# Nonlinear Acoustic Doppler Tomography of Flowing Liquid

Igor Didenkulov[1], Leonid Kustov[2], Alexander Martyanov[2], Nikolay Pronchatov-Rubtsov[2]
[1]*Institute of Applied Physics, Nizhny Novgorod, Russia; Email: din@hydro.appl.sci-nnov.ru*
[2]*Nizhny Novgorod State University, Nizhny Novgorod, Russia; Email: nikvas@rf.unn.ru*

## Introduction

Measurement of parameters of flowing liquid is one of problems, which have many applications. Traditionally it is solved by linear acoustic methods, *i.e.* the Doppler technique, which meets some restrictions in strong reverberation conditions. The nonlinear acoustic methods are based on receiving new frequencies from the medium compared to those which are input into it and therefore free from reverberation.

It is known that a bubble has prominent nonlinear properties. Nonlinear responses in scattered fields from a bubble are easily observed at the second or higher harmonics of the incident frequency, as well as at the subharmonics of the fundamental frequency and at the combination frequencies of the primary waves [12]. This opens up the possibility of using it for bubble detection and sizing. The advantage of nonlinear acoustic techniques are their high selectivity. Different nonlinear acoustical methods have been developed for bubble diagnostics [345].

For moving bubbles the specific Doppler frequency shift arises at the difference frequency, which can be used for bubble flow velocity measurement [6]. For low-density bubble populations such measurements can be done in the approximation of given primary waves. For dense bubble streams it is necessary to take into account attenuation of primary waves at bubbles inside the jet. The latter case is considered in this paper and called the difference frequency Doppler tomography method. In the present paper we use the difference frequency method for reconstruction of the flow parameters distribution across the submerged water jet filled with bubbles.

## Experiment

A scheme of the experiment is shown in Fig.1. Two plane piezoceramic transducers insonify the jet and the scattered acoustic wave at the difference frequency is received by the receiving system. The frequency of transducers 1 and 2 were around 1 MHz, and the difference frequencies varied from 100 to 200 kHz.

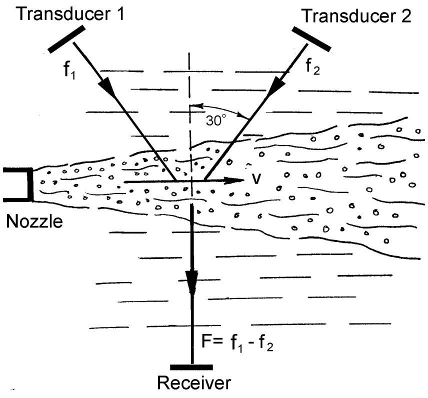

**Fig. 1: Dependence of the parameter *G* on ratio *R/δ*.**

In the experiment water flow was created by the centrifugal pump. The nozzle of 1 cm diameter ejected the jet with velocity of 25 m/s. Down the stream the jet diverges and the velocity decreases. The insonified part of the jet was at 60 cm distance from the nozzle. The mean diameter of the jet in the insonified zone was about 11 cm. Diameters of primary acoustic beams in the insonified zone are larger than the jet diameter. The nozzle, two transducers and the receiver were put at the depth of 41 cm from the water surface.

The receiving system represents a spherical hydrophone with the spherical reflector. The reflector of about 30 cm diameter has the focal distance 27.5 cm. The use of the reflector allows one to increase signal to noise ratio in the receiving acoustic signal. Heteronizing were used to transfer the receiving signal into low-frequency range for processing.

## Theory

The experimentally measured spectra of acoustic signals at the difference frequency were compared to the numerical model for reconstruction of water flow parameters. The theoretical model was based on the approximation of the axially symmetric bubble distribution in the cross section of the jet. We supposed that bubbles move with the stream with the mean local flow velocity. It is taken into account the attenuation of primary acoustic waves inside the jet by resonance bubbles as well as the attenuation of the difference frequency scattered wave. Geometry of numerical and physical experiments is shown in figure 2. Since diameters of primary acoustic beams in the insonified zone are larger than the jet diameter, the primary waves can be considered as plane.

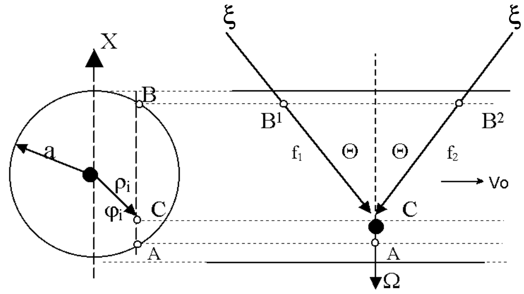

**Fig. 2. Geometry of the problem. Cross (left) and longitudinal (right) sections of the jet.**

Elementary scattering volume in point C radiates the difference-frequency wave, which intensity is proportional to the squared multiplication of primary wave amplitudes and bubble concentrations in the scattering volume. In calculation of primary wave amplitudes insonifying the scattering volume the attenuation of high-frequency primary waves of frequencies $f_2$ and $f_1$ along geometry-acoustic rays $B_1C$ and $B_2C$ is accounted. Insonified part of the jet is considered as cylindrical volume filled in with microbubbles moving with the flow. The intensity of the difference-frequency-scattered signal generated from an elementary volume around the point C and the amplitude of the received signal is calculated.

Supposing that signals of combination scattering from elementary volumes are incoherent and taking into account the attenuation of primary waves and the difference-frequency wave inside the jet along the path CA, one can write the intensity of the difference-frequency signal at the receiver as follows:

$$\Delta W(\rho_i) \sim \exp(-\beta\rho_i) \cdot \int_0^{2\pi} \exp\left[ -2\sigma_f \int_B^C n(\xi)\, d\xi - \sigma\Omega \int_C^A n(x) dx \right] d\varphi \ , \qquad \text{eq. 1}$$

where $\sigma_f$ ,$\sigma_\Omega$ - bubble extinction cross sections at the primary frequencies and the difference frequency, respectively, $n(\xi)$, $n(x)$ - distribution of bubble concentrations along the primary wave paths and the difference-frequency wave path. This expression allows one to calculate the intensity of spectral component of the difference-frequency signal in any frequency range. This intensity is the sum of intensities of nonlinear scattered signals from elementary volumes, which are at the same distance from the jet axis. Considering the flow velocity distribution across the jet dependent from the radial distance as

$$V = V_0 \exp(-\beta\rho^2) \ , \qquad \text{eq. 2}$$

one can obtain an expression for the frequency of the difference-frequency-scattered signal by the elementary volume as follows:

$$\Omega_i = 2f \cdot Sin\Theta \cdot \frac{V_0}{C_0} \cdot \exp(-\beta\rho_i^2) \ . \qquad \text{eq. 3}$$

Here $V_0$ – flow velocity at the jet axis, $\rho_i$ - radial distance from the jet axis to the elementary volume of scattering, $\beta$ - scaled coefficient describing the effective jet radius, $C_0$ – sound velocity. In paper [7] it was shown that the bubble distribution across the jet can be described by the function:

$$n = n_0 \exp(-2\beta\rho^2) \sim V^2(\rho) \qquad \text{eq. 4}$$

We used this model to calculate the form of spectral line. Free adjusting parameters of the model are the exponent power $\beta$ of the flow velocity distribution (and bubble distribution) across the jet and the velocity $V_0$ at the jet axis. Besides, values of the jet axis velocity and the bubble concentration were also varied in numerical simulations. Since the primary wave frequencies are closed to each other it was supposed that the primary waves have the same attenuation when propagate inside the jet.

In figure 3 it is shown normalized experimental spectrum of the difference frequency scattering (solid bold line) and theoretical spectra.

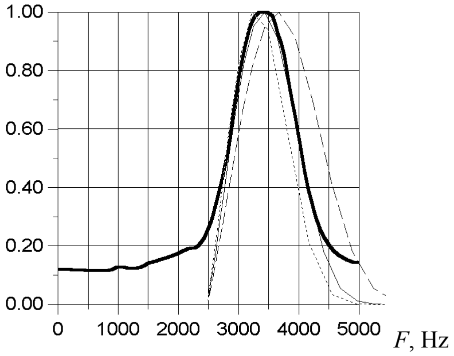

**Fig. 3. Experimental (solid bold line) and theoretically calculated spectra of the difference-frequency-scattered from the cavitation jet acoustic signal.**

Solid thin line in the figure 3 is the theoretical spectrum for the jet flow velocity $V_0$ = 3.2 m/s and for the coefficient $\beta$ = 0,2 cm$^{-2}$. Dashed line is the theoretical spectra for the jet flow velocity of 4м/c and for the same value of $\beta$ as in previous case. Dotted line represents theoretical spectrum for $\beta$=0,3 cm$^{-2}$ and $V_0$ =3.2 m/s. In the given example the primary frequencies $f_1$ and $f_2$ were 1000 kHz and 1100 kHz, respectively. The heterodyne frequency was 97.5 kHz. Consequently, the difference frequency $\Omega$=100 kHz corresponds to the frequency 2.5 kHz in figure 3. It is seen good agreement between experimental data and theoretical curve for $V_0$= 3.2 m/s, $\beta$= 0,2 cm$^{-2}$.

## Conclusion

The conducted experiments demonstrate feasibility of the difference-frequency Doppler tomography technique for measurements of the spatial distribution of bubbles and velocity across the stream. The use of the difference-frequency scattering for diagnostics of moving scatterers is more effective compared to the linear scattering due to less reverberation and due to higher sensitivity of the nonlinear scattered wave spectrum to the variation of diagnostic parameters.

This work was supported by RFBR (01-02-17653, 01-02-16938), and by the Russia Ministry of Education (grant E02-3.5-517).

---

[1] Zabolotskaya, E. Soluyan, S., "Emission of harmonic and combination frequency waves by air bubbles," Sov. Phys. Acoust., **18,** 396-398 (1972).

[2] Eller, A., Flynn, H., "Generation of subharmonics of order one-half by bubbles in a sound field," J. Acoust. Soc. Am. **46,** 722-727 (1969).

[3] Ostrovsky, L., Sutin, A., "Nonlinear acoustic diagnostics of discrete inhomogeneities in liquids and solids," *in Proc. 11th Int. Congr. Acoust.* (Paris, 1983), V.2, 137-140.

[4] Zverev, V., Kobelev, Yu., Selivanovsky, D., Sokolov, A., "On a method of detection of gas bubbles in a liquid," Sov. Tech. Phys. **50,** 1544-1545 (1980).

[5] Leighton, T., Walton, A., Field, J., "Acoustic bubble sizing by combination of subharmonic emission with imaging frequency," Ultrasonics, **29**, 319-323 (1991).

[6] Didenkulov, I. Yoon, S.W., Sutin, A., Kim, E.J., "Nonlinear acoustic Doppler effect and its use for bubble flow velocity measurement," J. Acoust. Soc. Am., **106**, 2431-2435 (1999).

[7] Barkhatov, A., Gavrilenko, V., Martyanov, A., "On form of spectrum of signal scattered by the sound absorbing turbulent jet," Sov. Acoust. Phys. **25,** 32-35 (1979).