Multiple scattering of acoustic waves and porous absorbing media

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Introduction

Sound propagation and absorption in fluid-saturated porous materials is well understood in the long-wavelength limit. For porous media like air-saturated polymer foams with open cells (see Fig. 1), this regime generally includes the whole audio-frequency range and some of the low-frequency ultrasonic range. The solid skeleton often remains motionless and the material behaves like an equivalent lossy fluid. Spatial averaging methods enable to express the complex frequency-dependent density and compressibility in terms of purely geometrical parameters : porosity, "viscous and thermal" permeabilities, tortuosity, and "viscous and thermal" characteristic lengths. We refer to this model as the equivalent fluid model *EFM* [1-2].

The situation is different at higher ultrasonic frequencies, when the wavelength in the air becomes comparable to the typical scale of scatterers. In this "multiple scattering" regime, while the propagation cannot be entirely followed at the macroscopic level, one may still speak in certain conditions, of an effective macroscopic wavenumber $k_e(\omega)$ characterizing the propagation of the coherent field [3]. However, no general modeling is available for this coherent wave propagation, taking into account both scattering effects and viscothermal losses, in the complicated microgeometry of materials found in practice.

In this presentation, experiments on the multiple scattering in air-saturated foams with open-cells are reported and a simple multiple scattering model is developed which gives useful hints on the coherent wavenumber in such materials.

First, using the classical independent scattering approximation *ISA* and allowing for viscothermal losses by means of appropriate viscothermal admittance factors β , we derive the coherent wave wavenumber $k_e(\omega)$ for the special case of dilute arrays of rigid parallel cylinders in air. This modeling is called $ISA\beta$ [5].

Second, ultrasonic experiments are performed on standard porous foams to determine the coherent wave wavenumber. It appears that, despite the complicated microgeometry, the measured wavenumber may be reasonably well fitted by the $ISA\beta$ two-parameters (radius and density of cylindrical scatterers).

Finally, adding "mesoscopic" scatterers in the host porous foam in the form of additional parallel cylinders, we note that the combination of *ISA* at the mesoscale and $ISA\beta$ at the host scale, allows to properly describe the experimental data on the effective wavenumber for the multi-scale medium with the same $ISA\beta$ parameters as before.

Multiple scattering model ISAβ

Explicit calculation of the coherent wavenumber is possible, replacing the complicated microstructure of the material by a simple 2D random arrangement of parallel cylinders of radius *R* and density n.

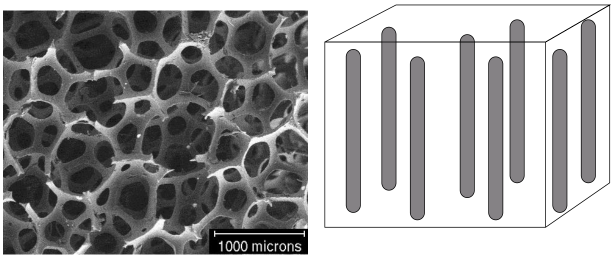

Fig. 1: The complicated structure of an air-saturated polymer foam (left) and its 2D idealization (right). The radius and density of cylinders are free parameters.

The solid volume fraction being generally small, the classical independent scattering approximation *ISA* can be used. In harmonic regime $e^{-i\omega t}$ it yields the following expression for the coherent wavenumber k_e [3] :

$$k_e^2 = k_0^2 + 4ni\sum_{m=0}^{\infty}(2-\delta_{m0})D_m \,, \qquad (1)$$

where k_0 is the wavenumber in air and D_m are the scattering coefficients. The latters are defined by considering the scattering of a plane wave on a 2D circular rigid scatterer placed at the origin of polar coordinates r, θ . The pressure field outside the boundary layers may be written

$$p(r,\theta) = \sum_{m=0}^{\infty} i^m (2-\delta_{m0})[J_m(k_0 r) + D_m H_m(k_0 r)]\cos m\theta \ .$$

For rigid scatterers without absorption, the boundary condition $\left.\frac{\partial p}{\partial r}\right|_{r=R} = 0$ yields $D_m = -\frac{J'_m(k_0 R)}{H'_m(k_0 R)}$. In presence of thin boundary layers at the scatterer surface, an admittance boundary condition $\left[\frac{\partial p_m}{\partial r} + i\beta_m k_0 p_m\right]\Big|_{r=R} = 0$, applies to each component m of the field. Because the boundary layers are considered thin compared to the scatterer radius, they appear locally plane. Thus, the expressions of the admittances β_m are deduced from that, well-known, for a plane rigid surface under plane wave excitation [4]. Substituting the value $\frac{m^2}{R^2}$ for the parallel wavenumber, it yields,

$$\beta_m = \frac{1-i}{2} k_0 \left[(\gamma - 1)\delta_h + \frac{m^2}{k_0^2 R^2} \delta_v \right]. \qquad (2)$$

These admittances determine the new coefficients D_m,

$$D_m = -\frac{J'_m(k_0 R) + i\beta_m J_m(k_0 R)}{H'_m(k_0 R) + i\beta_m H_m(k_0 R)}. \qquad (3)$$

Using (1-3), the $ISA\beta$ coherent wavenumber can be explicitly evaluated once given the radius R and density n of scatterers.

Experiments and discussion

The samples are polyurethane foam slabs of thickness L=5 cm and low flow resistivity. For the ultrasonic excitation and detection, the same wide bandwidth (20 kHz - 200 kHz) transducers are used. A reference signal associated to the acoustic propagation between the ultrasonic emitter and receiver (~ 20 cm) in air is first registered. Then, the same signal is sent to the slab of porous material placed between the transducers. The received signal spectrum is used to determine the phase velocity dispersion curve $v = \dfrac{\omega}{Re(k_e)}$, and the transmission coefficient $T = e^{-\mathrm{Im}(k_e)L}$ (neglecting the reflections at the slab interfaces). The same measurements of phase velocity and transmission coefficient are performed for the same polymer foam slabs in which full metallic cylinders (1.6 mm in diameter) have been embedded randomly but in a parallel manner. The transmission coefficient and phase velocity dispersion curve for the two kinds of samples - with and without metallic cylinders - are plotted on Fig. 2 and Fig. 3.

The '+++' lines correspond to the *EFM* prediction applied to the host porous material. As expected, significant deviations due to the scattering by the polymer foam microstructure appear at high frequencies. However, as shown by the continuous lines, this scattering is well represented by an ad-hoc utilization of $ISA\beta$. The two adjustable parameters $R = 70 \pm 2\ \mu m$, and $n = (45 \pm 1)\,10^5$, are fixed by fitting the experimental data. R is on the order of the transverse dimensions of the foam ribs. We note that a direct estimation of the solid foam volume fraction f by means of the expression $f = n\pi R^2$ would give $f = 0.07$. This is 3 times overestimated as compared to the true solid volume fraction of the polyurethane foam. This could be due to the difference between the simple geometry used in the $ISA\beta$ and the true geometry of the foam. Notwithstanding, having determined the $ISA\beta$ parameters R, n, of the porous foam, and now randomly adding in the medium a certain dilute amount of rigid parallel "mesoscopic" cylinders, we may predict with no adjustable parameters the new coherent wavenumber by combining *ISA* at the mesoscale, and the known $ISA\beta$ for the host. Good agreement with experimental data is observed. For the sample with mesoscopic scatterers (metallic cylinders), the experimental fluctuations observed are explained by the remaining 'incoherent field" contributions that are not eliminated with our limited spatial averaging (the sample is translated to probe different cylinder configurations).

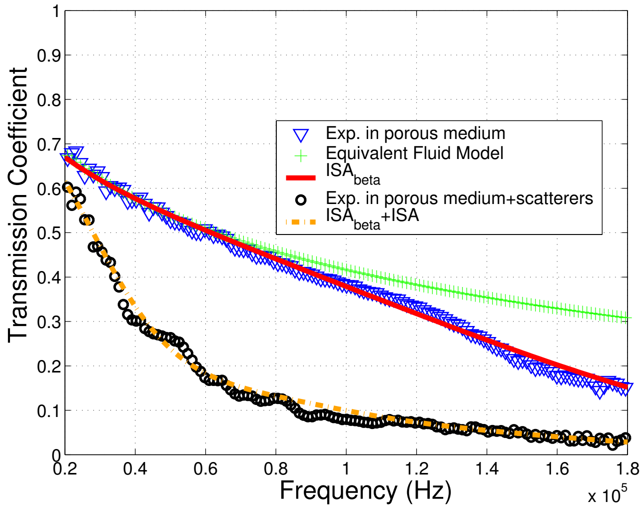

Fig. 2: Top : Transmission coefficient for the polyurethane foam slab. Bottom : Transmission coefficient of the same porous sample with additional parallel rigid cylinders of radius *R=0.8 mm,* with a filling ratio of *f~0.052*.

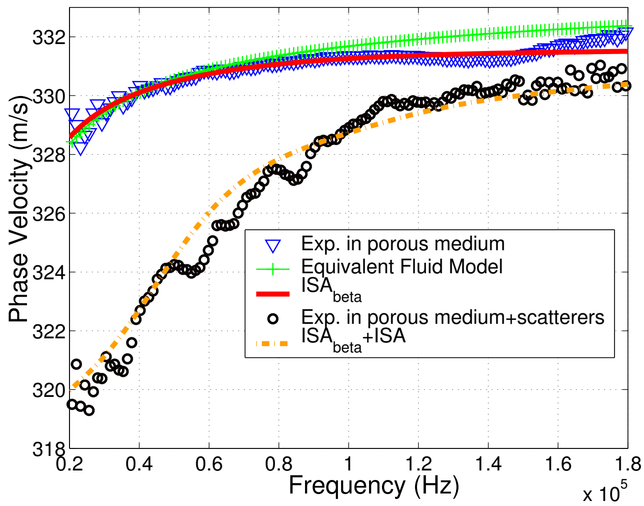

Fig. 3: Top : Phase velocity for the polyurethane foam. Bottom : Phase velocity for the same polyurethane foam with additional parallel rigid cylinders of radius *R=0.8 mm*, with a filling ratio of *f~0.052*.

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