $d$ is small in relation to wavelength, the resulting pattern is almost omnidirectional;
- at higher frequencies, path-length differences are responsible for sound constructive and destructive interferences, whose location varies with frequency.
- No analytical study can properly express what occurs in the near field.

Figure 2 gives the resulting coverage for a doublet, summing various frequencies.

Practically, when two “30° coverage” loudspeakers are assembled with a 30° angle, the resulting coverage is so complex that it is anything but 60° (figure 3).

**Acoustic antenna (or line source)**

Acoustical radiation of an antenna has been previously studied by various authors [2] [3]; the radiated pressure is given by the following equation:

$$p(r) = \frac{jk\rho CqL}{4\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\exp(-jkr)}{r} \, dx$$

(3)

where: $L$ is the antenna length (m).

Approximating this expression in the near field ($r < \frac{fl^2}{2C}$), the on-axis pressure becomes:

$$p_{ax}(r) = \frac{jk\rho CqL}{\sqrt{8\pi kdx}} \exp(-j(kr + \frac{\pi}{4}))$$

(4)

In the far field ($r > \frac{fl^2}{2C}$), the on-axis pressure is:

$$p_{ax}(r) = \frac{jk\rho CqL}{4\pi} \exp(-jkr)$$

(5)

and the far-field off-axis behaviour is described by the directivity function:

$$h(\theta) = \frac{\sin\left(\frac{kL}{2} \sin \theta\right)}{\frac{kL}{2} \sin \theta}$$

(6)

Properties deriving from equations (4), (5) and (6) are:

- In the near field, sound pressure level decreases at –3 dB per doubling of distance;
In the far field, sound pressure level decreases at –6 dB per doubling of distance (which is a good definition of the far field);

Limit distance between near field and far field is proportional to frequency and to the square of the antenna length (leading to an unbalanced frequency response over distance);

Coverage off-axis varies with frequency, and becomes extremely narrow at high frequencies; side lobes are minimized.

However, the off-axis coverage of an antenna varies so much, and becomes so narrow at high frequencies (figure 4) that most of its practical applications are in the military field (sonar etc…) ; some authors even state that such an object could never have applications in the audio-reproduction domain [3].

**Progressively curved line source**

Interesting results can be obtained by progressively curving an antenna in relation to the audience geometry.

In this case, equation (3) is still valid to compute the pressure, but no simplification can be done as in equations (4) and (5).

Numerical analysis shows that a properly designed, progressively curved antenna design has the following properties:

- constant level over the audience from one source (with restrictions in frequency related to the height of the line);

- high sound pressure level drops quickly off the covered zone (typically more than 15 dB within 2 or 3 meters);

- good frequency response balance over distance for sufficient line height.

Figure (5) illustrates these properties.

**Loudspeaker design**

**Waveguide prototype construction**

The intention is to develop a loudspeaker whose velocity profile is constant over its height (or width). At low frequencies, such a velocity profile can be efficiently approximated as long as \( d \) is smaller than half-wavelength; at higher frequencies – typically from 1 kHz to 20 kHz –, a constant velocity profile requires source spacing measuring less than a centimetre. Consequently, it is essential to create a device for both the mid and high frequencies.

The waveguide design described below was filed at the French Patent Office on September, 8, 2000. It was published on March, 13, 2002 (#1187094) and has also been internationally extended.

The Geo waveguide uses an acoustic mirror (infinite acoustic impedance surface) to modify the acoustic wave profile. This mirror is defined as a portion of a conicoid, which is a surface constructed by rotating a conic curve around its foci. When sound is generated from a point source at one focus, the velocity profile after reflection on the mirror is identical as if sound was generated from the second focus.

Three cases can be determined:

- the mirror is a portion of a hyperboloid: the spherical profile of the acoustic wave is converted after reflection into a new spherical profile with an increased radius (figure 6);

- the mirror is a portion of a paraboloid: the spherical profile is converted into a flat profile, the second focus being at infinite (figure 7);
- the mirror is a portion of an ellipsoid: the spherical profile is converted into a convex spherical profile with an increased radius (figure 8).

Figure 9 describes the design steps for a hyperboloid waveguide:

- a: the output desired profile is defined as a portion of a sphere; $S_v$ is the sphere centre (if the sphere radius becomes infinite, this profile becomes flat); its contour can be any shape, however it is rectangular in the described case.

- b: a radial horn is defined as being the envelope linking the sphere centre to the output contour;

- c: the point source (generator) is placed in $S_r$, typically close to the output;

- d: $S_v$ and $S_r$ are defined as the two foci of a hyperbole (red line);

- e: rotating this hyperbole around the $S_r$-$S_v$ axis defines a hyperboloid surface, whose intersection with the equivalent radial horn defines the acoustic mirror;

- f: the point source is connected to the mirror;

- g: the waveguide input is matched to the circular output of a compression driver

- h, i: pictures of a 10° wavefront waveguide prototype, loaded with a configurable 80°/120° directivity device in the non-coupling plane

First applications: Geo S805 & Geo S830

The waveguide has been implemented in the Nexo Geo S805 (5° output profile) and Geo S830 (30° output profile) after a three year R&D program. These cabinets were first disclosed at the New York AES in November 2001 and released at the Frankfurt Music Messe in March 2002.

The Geo S805 and S830 are two-way passively filtered cabinets, with an 8” direct radiating Neodymium driver, and a 1” Neodymium driver waveguide loaded. Each cabinet weights 12.5 kg. Dimensions are 406mm(H) x 250mm(W) x 219mm(D). Nominal impedance is 16 ohms, allowing connection of between 6 - 8 cabinets to a single amplifier channel. Recommended amplifier input is around 750 W per cabinet. Dispersion in the non-coupling plane is 80° / 120° (configurable). The rigging system allows angles from 0.31° to 5° in logarithmic steps (figures 10 & 11).

Measurements

The first validation measurements were achieved at Nexo’s R&D department during Spring 2001. These measurements concerned:

- The waveguide’s far-field directivity;
- The waveguide output velocity profile using a velocity probe;
- The coupled waveguide far-field directivity.

These measurements have been systematically compared with the theoretical equivalent antennas behaviour (straight or curved) in order to validate the technology.

Figures 12, 13 and 14 show some of the results:

- Figure 12: 30° waveguide angular coverage compared with a 30° antenna (white line);
- Figure 13: 5° waveguide far-field polar diagrams (blue) compared with a 5° antenna (red);
- Figure 14: Two 30° waveguide angular coverage compared with a 60° antenna (white line).
A second measurement session on large arrays was done at the NEC in Birmingham. This venue has two advantages: it is very reverberant, and has a flat floor of 50m x 50m with no obstacles. These measurements concerned:

- Sound pressure level and frequency response using a wide variety of arrays;
- Sound pressure level and frequency response versus distance;
- Gain in intelligibility criteria in relation to a conventional sound system;
- Mechanical accuracy of the rigging system;
- R&D simulation software validation (using a Matlab™ platform).

Figures 15, 16 & 17 show some of the results:

- Figures 15 and 16 show the frequency response evolution of an 8 * S805 straight array from 2 metres to 40 metres, compared with its theoretical equivalent (8 * 25cm / 5° curved antennas, aligned at 0°); measurements are referenced to the 40 metres frequency response.
- Figure 17 shows the pressure gain from 1 to 4 S805’s angled at 0°.

Differences above 8 kHz are related to air absorption - which is not taken into account in the simulation software – and to the measurement setup (microphone on the floor, that is not perfectly reflective at high frequencies). Once these effects are compensated for differences between theoretical model and measurements, the error does not exceed 1.5 dB up to 14 kHz. Acoustic summation is 6 dB SPL per doubling quantities with no interference.

An 8 * S805 array provides a consistent 98 dBA sound pressure level with a +/-2 dBA tolerance over distances varying from 8 to 80 metres. Measured RASTI varies from 0.8 to 0.65 over the same distance range.

Following these validation measurements, a user-driven software calculation interface, based on the R&D simulation algorithm, was developed; this software allows fast array geometry optimisation in relation to the audience geometry and gives predictable direct sound results (figure 18).


Figure 1: Point source radiation (3 dB per colour)

Figure 2: Two point sources radiation (3 dB per colour)

Figure 3: Two 30° cabinets assembled at 30° - coverage

Figure 4: Antenna radiation (3 dB per colour)

Figure 5: Progressively curved antenna radiation (3 dB per colour)
Figure 6: Hyperbolic reflector

Figure 7: Parabolic reflector

Figure 8: Elliptical reflector
figure 9: waveguide design and construction
figure 10: Geo S805 / S830 front view

figure 11: 6 Nexo Geo S805 + 1 Geo S830 array
Figure 12: Geo S830 coverage (white = 30° curved antenna)

Figure 13: Geo S805 waveguide (blue) & 5° equivalent antenna (red) polar diagrams – 5 dB/div.

Figure 14: Two Geo S830 coverage (white = 60° curved antenna)
Figure 15: Frequency response for 8 Nexo Geo S805 angled at 0° from 2 meters to 40 meters (referenced at 40 meters)

Figure 16: Frequency response for 8 25cm/5° antennas angled at 0° from 2 meters to 40 meters (referenced at 40 meters)

Figure 17: Frequency response for 1, 2 and 4 Geo S805 (referenced to 1 Geo S805)
figure 18: Matlab™ based simulation software