Ultrasonic wave propagation in human cancellous bone: Application of Biot theory.

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Introduction

Osteoporosis is a disease caused by biochemical and hormonal changes, affecting the equilibrium between the resorption and deposition of new bony tissue. It leads to modification of the structure (porosity and thickness of trabeculae) and composition (mineral density) of this material. There has been much discussion on changes in trabecular pattern due to osteoporosis, but general indications are that the trabeculae grow thinner, possibly disappearing, and are therefore more widely spaced. Early clinical detection of this pathology via ultrasonic characterization would be of fundamental interest. Since trabecular bone is an inhomogeneous porous medium, the interaction between ultrasound and bone will be highly complex. Modelling ultrasonic propagation through trabecular tissue has been considered using porous media theories, such as Biot's theory [1]. Applications of Biot's theory to trabecular bone have enjoyed varying degrees of success[2, 3, 4, 5, 6]. The theory predicts two compressional waves: a fast wave, where the fluid (blood and marrow) and solid (calcified tissue) move in phase, and a slow wave where fluid and solid move out of phase. Fast and slow waves were identified independently in bovine trabecular bone in the late 1990s by Hosokawa and Otani [6]. One important assumption of the Biot theory is that the wavelength must be large compared with the dimensions of a macroscopic elementary volume. This volume has well defined properties, such as porosity, permeability and elastic moduli, which are representative of the medium. Scattering effects are thus neglected. However when the sizes or features of the trabecular bone are close to the wavelength, scattering effects must be taken into account.

The modified Biot theory

The equations of motion for the frame and fluid are given by the Euler equations applied to Lagrangian density. A correction factor is introduced via the dynamic tortuosity $\alpha(\omega)$ [7]. Its theoretical expression in high frequency domain is $\alpha(\omega) = \alpha_\infty \left(1 + \frac{2}{\Lambda}\left(\frac{\eta}{j\omega\rho_f}\right)^{1/2}\right)$, where α_∞ the tortuosity, and Λ the viscous characteristic length. The function $\alpha(\omega)$ expresses the viscous exchanges between the fluid and the structure which plays an important role in damping the ultrasonic wave in cancellous bone. In this case, the equations of motion for compressional waves can be written in the following form.

$$\rho_{11}(\omega)\frac{\partial^2 \vec{u}}{\partial t^2} + \rho_{12}(\omega)\frac{\partial^2 \vec{U}}{\partial t^2} = P\vec{\nabla}.(\vec{\nabla}.\vec{u}) + Q\vec{\nabla}(\vec{\nabla}.\vec{U}),$$

$$\rho_{12}(\omega)\frac{\partial^2 \vec{u}}{\partial t^2} + \rho_{22}(\omega)\frac{\partial^2 \vec{U}}{\partial t^2} = Q\vec{\nabla}(\vec{\nabla}.\vec{u}) + R\vec{\nabla}(\vec{\nabla}.\vec{U}).$$

In these equations, P, Q and R are generalized elastic constants which are related, via gedanken experiments, to other, measurable quantities, namely ϕ (porosity), K_f (bulk modulus of the pore fluid), K_s (bulk modulus of the elastic solid), K_b (bulk modulus of the porous skeletal frame) and N (shear modulus of the composite as well as that of the skeletal frame). The Biot coefficients, ρ_{mn} are related to the densities of solid (ρ_s) and fluid (ρ_f) phases. The coefficient ρ_{12} represents the mass coupling parameter between the fluid and solid phases. When a sound wave in the fluid impinges upon a cancellous bone at normal incidence, part of it is reflected back into the fluid, part is transmitted into the porous medium as a fast wave, and part is transmitted as a slow wave. For non-normal angles of incidence, part of it is also transmitted as a shear wave. In this paper we consider only the reflection and transmission at normal incidence. The amplitudes of these reflected and transmitted waves are determined by the relevant boundary conditions.

Ultrasonic Measurements

Experiments are performed in water using two broadband Panametrics A 306S piezoelectric transducers with a central frequency of 2.25 MHz in water. 400 V pulses are provided by a 5058PR Panametrics pulser/receiver. The signals received are amplified to 90 dB and filtered above 10 MHz to avoid high frequency noise. The experimental setup is shown in Fig. 1. The parallel-faced cubic samples were machined from human cancellous bone in femoral heads. The liquid in the pore space (blood and marrow) is removed from the bone sample and substituted by water. The transmitting transducer insonifies the sample at normal incidence with a short (in time domain) pulse. When the pulse hits the front surface of the sample, part is reflected, part is transmitted as a fast wave, and a part is transmitted as a slow wave. When any of these components, travelling at different speeds, hit the second surface, a similar effect takes place: part is transmitted into the fluid, part is reflected as a fast or slow wave. Sample characteristics are measured using standard methods [3] and given in Table 1. The fluid (water) characteristics are: bulk modulus $K_f = 2.28$ GPa, density $\rho_f = 1000\text{Kgm}^{-3}$, viscosity $\eta = 10^{-3} Kg.m.s^{-1}$. The incident experimental signal generated by the transducer is shown in Fig. 2. A comparison between experimental transmitted signals (solid line) and simulated signals (dashed line) for bone samples M1, M2 and M3

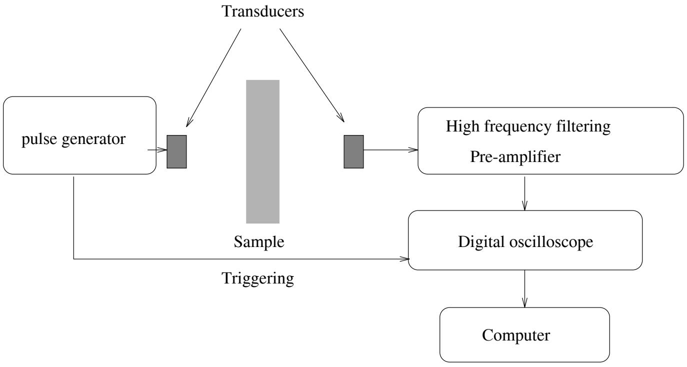

Figure 1: Experimental set-up for ultrasonic measurements.

Human bone samples	M1	M2	M3
Thickness (cm)	0.7	0.5	0.38
Density ρ_s (Kg/m^3)	1960	1960	1960
Porosityϕ	0.83	0.77	0.88
Tortuosity α_∞	1.05	1.01	1.02
Viscous length Λ (μm)	5	2.7	5
Solid mudulus K_s (GPa)	20	20	26
Frame mudulus K_b (GPa)	3.3	4	1.3
Shear modulus N (GPa)	2.6	1.7	0.35

Table 1: Biot's model parameters of cancellous bone

is given in Fig. 3, 4 and 5 respectively. The experimental transmitted waveforms are travelling through the cancellous bone in the same direction as the trabecular alignment. For some situations, fast and slow waves are superimposed, depending on the coupling between the two porous medium phases, so that this is due to the modified Biot's parameters for cancellous bone, which play an important role in the arrival time of the two waves. The experimental data and theoretical prediction are seen to match closely which allowed us to conclude that the modified Biot theory is quite suitable for describing the propagation of ultrasonic wave in cancellous bone.

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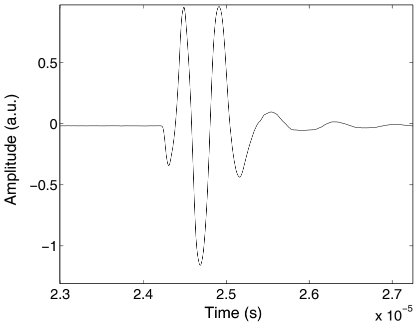

Figure 2: Experimental incident signal

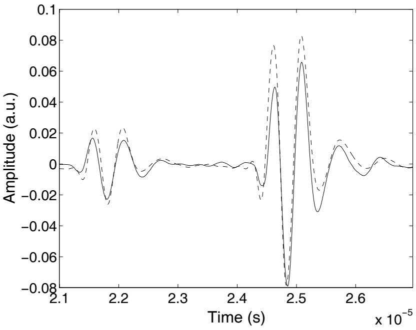

Figure 3: Comparison between experimental transmitted signal (solid line) and simulated transmitted signal (dashed line) for bone sample M1.

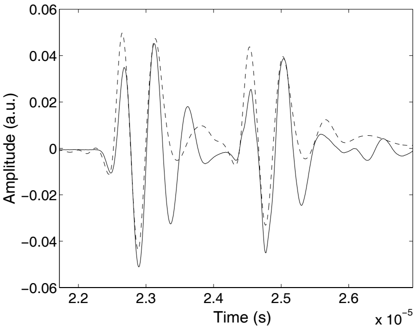

Figure 4: Comparison between experimental transmitted signal (solid line) and simulated transmitted signal (dashed line) for bone sample M2.

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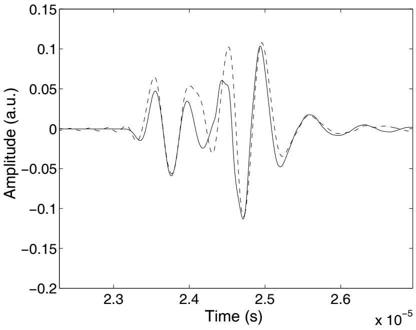

Figure 5: Comparison between experimental transmitted signal (solid line) and simulated transmitted signal (dashed line) for bone sample M3.