Sound field modelling in a narrow street by the transport theory

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Introduction

Noise is a major problem for people living in urban areas. Consequently, many mathematical and numerical models have been derived to predict the sound propagation in streets. However, because of the complexity of the façade effects, these models are not still satisfactory. The purpose of the present paper is to propose a general mathematical formalism, to predict the temporal and the spatial distribution of the diffuse sound field energy in urban areas, including diffuse reflections by building façades. The model is based on an application of the classical theory of particles transport applied to the concept of sound particles. As an example, an application to a narrow street is proposed.

Transport model

Sound particle concept

According to the geometrical acoustics assumptions, the sound propagation may be represented by sound particles or phonons. In this way, as mentioned by Joyce [1], geometrical acoustics is a special case of the classical-particle dynamic. A sound particle is then defined as a classical point particle by its elementary energy e, its position $\mathbf{x}$ and its velocity $\mathbf{v}$,whose norm is equal to the sound velocity c. Interactions and collisions between particles are neglected. The phonon follows a straight line until its impact with obstacles or building façades. During a collision, the velocity direction is deflected instantaneously. By assumption, urban areas are considered as ergodic. Hence, the description of the N particles system can be reduced to the knowledge of an artificial single particle system in a 6-dimension phase space Γ involving the three usual space and velocity coordinates ($\mathbf{x}$,$\mathbf{v}$). The statistical behavior of the system is then obtained from statistical mechanics, by using the single particle distribution function $f(\mathbf{x}, \mathbf{v}, t)$. It represents the amount of particles, at time t, with velocity $\mathbf{v}$ to within about $d\mathbf{v}$, in an elementary volume $d\mathbf{x}$ located at $\mathbf{x}$.

Transport equation

In this paper, scattering by urban objects in the street is neglected, so collisions of phonons only take place on the boundaries. The evolution of the sound particle density in urban areas is then similar to the evolution of the molecular density in a rarefied gas or Knudsen gas. Thus, the main equation of the model, can be derived from the transport theory to give:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} f = 0. \tag{1}$$

Boundary conditions

Typically, surfaces are made up of many irregularities due to window recesses, decorative structures, etc. According to the size of façade irregularities and frequency, reflections may be specular or not. The ratio of non-specular and specular reflection is expressed by the accommodation coefficient $d\,(\mathbf{x})$. It varies from 0, for non-specular reflection, to 1 for perfect specular reflection. This coefficient is similar to the well known diffuse-reflection coefficient $\delta = 1 - d$ [2]. If the reflection is specular, the reflection can be considered in a deterministic way. In the second case, surface reflection can be introduced in the model, by considering a probabilistic approach. Thus, a positive, integrable and smooth function $\mathcal{R}(\mathbf{x}, \mathbf{v}, \mathbf{v}')$, is introduced. It represents the probability that an incident sound particle with a velocity $\mathbf{v}'$ leaves the boundary, at position $\mathbf{x}$ after reflection, with a velocity $\mathbf{v}$. At last, in the present approach, the wall absorption is expressed in a probabilistic way by considering the probability $\alpha(\mathbf{x})$ for $\mathbf{x} \in \partial X$, that a sound particle hitting a façade at the position $\mathbf{x}$ is absorbed.

The boundary conditions express the flow of reflected particles by a building façade as a function of the incident particle flow. By considering the part of specularly and non-specularly reflected sound particles, the flow conservation is written

$$|\mathbf{n}\cdot\mathbf{v}|\, f^{\,-}(\mathbf{x},\mathbf{v},t) = (1-\alpha(\mathbf{x}))\Big[d(\mathbf{x})\,|\mathbf{n}\cdot\mathbf{v}^*|\, f^{\,+}(\mathbf{x},\mathbf{v}^*,t) + (1-d(\mathbf{x}))\int_{\Gamma^+} \mathcal{R}\left(\mathbf{x},\mathbf{v},\mathbf{v}'\right)|\mathbf{n}\cdot\mathbf{v}'|\, f^{\,+}(\mathbf{x},\mathbf{v}',t)\, d\mathbf{v}'\Big] \tag{2}$$

where the left member of this expression represents the reflected flow. The right member term weighted by the reflection coefficient $(1 - \alpha(\mathbf{x}))$, expresses the specular flow (first term) and the non-specular flow (second term). The $+$ an $-$ exponents are introduced to restrict the function $f(\mathbf{x}, \mathbf{v}, t)$ to the phase spaces Γ^+ and Γ^-, representing the incident and reflected sound particles respectively.

Application to narrow streets

As an example, the model is applied to the sound propagation in an empty narrow street with partially diffusely reflecting surfaces characterized by a Lambert's Law. In this case, the problem is completely defined by the transport equation (1) with the boundary conditions (2). However, nowadays, there is no exact analytical solution for such a system of equations. Although the problem could be simulated by numerical Monte-Carlo algorithm, the choice was done to find an asymptotic solution, using probabilistic considerations.

Let us consider a narrow street, where the width ℓ is much smaller than length and height. The sound absorption due to the pavement and the building façades, is neglected. If the sound source is located on the ground, the sound propagation in a narrow street is then similar to the propagation between two parallel planes. If the reflection law is symmetrical with respect to the normal to the façades and does not produce grazing reflection (as the Lambert's law for example), the problem can be solved by an asymptotic approach derived by Börgers *et al.* [3]. The aim of this approach is to show that the transport equation may be reduced to a diffusion equation. This result can be proved using extensive mathematical developments that are detailed in reference [4]. It shows that the distribution function $f(\mathbf{x},\mathbf{v},t) = f(x,y,z,u,v,w,t)$ may be expressed as a product of two functions $q(x,y,t)$ and $\phi(z,u,v,w)$. The last function can be easily calculated, leading to a constant. It suggests that the sound energy is uniform along the street section z. Moreover, it can be shown that the function $q(x,y,t)$, representing the sound field distribution in the plane (xOz) parallel to the façades, is solution of a diffusion equation

$$\frac{\partial q}{\partial t} - \mathcal{K}\left[\frac{\partial^2 q}{\partial x^2} + \frac{\partial^2 q}{\partial y^2}\right] = 0. \qquad (3)$$

$\mathcal{K}$ is defined as the diffusion coefficient: $\mathcal{K} = [(1+d)/(1-d)] \times [\ell\, c/4]$. In order to have a more realistic solution of the temporal and spatial distribution of sound energy in the street, the sound absorption by the openings, the pavement and the building façades have also to be introduced in the diffusion model. This is done by introducing an exchange coefficient h in the classical boundary conditions of the diffusion equation, at both extremities and at the opened top of the street [4].

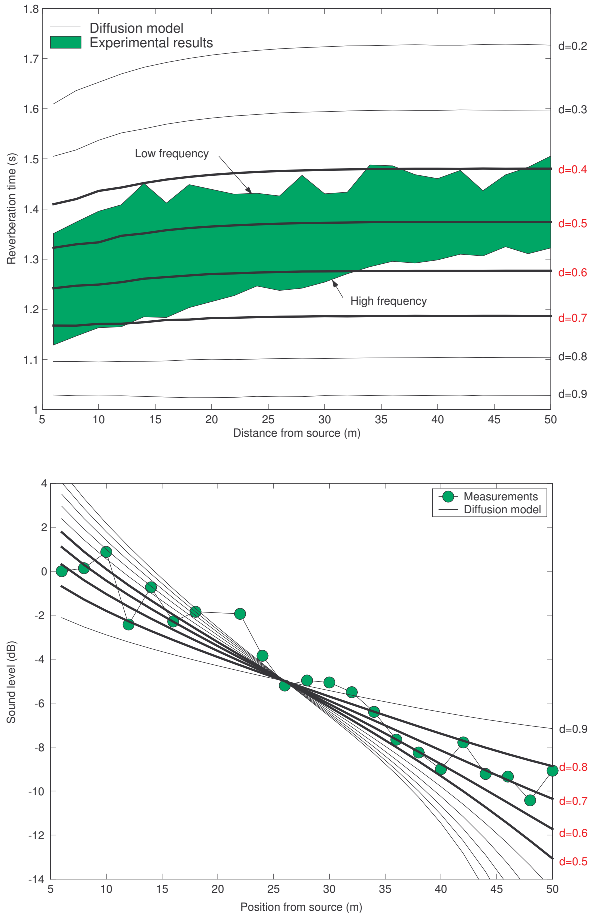

Figure 1: Mean reverberation times (upper curve) and mean sound level (lower curve), along the street, calculated by the diffusion model for $d = 0.2, 0.3, \ldots 0.9$ compared with the experimental data.

Experimental validation

Experiments were carried out in a pedestrian street of 210 m long, 18 m high and 7.90 m wide [5]. Sound levels and reverberation times were measured, along the street, for each 1/3 octave bands between 500 and 5000 Hz. First, these experiments confirm that the reverberation times and sound pressure levels are quite uniform inside a street section, even very close to the source. The model is then in agreement with the experimental results. Moreover, Figure 1 shows the respective comparison between the reverberation times and sound attenuation along the street, to the theoretical solutions for several of d. It can be observed that the model is quite in agreement with experiments for d between 0.4 and 0.8.

Conclusion

This paper shows that the transport model may be a solution to predict the diffuse sound field propagation in urban areas. The expression of the diffusion equation is given for a Lambert's law, but similar developments could be also investigated for any reflection law with a symmetry around the normal to the wall, and without grazing reflection. In a practical point of view, it would be interesting to consider real reflection laws and accommodation coefficients for the building façades, in order to validate and apply the model in all types of urban areas. However, it is then necessary to estimate or to measure these two parameters in urban areas.

References

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