On the use of the wave based method for the dynamic analysis of flat plate assemblies

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Introduction

In recent years, the vibro-acoustic behaviour of a product has become a design criterion of growing importance. This behaviour is mainly determined by the steady-state dynamic deformations in the mechanical structure of the product. Commonly, the finite element method (*FEM*) is used to predict the dynamic behaviour of structures. This method expands the dynamic field variables, within each element, in terms of local, non-exact shape functions. As a result, the size of the model becomes prohibitively large for increasing frequencies, thereby leading to a practical frequency limit. The newly developed wave based method (*WBM*), based on the indirect Trefftz method [1], expands the field variables in terms of global wave function expansions, which exactly satisfy the governing dynamic equations. The *WBM* exhibits a higher computational efficiency, such that it will be applicable also for higher frequencies. Although the elastodynamic calculations are the computationally most demanding part in a vibro-acoustic problem, until now the research focused mostly on acoustic systems.

Current research extends the applicability of the *WBM* towards three-dimensional elasto-dynamic applications. This paper discusses the development of the *WBM* for analysing the behaviour of an assembly of flat plates, coupled at arbitrary angles. Numerical validations confirm that the *WBM* achieves high accuracy with substantially smaller models in comparison with the finite element method.

Problem Definition

The problem case considered in this paper consists of two flat plates coupled under an angle of 45 °, as shown in Figure 1. All plate boundaries are clamped and the first plate is excited by a harmonic normal point force F applied at position (x_F, y_F). According to the thin plate theory [2], the *steady-state out-of-plane displacements* w_{zi} $(i = 1, 2)$ are governed by the following differential equation:

$$\nabla^4 w_{zi}(x_i, y_i) - k_{bi}^4 w_{zi}(x_i, y_i) = \frac{F_i}{D_i}\delta(x_{Fi}, y_{Fi}) \qquad (1)$$

where $\nabla^4 = \frac{\partial^4}{\partial x_i^4} + 2\frac{\partial^4}{\partial x_i^2 \partial y_i^2} + \frac{\partial^4}{\partial y_i^4}$. The plate bending wavenumber k_{bi} and the plate bending stiffness D_i are defined as

$$k_{bi} = \sqrt[4]{\frac{\rho_i h_i \omega^2}{D_i}}\ , \quad D_i = \frac{E_i h_i^3}{12(1-\nu_i^2)}\ , \qquad (2)$$

with h_i, E_i, ν_i and ρ_i, respectively, the plate thickness, the elasticity modulus, Poisson coefficient and the plate material density. The *steady-state in-plane displacements* are described by the dilatational and rotational strains e_i and Ω_i $(i = 1, 2)$,

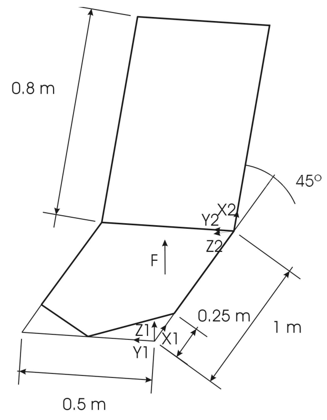

Figure 1: flat plate assembly.

$$e_i = \frac{\partial w_{xi}}{\partial x_i} + \frac{\partial w_{yi}}{\partial y_i} \quad \text{and} \quad \Omega_i = \frac{\partial w_{yi}}{\partial x_i} - \frac{\partial w_{xi}}{\partial y_i}. \qquad (3)$$

The dilatational and rotational strain are governed by the following differential equations,

$$\nabla^2 e_i + k_{li}^2 e_i = 0 \quad \text{and} \quad \nabla^2 \Omega_i + k_{ti}^2 \Omega_i = 0, \qquad (4)$$

where the in-plane longitudinal and shear wavenumbers are defined as,

$$k_{li} = \omega\sqrt{\frac{\rho_i(1-\nu_i^2)}{E_i}} \quad \text{and} \quad k_{ti} = \omega\sqrt{\frac{2\rho_i(1+\nu_i)}{E_i}}. \qquad (5)$$

The interface between the two plates is modelled by imposing the force equilibrium and displacement compatibility, thus leading to eight interface conditions.

Basic Concept of the *WBM*

This section will present very briefly the basic concepts of the *WBM*. For further details the reader is referred to [3] and [4].

The different field variables w_{zi}, e_i and Ω_i are approximated as a linear expansion of wave functions, which exactly satisfy the governing homogeneous equations, extended with some particular solutions to account for the inhomogeneous part of the governing equations. As a result, the field variable approximations satisfy a priori the governing differential equations, irrespective of the contribution factors of the wave functions. These contributions are determined through minimization of the approximation error of the boundary and interface conditions in an integral sense. To do so, the boundary conditions have to be transformed into a weighted residual or a least-squares formulation. Furthermore a complete set of wave functions must be selected from the infinite set of wave functions, which satisfy the dynamic equations, to ensure the convergence of the *WBM*.

Comparison of the *WBM* with the *FEM*

This section shows the prediction results for the problem defined in Figure 1 and compares the efficiency of the *WBM* with the *FEM*. The considered structure consists of two flat aluminium plates with thickness $h1 = h2 = 0.005m$. The different material constants of aluminium are $E = 70 \cdot 10^9 N/m^2$, $\nu = 0.3$ and $\rho = 2790 kg/m^3$. All the plate boundaries are clamped and a unit normal point force is applied at position $(x_F, y_F) = (0.5625m, 0.125m)$ of plate 1.

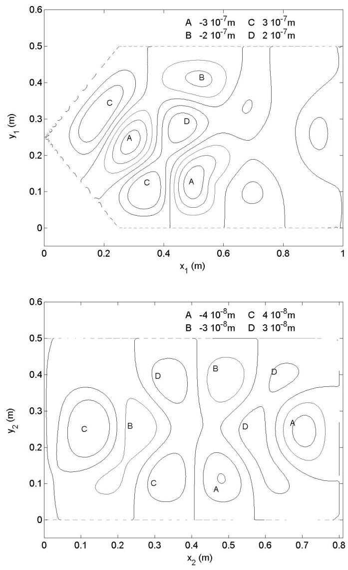

Figure 2: prediction result for the out-of-plane displacement at 900Hz for the first and second plate respectively.

Figure 2 shows the out-of-plane displacement of the two plates when excited at $900Hz$. These predictions result from a weighted residual wave model with 1008 bending wave functions and 848 dilatational and rotational wave functions. The least-squares wave model yields the same result.

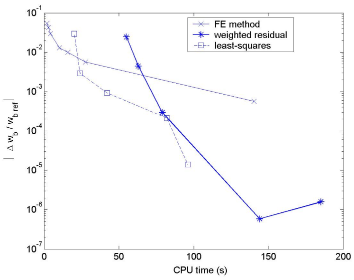

Figure 3: convergence curves for the out-of-plane displacements at $900Hz$.

The advantage of the smaller *WBM* prediction models is to some extent annihilated by the fact that the model matrices are fully populated, complex and frequency dependent. To make a fair comparison between the *WBM* and *FEM*, Figure 3 displays the relative prediction error of the out-of-plane displacement of the point $(x_1, y_1) = (0.7m, 0.125m)$ against the CPU times. For the *WBM* the indicated times include both the times needed for the construction of the model as well as for solving the model. For the *FEM*, only the direct solution time is included in the indicated CPU times. A FE model with 1 326 405 DOF's and a wave model with 1680 bending wave functions and 1424 dilatational and rotational wave functions are used as reference solutions, respectively, for the FE models and the *WBM* models. The FE predictions were calculated using the direct solution method and a 4-noded quadrilateral shell discretization. Figure 3 clearly indicates the beneficial convergence rate of the *WBM* in comparison with the *FEM*. Earlier research regarding acoustic and single plate problems has identified the weighted residual method as the most efficient method when compared with the least-squares method. For this example, however, this conclusion no longer holds, although an improvement of the weighted residual method can be expected by adding additional corner point residuals, as was done for single plate problems[3].

Conclusions

This paper applies the *WBM* for the steady-state dynamic analysis of coupled plates. The numerical example illustrates the beneficial convergence rate compared with the *FEM*. Another major advantage of the *WBM* compared with the *FEM* is the possibility to easily identify the prediction accuracy by evaluating the field variables on the boundaries.

Due to the enhanced convergence properties, the practical frequency limitation of the proposed prediction method can be shifted towards much higher frequencies than possible with the *FEM*.

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