Acoustic Wave Propagation in Functionally Graded Material (FGM) Cylinders

L. ELMAIMOUNI, J. E. LEFEBVRE, T. GRYBA, V. ZHANG,
IEMN-OAE, Université de Valenciennes, Le Mont Houy, 59313-Valenciennes Cedex, France
Lahoucine.elmaimouni@univ-valenciennes.fr

We present a numerical method for calculating the guided wave propagation in Functionally Graded Material (FGM) cylinders. The FGM cylinder made up of stainless steel and silicon nitride is described. The method uses the wave equation with the elastic moduli and mass density assumed to be functions of position. We developed the necessary formulas to calculate the dispersion curves in FGM cylinders of various geometries. The radial variation laws of Young's modulus, Poisson's ratio and mass density are given. The position-dependent stiffness constants are deduced. The equations of motion are solved by expanding each mechanical displacement component using Legendre polynomials and harmonic functions. The dispersion curves for axisymmetric modes are presented.

Introduction

The concept of functionally graded materials was the first introduced in 1984 by a Japanese scientist group [1-2]. FGMs are composite materials intentionally designed so that they possess desirable properties for specific applications, often to withstand high-temperature-gradient environments while maintaining their structural integrity. The mechanical properties of cylindrical structure made of FGM vary continuously in the macroscopic sense from one surface to the other, for example all ceramic to all metal. Many applications of FGM have been reported, including the most recent focus on the solar energy conversion devices [3], naturally occurring biological FGM [4], aerospace, nuclear and automobiles industries. Several researches have been realized on analysis of thermal stress and deformation of FGM. The study of the acoustic propagation in FGM cylindrical structures is important for non destructive evaluation of properties of composite materials.

In the present approach, we adopt the Legendre polynomials to calculate the dispersion curves in FGM cylinders. The application of the boundary conditions on the inner and the outer surfaces of the structure coupled with the constitutive equation of materials and Newton equation makes it possible to study the guided acoustic modes.

Numerical Results

Consider FGM cylinder with varying material properties in the thickness direction. The thickness, mean radii, inner and outer radius of cylinder are respectively denoted by H, R, R_1 and R_2. The problem will be dealt with in the system of cylindrical coordinates Orϕz. The mechanical displacement components u, v, and w are assumed in the form:

$$u(q_1,q_2,q_3,t)=\cos(nq_2)\cos(\omega t-q_3)\sum_{m=0}^{N}p_m^1 Q_m(q_1)$$

$$v(q_1,q_2,q_3,t)=\sin(nq_2)\cos(\omega t-q_3)\sum_{m=0}^{N}p_m^2 Q_m(q_1)$$

$$w(q_1,q_2,q_3,t)=\cos(nq_2)\sin(\omega t-q_3)\sum_{m=0}^{N}p_m^3 Q_m(q_1)$$

where q_1=kr, q_2=ϕ, and q_3=kz are respectively the radial, circunferential and axial direction, k is the magnitude of the wave vector in the propagation direction, t is time, ω is the angular frequency of vibration, and n is the circumferential wave number. When n=0, the vibration becomes axisymmetric, and n=1,2,3…., the vibration is non-axisymmetric :

$$Q_m(q_1)=\sqrt{\frac{2m+1}{D}}P_m\left(\frac{2q_1-S}{kH}\right)$$

where P_m represents the m^{th} Legendre polynomial, and $kH=kR_2-kR_1$, $S=kR_2+kR_1$.

For an FGM, the stiffness tensors and mass density can be expressed as:

$$C_{ij}^M(q_1)=C_{ij}^{(0)}+C_{ij}^{(1)}\left(\frac{q_1}{kh}\right)^1+\dots\dots\dots+C_{ij}^{(N)}\left(\frac{q_1}{kh}\right)^N \quad (1)$$

$$\rho^M(q_1)=\rho^{(0)}+\rho^{(1)}\left(\frac{q_1}{kh}\right)^1+\dots\dots\dots+\rho^{(N)}\left(\frac{q_1}{kh}\right)^N \quad (2)$$

Using the gradient law of FGM cylinder and the strain-displacement in the cylindrical coordinate system, the field equations governing wave propagation are given by:

$$q_1^2\frac{\partial}{\partial q_1}\left(\frac{T_{rr}}{k}\right)+q_1^2\frac{T_{rr}}{k}\frac{\partial BC(q_1)}{\partial q_1}+q_1\frac{\partial}{\partial q_2}\left(\frac{T_{r\phi}}{k}\right)+q_1^2\frac{\partial}{\partial q_3}\left(\frac{T_{rz}}{k}\right)+q_1\left(\frac{T_{rr}-T_{\phi\phi}}{k}\right)=\rho\frac{q_1^2}{k^2}\frac{\partial^2 u}{\partial t^2}$$

$$q_1^2\frac{\partial}{\partial q_1}\left(\frac{T_{r\phi}}{k}\right)+q_1^2\frac{T_{r\phi}}{k}\frac{\partial BC(q_1)}{\partial q_1}+q_1\frac{\partial}{\partial q_2}\left(\frac{T_{\phi\phi}}{k}\right)+q_1^2\frac{\partial}{\partial q_3}\left(\frac{T_{\phi z}}{k}\right)+2q_1\left(\frac{T_{r\phi}}{k}\right)=\rho\frac{q_1^2}{k^2}\frac{\partial^2 v}{\partial t^2}$$

$$q_1^2\frac{\partial}{\partial q_1}\left(\frac{T_{r\phi}}{k}\right)+q_1^2\frac{T_{rz}}{k}\frac{\partial BC(q_1)}{\partial q_1}+q_1\frac{\partial}{\partial q_2}\left(\frac{T_{\phi z}}{k}\right)+q_1^2\frac{\partial}{\partial q_3}\left(\frac{T_{zz}}{k}\right)+q_1\left(\frac{T_{rz}}{k}\right)=\rho\frac{q_1^2}{k^2}\frac{\partial^2 w}{\partial t^2}$$

where the boundary functions $BC(q_1)$ is given by:

$$BC(M)=\pi(kR_1,kR_2) \qquad \textit{for a hollow cylinder}$$

Combining the system equations defined above, using the boundary conditions for the normal stress components and taking into account the orthogonality of the Legendre polynomials yields a form of the eigenvalue problem :

$$[A]_{mj}^{\alpha\beta}P_m^{\beta}=\lambda^2 P_j^{\alpha} \quad (3)$$

The eigenvalue $\lambda^2 = \rho(\omega / k)^2$ gives the guided wave velocity and the eigenvectors p_m^α (β=1,2 and 3) the particle displacement components.

Gradient law of FGM cylinder

The material properties of the FGM cylinder composed of stainless steel on its inner surface and silicon nitride on its outer surface are presented in table.1.

Stainless steel			Silicon nitride		
E_1(GPa)	ν_1	ρ_1(kg/m^3)	E_2(GPa)	ν_2	ρ_2(kg/m^3)
207.82	0.317	8166	322.4	0.24	2370

Table 1 : Material properties of FGM cylinder.

The gradient law of mixture is given by [5]:

$$E(q_1) = E_2 + (E_1 - E_2)(\frac{q_1}{kH} - 1)^s$$
$$\nu(q_1) = \nu_2 + (\nu_1 - \nu_2)(\frac{q_1}{kH} - 1)^s \qquad (4)$$
$$\rho(q_1) = \rho_2 + (\rho_1 - \rho_2)(\frac{q_1}{kH} - 1)^s$$

with s is the law-power exponent.

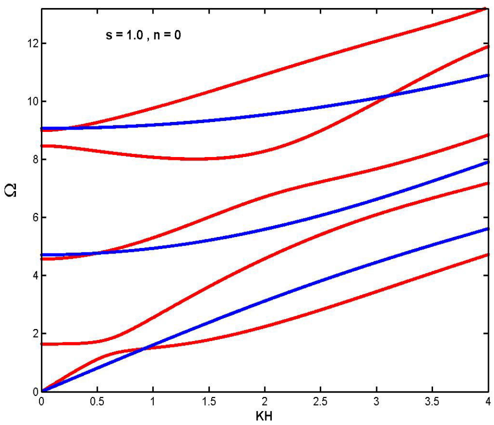

Figure 1: Dispersion curves for axisymmetric modes of FGM cylinder, (R_1=H, s=1, n=0)

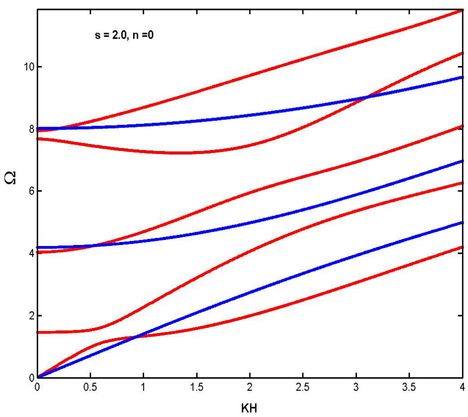

Figure 2: Dispersion curves for axisymmetric modes of FGM cylinder, (R_1=H, s=2, n=0)

For numerical calculation, the order of the expansion is truncated to some value N. The solution converges within a very low number N of terms. Figs. 1 2 and 3 gives respectively the dispersion curves (normalized frequency Ω) of FGM cylinder as a function of kH calculated by our approach for kR_1=kH and for s=1,2,4 with a truncation number equal to 8. The presented method has been validate earlier though comparison with result from literature [6-7].

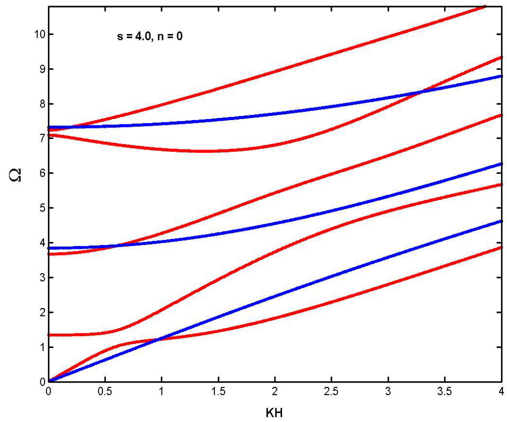

Figure 3 : Dispersion curves for axisymmetric modes of FGM cylinder, (R_1=H, s=4, n=0)

Conclusion

We have developed a polynomial approach for the calculation of guided wave propagation in Functionally Graded Material FGM cylinder. The method is capable of accurately determining the dispersion curves of mixture.

We anticipate applications of this method to guided wave with various sections of cylinders, such as hollow and solid, in particular to the non-destructive testing evaluation of properties of composite materials.

References

[1] M. Yamanouchi, M. Koizumi, T. Hirai and I. Shiota, “Proceedings of the First International symposium on functionally Gradient Materials” Japan (1990).

[2] M. Koizumi, “The concept of FGM Ceramic Transactions, Functionally Gradient materials”, **34** (1993), 3-10

[3] Koizumi. “FGM activities in Japan” Composites Part B **28** (1997), 1-4

[4] S. Amada, Y. Ichikawa, T. Munekata, Y. Nagase, H. Shimizu, “Fiber texture and mechanical graded structure of bamboo”, Composites Part B **28** (1997), 13-20

[5] S. W. Gong, K. Y. Lam, J. N. Reddy, “The elastic response of functionally graded cylindrical shells to low-velocity impact” International Journal of Impact Engineering, **22**, (1999) 379-417

[6] X. Han, G. R. Liu, Z. C. Xi and K. Y. Lam, “Characteristics of waves in a functionally graded cylinder”, International Journal for numerical methods in Engineering, Int. J. Numer. Meth. Engng , **53** (2002), 653–676

[7] L. Elmaimouni, J. E. Lefebvre, T. Gryba, V. Zhang, “Polynomial Method Applied to Acoustic Waves in Inhomogeneous Cylinders” IEEE, International Ultrasonics Symposium, October 5-8, (2003), Hawaii (USA).