Patch Transfer Function (PTF): a substructuring approach for linear acoustics

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Introduction

For industrial structures, large acoustics problems like noise radiated by cars are very difficult to solve using classical methods like full BEM, because of the complexity of the structure. The coupling between FEM and BEM procedures and the ways to reduce calculation time have been developed from the 70's [1] and this is still a research area of first interest [2]. The PTF approach has been first presented in reference [3]: it considers several acoustic subsystems, which are analysed separately, considering each coupling area as a rigid surface, which is divided in elements called patches. For these calculations, any available method (FEM, BEM…) can be used in order to build a database of transfer functions between sources and patches, which are acoustic impedance transfer functions. Then, continuity relations can be written on the interface in order to couple the sub domains: this approach is close to the mobility and impedance concepts which are widely used in mechanics, it is based on linearity properties of acoustic phenomenon.

Patch Transfer Functions

The PTF method has been described in details in ref. [3], to which readers are invited to reefer for more details. The coupling area between the two considered subsystems has to be divided in surface elements called "patches". For a sake a simplicity, one will suppose here that there is an internal medium, with rigid faces opened by holes allowing the sound to be radiated to an external medium. Analyses of uncoupled subsystems are performed using classical tools (FEM, BEM…), in order to built a database of Patch Transfer Functions $Z_{\alpha\beta}$, which are defined as the ratio between mean pressure on patch α and mean normal velocity on patch β, like indicated in equation 1. This calculation is done considering a normal unit velocity imposed on patch β while other patches are considered as rigid surfaces.

$$Z_{\alpha\beta} = \frac{\langle p \rangle_\alpha}{\langle V_n \rangle_\beta} \tag{1}$$

Some PTFs are also defined between excitation areas (which are vibrating surfaces divided in patches) and coupling patches, and between patches and listening points (microphones locations). Writing continuity relationships on the coupling area leads to a N_p by N_p linear system if the coupling area is divided in N_p patches:

$$\sum_{\alpha=1}^{N_p} \left(Z_{\alpha\beta}^{ext} + Z_{\alpha\beta}^{int} \right) \left\langle V_n^{ext} \right\rangle_\alpha = \sum_{\gamma=1}^{N_s} \tilde{Z}_{\gamma\beta}^{int} \left\langle \overline{V}_n \right\rangle_\gamma \tag{2}$$

In this equation ext and int indicates that PTFs calculations are obtained from internal and external analyses. The unknowns are the mean coupling velocities $\left\langle V_n^{ext} \right\rangle_\alpha$ on coupling patches. The right term is related to the N_s source patches. Inversion of the linear system leads to the value of pressure at listening points. One should notice that this linear system is full, but its reduced size does not result in numerical difficulties. This analysis is performed at each considered frequency step.

Convergence: number of modes

When one of the sub domains is closed with only some holes, the internal analysis can be efficiently done using modal analysis: analytic modes for simple geometry, FEM for complex one. Then, the modal composition can be used to calculate the PTFs. When such a method is used, one has to take care of the number of modes used for this calculation, since they are used to built a linear system which should not be singular. An example is given on Figure 1: a parallelepiped box is considered, divided in two boxes with the coupling area between: this is a strong coupling case. In that situation the mode order in the longitudinal direction of the higher considered mode has to be larger than the number of patches, otherwise the PTFs will be linearly dependant. The figure shows the limit case between the two configurations, while the reference calculation comes from the analytical modal analysis of the box.

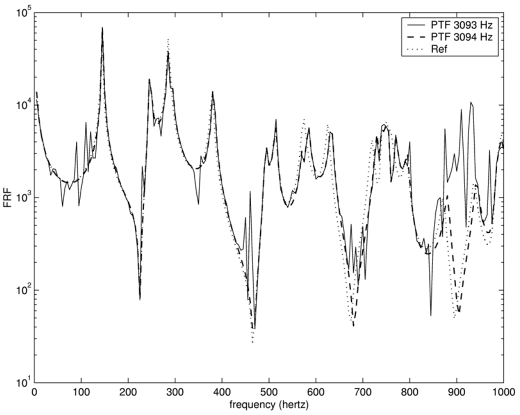

Figure 1: Influence of the number of modes

Convergence: patch size

Another parameter of importance in the PTF approach is the patch size. On Figure 2, the same structure as the previous one is considered, using a large number of modes, while the patch size varies. One can observe that the meshing size of the coupling area which is necessary for convergence is the half wavelength. This criterion is less severe than the 1/6th wavelength practical criterion, widely used in FEM for example.

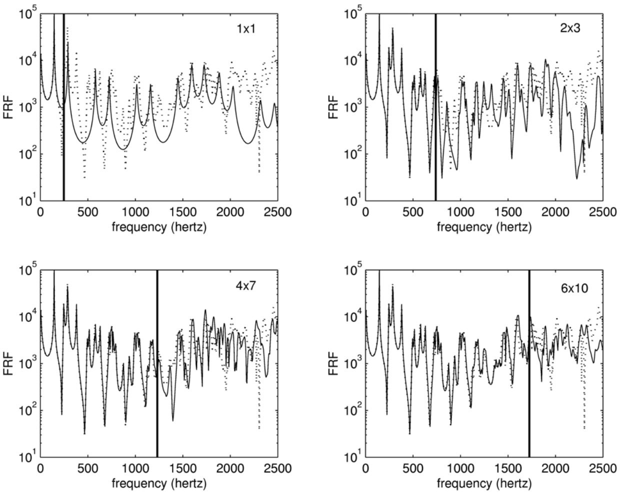

Figure 2: Influence of the patch size (the vertical line indicates the λ/2 limit).

Automotive application

The PTF approach has been developed within the European project VISPeR to estimate the noise radiated through apertures of a vibrating car engine cavity. A finite element analysis of the cavity (including engine and all accessories) has been performed, considering rigid coupling areas, while a simplified Rayleigh approach has been used to built PTFs for the external problem. This model includes reflection on the ground, using source images theory. Inputs for the model are 14 equivalent vibrating sources located all around the engine, using a monopole substitution technique. Some absorbing materials are also considered inside the engine cavity. The test configuration correspond to this situation: the car is placed in a semi-anechoic chamber with reflecting ground, in rolling condition (3rd gear, 3600 RPM) using slick tyres. Measurements have been carried out by CRF.

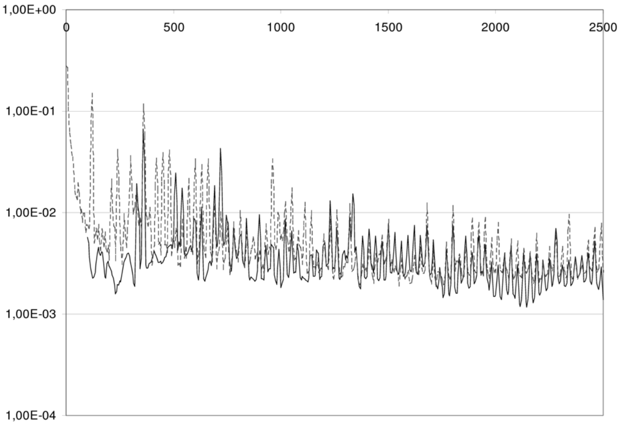

Figure 3: Pressure radiated by an engine in its cavity to a microphone located outside of the car: comparison of numerical results (continuous line) and experiments (dot line) on real vehicle.

Figure 3 shows comparison between experiments and numerical simulations, in terms of acoustic pressure, at a listening point located outside of the car, 1.2 meter height. One can observe on this picture that the method is efficient up to 2.5 kHz: such a frequency range would require an extremely high calculation cost using classical coupled FEM/BEM approach. The differences observed on the lower frequency range correspond to the accuracy of the sources: the method used for their determination seems to be not very accurate for low frequencies. However the global results are very satisfactory comparing with the complexity of the considered problem.

Conclusion

The PTF approach is a substructuring technique for linear acoustics. The coupling area has to be divided in patches, using half-wavelength criterion. Then a database of impedance functions between patches, sources and microphones is calculated using any available method. Writing continuity relationships on the discretized area leads to the coupling of the subsystems with a low calculation cost: since problems are analysed separately, this cost is highly reduced. Moreover, taking into account some changes in the model does not require to do the whole calculation once again: only the modified subsystem has to be analysed again, which is an advantage in a design stage. For some particular changes like absorbing materials characteristics, only the final assembly has to be redone resulting in a very fast estimation of radiated noise, since this step requires only the inversion of a small linear system for each frequency step.

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