Efficient Simulation of Sound Insulation in Building Acoustics

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Introduction

The results of a round robin test performed by the Physikalisch-Technische Bundesanstalt show the necessarity of enquiring damping mechanisms in building acoustics. The measured results of the institutes differ within a large range. Therefore, it is necessary to develop a tool based upon Finite Element method to study different damping mechanisms and their effects on sound transmission in buildings. Within this work different procedures to improve the efficiency of the finite element simulation of sound insulation are presented. First step is to use only the Mindlin plate theory for moderately thick as well as for thin plates by using the *discrete shear gap method* [2]. The next step is the use of efficient storage schemes for matrices and the use of iterative solvers. The positive effects of these procedures are presented within a numerical example.

Finite Element Formulation

The calculations will be performed in the frequency domain where a steady state is implied. The fluid is assumed to be ideal and compressible and the structure is modelled using the Mindlin plate theory. Based on Hamilton principle a finite element formulation for the treated domains has been derived. After discretisation one obtains

$$\left(\mathbf{K} - \omega^2\mathbf{M}\right)\mathbf{w} = +\mathbf{C}\mathbf{p}^{(i)} \qquad (1)$$

for the structural domain and

$$\left(\mathbf{K}_1 - \kappa^2\mathbf{K}_2\right)\mathbf{p} = +\rho\omega^2\mathbf{C}^T\mathbf{w}^{(i)} \qquad (2)$$

for the fluid domain. $\mathbf{K}$ denotes the stiffness matrix, $\mathbf{K_1}$ the compressibility matrix, $\mathbf{M}$ and $\mathbf{K_2}$ are the mass matrix of the structure and the fluid respectivily and κ denotes the wave number. The unknowns are the pressure $\mathbf{p}$ and the deflection $\mathbf{w}$. The coupling matrix $\mathbf{C}$ results from the discretised form of the virtual work on the interfaces $\Gamma^{(i)}$ caused by the interaction forces

$$\mathbf{C} = \int_{\Gamma^{(i)}} \mathbf{N_S^T}\mathbf{N_F}\,\mathrm{d}\Gamma^{(i)}. \qquad (3)$$

There, $\mathbf{N_S}$ denote the shape functions of the structure and $\mathbf{N_F}$ those of the surface of the fluid. This is a strong coupling procedure that represents all interactions between structure and fluid and leads to one large equation system [4]. Another alternative would be an iterative coupling procedure which may be investigated in the future. Because of the slenderness of the plate transverse shear locking is a problem. To get rid of this problem the *discrete shear gap method* [2] is applied to the element formulation. This leads to a modified stiffness matrix of the structure. The discrete shear gap method (DSG) leads in case of a four node element to a formulation which is equivalent to the well known MITC element [1]. In contrast to MITC, the DSG method can easily be applied to triangular or rectangular elements of arbitrary polynominal order.

Numerical Example

Two fluid domains which are assumed to be air (c=346 m/s, ρ=1.2 kg/m^3) are connected by a wall which consists of glass (E=3.2·10^9 N/m^2, ν=0.24, ρ=2300 kg/m^3) (see fig. 1). The system is excited by a normal velocity v_n on the front of the first fluid domain. As the slenderness of the plate is $h/l = 0.025$ the Kirchhoff theory is applicable. Hence, the Kirchhoff result can be used as reference solution for the DSG element.

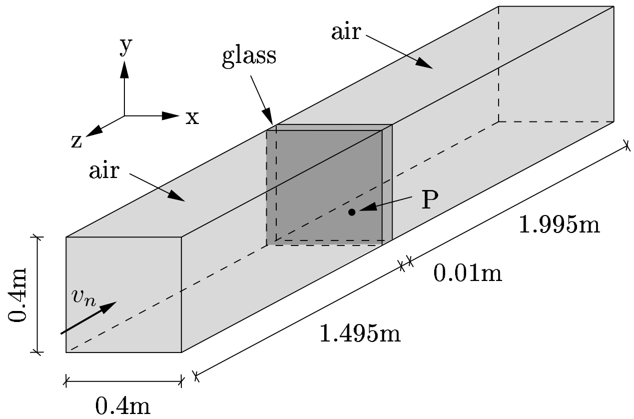

Figure 1: Study for the exactness of DSG elements: two fluid domains (air) connected by a wall (glass)

The plate has been discretised in three different ways: a very coarse mesh with 5·5 elements, one with 8·8 elements, and a very fine mesh with 10·10 elements. Because the four node Kirchhoff elements have cubic ansatz functions 5·5 elements already lead to the final solution. Clearly, the four noded bilinear DSG element can not be expected to lead to the same solution. In fig. 2, the displacement at point P is plotted versus frequency to compare the results obtained from Kirchhoff theory with the DSG elements. It can be seen from fig. 2 that the displacements at the first eigenfrequency conincide. The higher eigenfrequencies converge only for finer meshes against the solution of the Kirchhoff plate. So, it is possible even to compute very slender walls using finite elements based upon Mindlin plate theory. So it is possible to consider problems that consist of moderately thick plates as well as problems of thin plates with

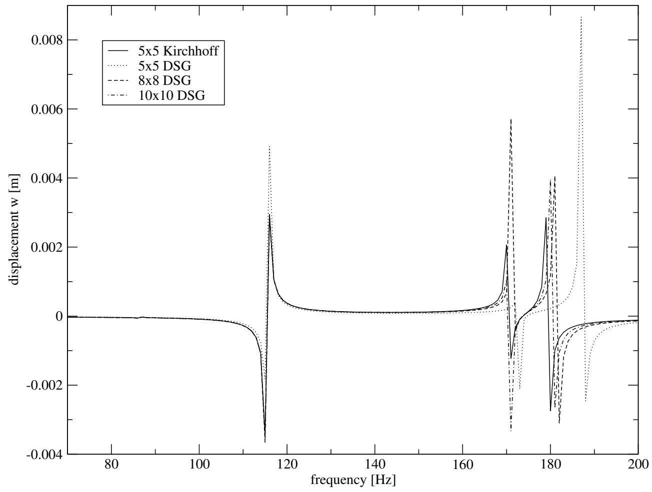

Figure 2: Displacement at point P versus frequency: comparison of Kirchhoff theory with Mindlin plate theory (DSG)

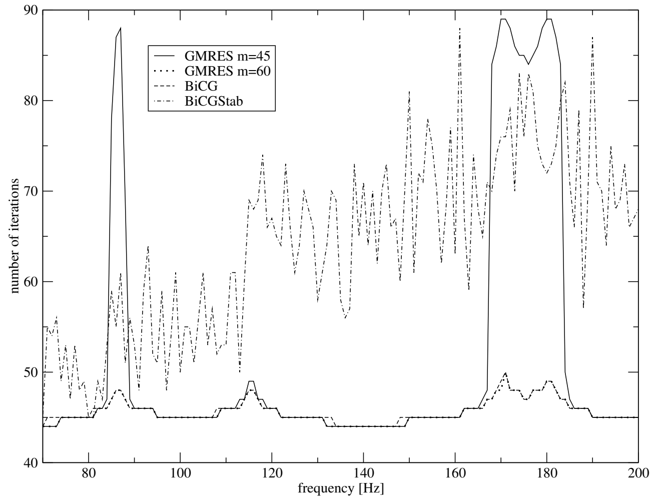

Figure 3: Number of iterations versus frequency: study of the performace of different iterative solvers

one kind of plate element. Both types of plates occur in building acoustics: walls are generally speaking moderately thick and window glasses can be treated as thin plates.

An important point to increase the efficiency of the finite element code is the matrix storage scheme. Clearly, dense storage makes no sense in finite element analysis. A

storage scheme	number of entries	
dense storage	36905625	100%
band storage	3165075	8.6%
compressed row storage	140000	0.4%

Table 1: Memory requirements of different matrix storage schemes

simple band storage drastically decreases memory usage (see tab. 1). Further, the compressed row storage reduces the required memory but it has to be mentioned that the effort for accessing the matrix entries is increased [5]. Another important fact is the bandwidth of the matrix even for matrices stored in compressed row storage. Many algorithms exists to improve the bandwidth. Very popular are the Cuthill-McKee-algorithm [3] and its variants, e.g., the Reverse Cuthill-McKee, which has been applied here.

In a last step, the convergence rate of different iterative solvers is compared. Due to the coupling unsymmetric matrices appear. Therefore, CG-methods which are based on symmetric and positiv definite matrices can not be used. For preconditioning the incomplete LU decomposition (ILU) has been used. As known from the literatur [5] this method requires only a few operations and is very efficient. In fig. 3, the number of iterations, which is a sign for the efficiency of an iterative solver, is depicted versus frequency. According to fig. 3, the BiCG seems to be the best choice. Its convergence rate is almost constant along the whole frequency range. The BiCGStab needs many more iterations and the convergence rate is non-uniform. For some frequencies the BiCGStab needs up to two times more iterations than BiCG. The use of GMRES is not recommended because the efficiency of this algorithm depends heavily on the restart parameter m. If m is chosen too small the GMRES won't converge. For a large value of m the increase of memory usage and the number of vector operations is unjustifiable.

Conclusions

The DSG element has been shown to give good results even for thin plates. Further, efficient storage schemes for the system matrix can dramatically decrease memory usage. The compressed row storage saves up to 99% of memory. Bandwidth reduction is another important fact to save memory and increase effiency. The best choice for solving the equation system in our field of application is the BiCG. Compared to other methods it needs less operations and, therefore, is much faster. The use of GMRES is not recommended due to the necessarity to find a proper restart parameter m.

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