

## Representing outdoor sound propagation effects with a BEM model

Sebastian Hampel, Sabine Langer, Heinz Antes

Institut für Angewandte Mechanik, D-38106 Braunschweig, Germany, Email: sebastian.hampel@tu-bs.de

### Introduction

Outdoor sound propagation is influenced by numerous different factors which should be implemented in a realistic simulation model. In acoustics, for infinite domains in particular, the Boundary Element Method (BEM) has established as a suitable and powerful numerical method. Hence, it is shown how important effects can be included in a BEM model. The focus is on damping in the air, on the effect of a mean flow, and on refraction, respectively.

### Damping in the air

Beside geometric damping, the most important reason for transmission loss in outdoor sound propagation is the absorption in the air. However, inhomogeneous phenomena like turbulence or fog are not subject to investigation here. Considering absorption in the homogeneous domain leads to a modified Helmholtz equation [1]

$$\nabla^2 \phi + \bar{k}^2 \phi = a \quad (1)$$

with a complex wave number  $\bar{k} = k(1 + i\mu)$ , where  $k = \omega/c$  is the original wave number and  $\mu$  is the absorption coefficient. It can be given for different damping causes: Internal friction, thermal conduction, and molecular damping where the last dominates the others. Values for  $\mu$  depending on frequency, relative humidity, and temperature can be taken from standard norms, e.g. [2]. For the implementation in the BEM model one simply has to replace  $k$  by  $\bar{k}$  in the fundamental solution.

Studies show a transmission loss caused by molecular damping which is linearly increasing with distance [dB/m], for spherical as well as for planar waves. The influence of molecular absorption compared to geometric damping increases with frequency and furthermore depends strongly on humidity and temperature. In outdoor sound propagation considering long distances and/or higher frequencies, molecular damping easily reaches or overcomes the effect of geometrical damping. Hence, it is important to be considered for these cases.

### Mean flow

The first step to include the effect of wind may be considering a constant mean flow in the domain, e.g., in  $z$ -direction. The mean flow velocity is  $v_z = \text{const}$  and the Mach number is  $M_z = v_z/c_0$ , respectively. The standard Helmholtz equation changes into

$$\nabla^2 \phi + k^2 \phi - 2ikM_z \frac{\partial \phi}{\partial z} - M_z^2 \frac{\partial^2 \phi}{\partial z^2} = a, \quad (2)$$

as shown in detail in [3]. The relation between pressure  $p$  and velocity potential  $\phi$  becomes

$$p = i\rho\omega\phi + \rho v_z \frac{\partial \phi}{\partial z} \quad (3)$$

and

$$\frac{\partial p}{\partial n} = i\rho\omega \frac{\partial \phi}{\partial n} + \rho v_z \frac{\partial}{\partial n} \left( \frac{\partial \phi}{\partial z} \right). \quad (4)$$

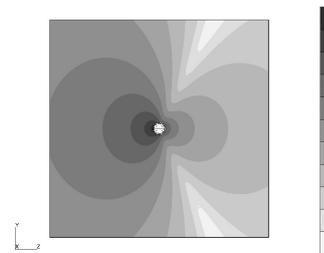
The second terms in eqn.(3) and (4) include the mean flow effect and vanish in case of a medium at rest. It has to be noticed that in case of pressure formulation the Neumann boundary condition (pressure flux) will change compared to the standard no-flow case, e.g., considering an acoustically rigid boundary leads to

$$\frac{\partial \phi}{\partial n} = 0 \quad \rightarrow \quad \frac{\partial p}{\partial n} = \rho v_z \frac{\partial^2 \phi}{\partial z \partial n} \neq 0. \quad (5)$$

To overcome this difficulty, the problem has to be solved in potential formulation, before the pressure can be computed with eqn.(3) at the points of interest.

One way to solve the modified eqn.(2) is to apply the so-called Prandtl-Glauert transformation [4]. With this transformation, eqn. (2) passes back into the standard Helmholtz equation for which solving the integral equation is well-known and implemented in numerous BEM programs. This transformation affects the coordinate  $z$  in direction of the mean flow, the wave number  $k$  as well as the field quantities  $\phi$  and  $\partial \phi / \partial n$  or pressure  $p$  and flux  $\partial p / \partial n$ , respectively.

An example for a dipole source in a mean flow of  $\text{Ma}=0.5$  in positive  $z$ -direction is shown in figure 1. It is obvious that the pressure tends to be higher on the upwind (left) side for the constant flow. This tendency does not change taking monopole sources or different frequencies. So, the effect of increasing pressure for downwind situation can not be shown with a constant wind field. This effect must refer to refraction (see next section).



**Figure 1:** Characteristic pressure field around a dipole source in a mean flow (from left),  $k = \frac{\omega}{c} = 1$ ,  $\text{Ma} = 0.5$ .

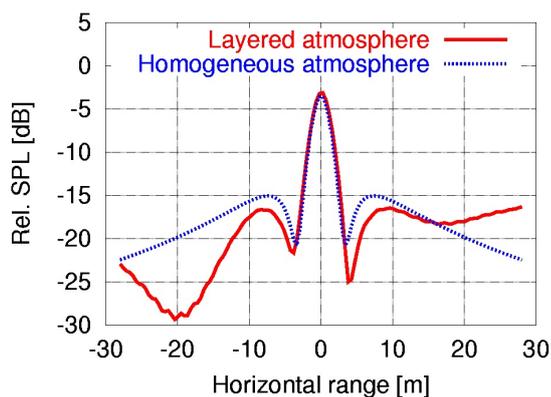
An alternative way to solve eqn.(2) is given in [4] where a fundamental solution is derived for the mean flow case.

However, this leads to an integral equation which contains terms of  $\partial\phi/\partial z$  additionally to the considered field quantity  $\phi$  and its derivation  $\partial\phi/\partial n$  normal to the boundary.

## Refraction

In outdoor sound propagation, refraction is caused by a gradient in the temperature or wind speed profile, as both affect the efficient sound speed profile. Including refraction in a BEM model is a challenging task because the method is usually restricted to deal with homogeneous domains. Two approaches are investigated in the following to include this effect in a BEM computation.

One possibility is describing a layered atmosphere by homogeneous substructures with different material parameters such as mean flow or temperature. The substructure technique is a common method to couple two separate domains or, vice versa, to split one domain in parts. For the following example, a three-layer atmosphere was modeled approximating the wind speed profile as follows:  $v_1=0\frac{m}{s}$  in the bottom layer ( $0m < z < 2m$ ),  $v_2=5\frac{m}{s}$  for  $2m < z < 4m$  and  $v_3=10\frac{m}{s}$  above. The infinite ground is assumed to be acoustically rigid and a monopole source is placed at  $(x_s=0, z_s=3m)$  with  $f=100Hz$ . Figure 2 shows the computed relative sound pressure level for the described layered atmosphere as well as for the homogeneous atmosphere at rest. The rel. SPL is defined as the level difference between a receiver point ( $h_r=1.3m$ ) and a reference point 1m from the source in free space. It can be seen that refraction at the layer interfaces has a significant influence. It seems to increase the SPL downwind compared to upwind. Hence, the layered atmosphere approximation shows the same effect as known from outdoor measurements.



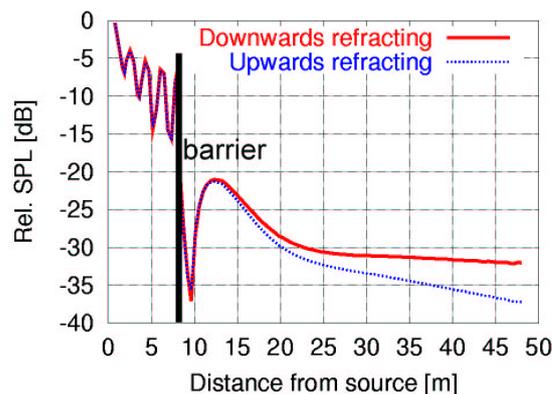
**Figure 2:** Rel. SPL for monopole source ( $h_s=3m$ ,  $f=100Hz$ ) over rigid ground, receiver height  $h_r=1.3m$ : Layered (wind from left) and homogeneous atmosphere ( $v=0$ ).

Another possibility to overcome this restriction of a homogeneous domain is to use the analogy between a refracting medium over a flat terrain and a homogeneous medium over a curved terrain as studied in [5].

The medium is assumed to be stratified and to have the sound speed profile  $c(y) = c_0 e^{y/R_c}$ , where  $y$  denotes the

height over the ground and  $R_c$  is the radius of curvature of the sound rays. For  $y \ll R_c$  this approximates a sound speed which is linearly increasing with height. For this case, the sound rays, travelling from a source to a receiver at the same height, will follow curves with constant radius  $R_c$ . Introducing a conformal coordinate transformation, as shown in [5], will then curve the flat terrain into a cylindrical surface and refer the problem to the homogeneous case.

As a sample, an acoustically rigid ground is considered with a rigid barrier ( $h_b=1.7m$ ,  $x_b=8m$ ). A monopole source ( $h_s=1m$ ,  $f=100Hz$ ) is placed at  $x_s=0$ . Figure 3 shows the computed transmission loss for both upwards and downwards refracting atmosphere for  $R_c=300m$ . Receiver points are at  $h_r=1m$ . It can be seen that refraction nearly does not effect the area in front of the barrier, but has increasing influence for increasing distance from the source. It has to be noticed that the approximation of a constant gradient of sound speed holds for a linear temperature profile, but for simulating wind it can be just a first estimation because the barrier will influence the wind field in the neighbourhood.



**Figure 3:** Rel. SPL for monopole source ( $h_s=1m$ ,  $f=100Hz$ ) with barrier ( $h_b=1.7m$ ), receiver height  $h_r=1m$ : Influence of upwards and downwards refracting atmosphere ( $R_c=\pm 300m$ ).

## References

- [1] S.Langer: BEM-studies of sound propagation in viscous fluids, Proceedings of ECCOMAS '04, Eds.: P.Neittaanmäki et al., to appear 2004.
- [2] VDI-Richtlinie 2714: Schallausbreitung im Freien, Anhang C: Dämpfung durch Luftabsorption.
- [3] T.Tsuji, T.Tsuchiya and Y.Kagawa: Finite element and boundary element modelling for the acoustic wave transmission in mean flow medium, Journal of Sound and Vibration (2002) **255**(5), p.849-866.
- [4] T.W.Wu and L.Lee: A direct boundary integral formulation for acoustic radiation in a subsonic uniform flow, JSV(1994) **175**, p.51-63.
- [5] K.M.Li and Q.Wang: A BEM approach to assess the acoustic performance of noise barriers in a refracting atmosphere, JSV(1998) **211**(4), p.663-681.