Transient Sound Generation of a Plate Coupled to an Acoustic Cavity

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Introduction

The sound radiated into a cavity by the surrounding walls depends on several factors. The signal type, wall construction, cavity geometry and damping are of the most important. In this paper results that illustrate many of the basic features of vibro-acoustic coupling in the transient regime are presented. One shows that the efficiency by which the plate radiates sound into the room can in general qualitatively be explained by the speed of the Fourier frequency components of the impulse type signal. Subsonic phase velocities generate a near acoustic field, and only transfer acoustic energy to the room from the region close to the plate boundaries. For supersonic bending wave speeds energy is transferred to the room all along the trajectory.

Statement of the problem

In the rest of this paper, small case letters correspond to time functions while capital letters denote time harmonic functions with time dependence $\exp(-\imath\omega t)$. Let us consider a rectangular cavity, filled by air, defined in cartesian co-ordinate (O, x, z) by $x \in [0, L_x] \times z \in [0, L_z]$. A thin one dimensional steel plate of length $L_x = 1m$, thickness $h = 5mm$, with critical frequency 2517 Hz, and simply supported at its boundary occupies the upper wall. The other walls are considered as rigid. The plate is damped by a proportional damping given by $\eta_p = 1ms^{-1}$. The geometry of the cavity is given by the side lengths L_x and $L_z = 0.8m$. The plate is excited by a point-mechanical Ricker wavelet $f(x,t)$ of unit maximum amplitude. This signal is of finite duration between 0 and $Te = 2/f_0$ and of finite spectrum between 0 and $3f_0$. $u(x,t)$, the normal displacement of the plate, is the solution of the usual Kirchhoff plate equation. In the cavity, the acoustic pressure $p(x,z,t)$ is governed by the d'Alembert equation.

Modal expansion and time domain solution

In the harmonic regime, using Green's representation of the pressure, $U(x,\omega)$ is the solution of the following integrodifferential equation

$$D\Delta^2 U(x,\omega) - \rho_p h\omega^2 \Big(U(x,\omega) + \imath\frac{\epsilon_p}{\omega}U(x,\omega) - \epsilon_f \int_0^{L_x} U(x',\omega)G(x,0,x',0,\omega)\mathrm{d}x'\Big) = F(x,\omega) \quad (1)$$

where $\epsilon_p = \eta_p/\rho_p h$ and $\epsilon_f = \rho_f/\rho_p h$ are small parameters for a slightly damped plate in air. The Green's function of the rigid cavity is given by the usual modal expansion $G(x,z,x',z',\omega) = \frac{c_f^2}{L_x L_z}\sum_{m,n=0}^{\infty}\frac{\Psi_{mn}(x,z)\Psi_{mn}(x',z')}{\omega^2-\omega_{mn}^{02}}$. ω_{mn}^0 is the mn-th eigenpulsation and $\Psi_{mn}(x,z)$ is the mn-th eigenmode of the rigid cavity. The pressure inside the cavity is given by the integral equation $P(x,z,\omega) = \omega^2\rho_f\int_0^{L_x}U(x',\omega)G(x,z,x',0,\omega)\mathrm{d}x'$. It is easy to see that, because $G(x,z,x',z',\omega)$ depends on the frequency, both the eigenmodes and the eigenpulsations are frequency dependent. As ϵ_p and ϵ_f are small parameters, to compute these eigenmodes and eigenpulsations one can use pertubation expansions [2]. The fluid loaded eigenmodes are given by $\tilde{U}_l(x,\omega) = U_l^0(x) + \epsilon_f\sum_{s=1,s\neq l}^{\infty}\frac{\omega_l^{02}}{\omega_s^{02}-\omega_l^{02}}\beta_\omega(U_l^0,U_s^{0*})U_s^0(x)$, and $\tilde{\omega}_l(\omega) = \omega_l^0(1 - \imath\frac{\epsilon_p}{2\omega} + \frac{\epsilon_f}{2}\beta_\omega(U_l^0,U_l^{0*}))$, where $U_l^0(x)$ and ω_l^0 are the usual l-th eigenmode and eigenpulsation of the simply supported elastic plate in vacuum. The intermodal impedance $\beta_\omega(U_l^0,U_s^{0*}) = \int_0^{L_x}\int_0^{L_x}U_l^0(x')U_s^0(x')G(x,0,x',0,\omega)\mathrm{d}x'\mathrm{d}x$ describe the energy exchanged with the fluid. One shows that the solution reads

$$U(x,\omega) = \sum_{l=1}^{\infty}\frac{1/(\rho_p h)\langle F,\tilde{U}_l^*\rangle\tilde{U}_l(x,\omega)}{\omega_l^{02} - \imath\epsilon_p\omega - \omega^2(1+\epsilon_f\beta_\omega(U_l^0,U_l^{0*}))}, \quad (2)$$

where $\langle\tilde{U}_l,\tilde{U}_l^*\rangle$ is the usual inner product. The response of the plate and its pressure radiated into the cavity is computed using inverse time Fourier transform. Using residu integration theorem, one can shows that the response of the system involves resonance mode series expansion

$$u(x,t) = \frac{-2}{\rho_p h}\Im\sum_{l=1}^{\infty}\langle f,\hat{U}_l^{*+}\rangle\hat{U}_l^+(x)\frac{\mathrm{e}^{-\imath\hat{\omega}_l t}}{\hat{\omega}_l^+ - \hat{\omega}_l^-}, \quad (3)$$

$$p(x,z,t) = \rho_f\int_0^{L_x}\frac{d^2u(x',t)}{dt^2} * g(x,z,x',0,t)\mathrm{d}x', \quad (4)$$

where $*$ stands for the usual time convolution product. The resonance modes $\hat{U}_l(x) = \tilde{U}_l(x,\hat{\omega}_l^+)$ and resonance pulsations $\hat{\omega}_l^\pm$ are the free oscillation of the system [1]. $\hat{\omega}_l^\pm$ are the zeros of $\omega_l^{02} - \imath\epsilon_p\omega - \omega^2(1+\epsilon_f\beta_\omega(U_l^0,U_l^{0*}))$ and have negative imaginary part. Care must be taken when applying the residue integration around the first resonance frequency of the cavity which is zero for a rigid cavity. For an excitation without continuous component the solution exist and is unique.

Finite difference solution

Finite difference expressions of second order accuracy were used to replace the differential operators in the diffenretial equations. $\triangle x$, $\triangle z$ and $\triangle t$ are the spatial and

time discretization distances. For the finite difference grid, the appropriate equations were written in a finite difference form for each node, thereby obtaining a system of simultaneous algebraic equations in the unknowns u and p . Outside points are generated when the equations are written for points lying on the domain boundaries. These are however eliminated when combining the system equations with the boundary conditions. Recall that in the frequency domain, the bending wave phase velocity on an undamped beam is written as $c_b = \sqrt{\omega}\sqrt[4]{D/(\rho_p h)}$. In the time domain, care must therefore be taken to ensure that the wavelengths of the high frequency components of the pulse are meshed with enough nodes.

For a finite grid, too long time steps with respect to space steps will introduce instabilities in the system. It has been suggested by Crandall [3], that the second order accuracy finite difference explicit version of a 4th order equation of the type

$$\frac{\partial^4 \widetilde{\phi}}{\partial \widetilde{x}^4} + \frac{\partial^2 \widetilde{\phi}}{\partial \widetilde{t}^2} = 0, \tag{5}$$

where $\widetilde{\phi}$, $\widetilde{x}$, and $\widetilde{t}$ are non dimensional parameters, will be stable for $\triangle\widetilde{t}/(\triangle\widetilde{x})^2 \leq 0.5$. For the present system, regarding a fixed frequency, this stability criterium for bending wave propagation on the beam is given by

$$\frac{c_b \triangle t}{\triangle x} \leq \frac{\triangle x}{4\pi\lambda_b}, \tag{6}$$

where c_b and λ_b, designates the phase velocity and wavelength of the propagating wave at the frequency in question. When exciting the beam for the time domain calculations, one must therefore make certain that the higher Fourier components of the signal obeys the above criterium. In addition, the criterium $\frac{c_f \triangle t}{\triangle x} \leq \frac{1}{\sqrt{2}}$ for a 2 dimensional, equal spacing ($\triangle x = \triangle z$) acoustic system must be observed.

Results and discussion

For the finite difference investigation it was first of interest to study the behavior of bending waves travelling on infinite beams in vacuum. This to ensure that stable solutions were calculated numerically. Because the dispersive character of the signal was evident, to minimize numerical dispersion it important to have a fine enough grid to represent the shortest bending wavelengths. The stability criterium (equation 6), was judged to be valid for this type of calculations.

In figure 1, we present two pressure fields inside the cavity 1 ms after the beginning of the excitation ($L_x = 1m$ and $L_z = 0.8m$). The first is obtained for an excitation with $f_0 = 500$ Hz and the second with $f_0 = 3000$ Hz. Radiation from sub- and super critical waves are also presented in figure 2 for a later time. The figure shows that edge radiation is a predominant feature for the sub critical waves and that supersonic bending waves generate radiation where Mach angles are clearly visible; both for the waves travelling away from the excitation point, and the waves reflected back from the beam boundaries. The study has shown that both give physically reasonable results in the time domain. It was shown that the wave motion is dispersive with the highest frequency components propagating the fastest. Sub- and super-critical wave components will couple differently to the fluid in the cavity. The sub-critical waves will principally create a near field and only generate efficient radiation in the vicinity of the edges, while the super-critical waves radiate efficiently along the whole of the beam length, in the form of Mach angle waves. While the finite difference method gives a flexibility in geometrical modelling, computer space requirements does however limit the applicability when considering high frequencies and three dimensional models. Such limitations are much less of a problem using the analytical approach.

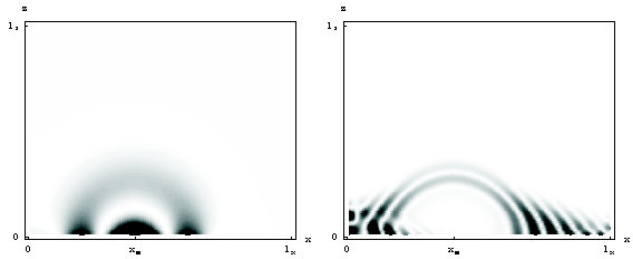

Figure 1: Pressure fields inside the cavity at 1 ms for $f_0 =$ 500Hz (left) and $f_0 = 3000$Hz (right), analytical model.

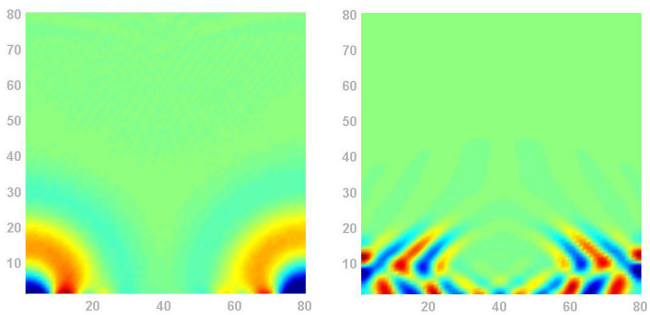

Figure 2: Pressure fields inside the cavity for sub- and super critical waves, finite difference model.

References

[1] D. Habault and P.J.T. Filippi, 2001. *Journal of Sound and Vibration, Phil Doak 80th Birthday Issue*, **239**(4):639-663. The role of the resonance modes in th response of a fluid-loaded structure.

[2] A. H. Nayfeh, 1993. *Introduction to Perturbation Techniques.* New-York : John Wiley.

[3] S. H. Crandall, 1956.*Engineering Analysis, a Survey of Numerical Procedures.* New-York: McGraw-Hill Book Co.

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