Analysis of energy flow and energy densities for one-dimensional acoustic fields

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Introduction

Energy methods have proved to be an efficient technique to model acoustics or structure vibrations in the high frequency range. Some of them assume that structural intensity is linked to total energy density and lead to a diffusion equation for structural intensity [1]. These approximate methods produce good results for one-dimensional systems [2], but the extension to two-dimensional systems fails [3]. Our purpose is to present an exact energy formulation for any frequency in one-dimensional systems. This formulation is able to model power transfers for structural acoustics in dissipative media.

Configuration

In this paper we consider a one-dimensional system in which only longitudinal waves can propagate (see Figure 1). Only four assumptions are put forward: small displacement and small strain, homogeneous and isotropic medium, steady state harmonic waves and hysteretic damping material.

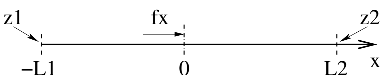

Figure 1: Configuration of the studied system.

For such a configuration, the displacement formulation gives the solution. The displacement potential $\phi(x)$ is the solution of a propagation equation with a second member due to the concentrated load $\boldsymbol{f_x} = f_x\ \boldsymbol{e_x}$ at $x = 0$, and is given by

$$\phi(x) = C_1 cos(kx) + C_2 sin(kx) + \frac{f_x H_0}{\rho\omega^2}(cos(kx) - 1) \quad (1)$$

where H_0 is the Heaviside function. The two coefficients C_1 and C_2 are given by the two specific impedances z_1 and z_2 (normalized by the characteristic impedance of the medium) that traduce mixed boundary conditions [4], respectevely at $x = -L_1$ and at $x = L_2$

$$\left\{ \begin{array}{rcl} \phi_{,x^2}(-L_1) - j\ k\ z_1\ \phi_{,x}(-L_1) & = & 0 \\ -\phi_{,x^2}(L_2) - j\ k\ z_2\ \phi_{,x}(L_2) & = & 0. \end{array} \right. \quad (2)$$

The displacement $\boldsymbol{u}(x)$ is obtained from the potential $\phi(x)$ by derivation: $\boldsymbol{u}(x) = \phi_{,x}(x)\ \boldsymbol{e_x}$. From $\boldsymbol{u}$ and ϕ we can obtain quadratic quantities, the kinetic energy density T and the strain energy density U:

$$T = \frac{\rho\omega^2}{4}\boldsymbol{u}.\boldsymbol{u}^*,\ \ U = \frac{\lambda + 2\mu}{4}\Delta\phi\ \Delta\phi^*. \quad (3)$$

Energy densities lead to the structural intensity $\boldsymbol{I}$ by integration of

$$div\ \boldsymbol{I} = -2j\omega(T - U) - \frac{j\omega}{2}\boldsymbol{f_x}.\boldsymbol{u}^*. \quad (4)$$

Quadratic formulation

Another way to obtain exact quadratic variables is to directly compute them from the linear equations linking them. In our configuration, energy densities T and U satisfy the system of equations

$$\left\{ \begin{array}{rcl} \Delta T + (k^2 + k^{*2})T - 2k^2 U & = & s_0\ \delta_0 \\ \Delta U + (k^2 + k^{*2})U - 2k^{*2} T & = & s_1\ \delta_0 + s_2\ \delta_{0,x} \end{array} \right. \quad (5)$$

where s_0 is the jump of $T_{,x}$, s_1 the jump of $U_{,x}$ and s_2 the jump of U. It must be noticed that these three discontinuities are given by the displacement formulation. The four mixed boundary conditions verified by T and U are derived from equations 2 and 3.

Simulations

Parameters

Simulations were carried out for longitudinal waves propagating in a steel medium between -$L_1 = -10\ m$ and $L_2 = 10\ m$ at the frequency of $5\ kHz$. Properties of the material are given below (see Table 1). At $x = 0$, $f_x = 1\ Pa$. Boundary conditions were chosen disymmetric: $z_1 = 0$ and $z_2 = 1$. Such values mean there is

- a total reflexion of the wave at $x = -L_1$ (due to $\boldsymbol{u}_{,x}(-L_1).\boldsymbol{e_x}$=0),
- an anechoic end at $x = L_2$.

Density ρ ($kg\ m^{-3}$)	7800
Young's modulus E (Pa)	$2.1\ 10^{11}(1 + j\ 0.01)$
Poisson's ratio ν	0.3

Table 1: Properties of the material

Results

The quadratic variables obtained by the displacement formulation and those obtained by the quadratic formulation are superimposed (see Figure 3, Figure 4 and Figure 5). Such results confirm that the quadratic formulation, constituted by equations 5 and mixed boundary conditions for T and U, leads to the exact quadratic variables. Quadratic variables present two scales of variations (for instance see Figure 3). Large scale spatial variations can be observed in the entire dissipative structure. They correspond to propagative components whose decreasing from $x = 0$ is related to the hysteretic damping η. Note that in the part $x > 0$ there is nothing but propagative components because of the anechoic end at $x = L_2$. Small scale spatial variations are only present in the part $x < 0$. When compared to the displacement field (Figure 2), the size of those variations is the half wavelength. They correspond to stationnary components due to the total reflexion condition at $x = -L_1$. This can also be interpreted in words of active and reactive components in viewing the imaginary part of the structural intensity (Figure 5): for $x > 0$, structural intensity is purely active since there is no backward propagating wave. As far as the external load is concerned, the injected power density related to f_x appears in the jump of $\mathrm{Re}(\boldsymbol{I})$ at $x = 0$ (Figure 4).

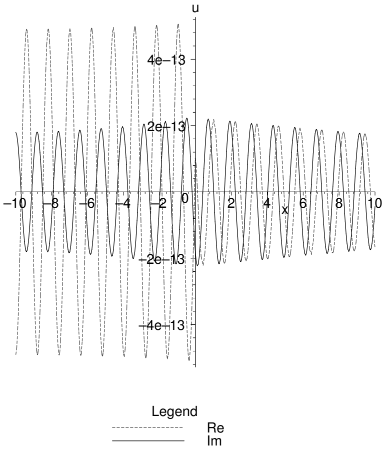

Figure 2: Displacement field.

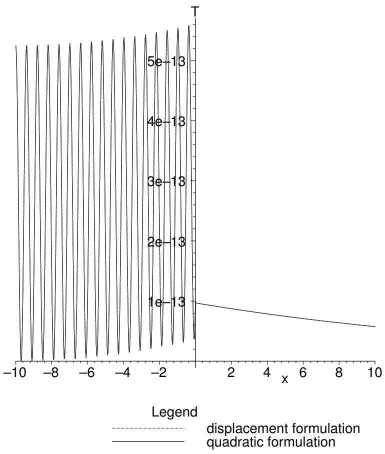

Figure 3: Kinetic energy density.

Conclusion

The quadratic variables (such as energy densities and structural intensity) obtained by the displacement formulation on the one hand and by the quadratic formulation on the other hand are exactly the same. Thus we have at our disposal an exact quadratic formulation to describe power transfers for any frequency in one-dimensional systems. These quadratic variables present two different spatial scales. Our objective is to keep only large scale components. So the next stage of this work will consist in traducing this objective in words of boundary conditions and discontinuities involved by external loads.

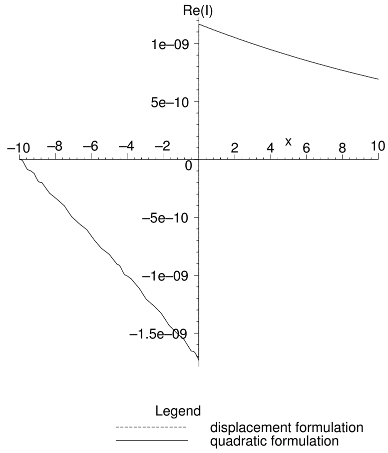

Figure 4: Real part of the structural intensity.

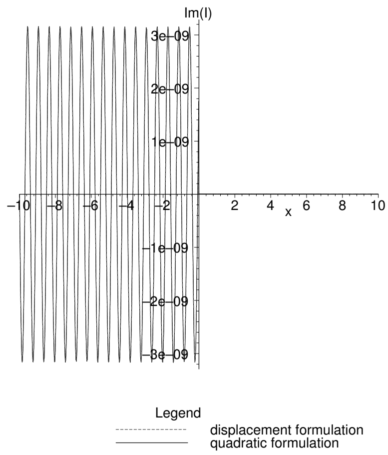

Figure 5: Imaginary part of the structural intensity.

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