Synthèse Modale pour les problèmes poroélastiques

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Introduction

Multilayered structures including porous materials are commonly used in many applications in order to attenuate vibrations and sound waves radiated by structures. Dazel *et al.* [1, 2, 3] recently proposed a generalized complex modes (GCM) technique for finite-element poroelastic problems and applied it to a finite-element three-dimensional problem. This technique consists in making a Taylor expansion of the frequency dependent coefficients of the model and then solving the associated polynomial eigenvalue problem in a generalized State Space extending the classical one. Generalized Complex modes are obtained and used as a basis to reduce the original finite element system. It has been shown that this technique allows for the convergence of the solution and yields a significant reduction of the number of degrees of freedom of the problem for single porous structures.

The objective of this paper is to apply to multilayered structures involving porous materials a Component Mode Synthesis (CMS) technique [5] based on GCM. A more detailed presentation is provided in [3, 4].

Ritz vectors associated with GCM

In Component Mode Synthesis, the modal basis of the whole structure is deduced by keeping the first modes of each substructure completed with *static* vectors which accounts for the flexibility of the modes which are no retained. All the extensions of these vectors (called Ritz vectors) are now introduced.

Free interface GCM

The free-interface modes are obtained through the resolution of a generalized eigenvalue problem:

$$\sum_{i=0}^{d}[\mathbf{M}_i](j\omega)^i\mathbf{v} = \mathbf{0} \Leftrightarrow [\mathbf{A}]\mathbf{V} + (j\omega)[\mathbf{B}]\mathbf{V} = \mathbf{0}. \qquad (1)$$

In the relation, the left hand side is the polynomial eigenvalue problem (of dimension n) associated with the free interface problem associated to the porous structure. The right hand side presents the linearized eigenvalue problem which stands in the Generalized State Space.

Rigid-body GCM

The n_R rigid-body correspond to null eigenvalue. It can be shown that they can be written:

$$[\mathbf{\Phi}_r] = \left[\frac{[\mathbf{0}_{n(d-1)\times n_R}]}{[\boldsymbol{\psi}_r]}\right] \text{ with } [\mathbf{M}_0][\boldsymbol{\psi}_r] = [\mathbf{0}]. \qquad (2)$$

Hence, it is shown that the rigid-body modes corresponds to the kernel of the stiffness matrix.

Attachment GCM

As in eq. (2), it is possible to show that the static behaviour of the porous structure is linked to $[\mathbf{M}_0]$ matrix. The attachment GCM is defined by

$$\hat{\mathbf{\Phi}}_E = \left[\frac{[\mathbf{0}_{n(d-1),n_E}]}{[\mathbf{M}_0]^{-1}[\mathbf{F}_E]}\right] = [\mathbf{G}][\boldsymbol{\mu}_E]. \qquad (3)$$

Inertia relief attachment GCM

When a component has rigid-body motion, inertia relief modes are required to represent the complete static response. This notion has been extended for GCM by introducing a projection matrix $[\mathbf{P}]$ with the following properties. On the one hand, when a force vector is pre-multiplied by the projection matrix $[\mathbf{P}]$, a self-equilibrated matrix force vector is obtained which take into account d'Alembert forces due to rigid body motion. On the other hand, when a displacement vector $\mathbf{u}$ is pre-multiplied by $[\mathbf{P}]^t$, the rigid-body component of motion are removed from $\mathbf{u}$ so as to form a matrix which is orthogonal to the rigid body modes.

Hence, a General Flexibility matrix $[\mathbf{G}_f]$ through this projection matrix can be introduced and it is shown that

$$[\mathbf{G}_f] = [\mathbf{P}]^t[\mathbf{G}][\mathbf{P}] = [\mathbf{\Phi}_f][\mathbf{\Lambda}_f]^{-1}[\mathbf{\Phi}_f]^t, \qquad (4)$$

where $[\mathbf{\Phi}_f]$ and $[\mathbf{\Lambda}_f]$ respectively correspond to elastic eigenvectors and eigenvalues.

Residual GCM

Matrix $[\mathbf{G}_f]$ can be partitioned in two blocks corresponding to the set of preserved modes $[\mathbf{\Phi}_p]$ and the set of higher modes $[\mathbf{\Phi}_h]$. Hence one defines:

$$[\mathbf{G}_p] = [\mathbf{\Phi}_p][\mathbf{\Lambda}_p]^{-1}[\mathbf{\Phi}_p]^t\,,\ [\mathbf{G}_h] = [\mathbf{\Phi}_h][\mathbf{\Lambda}_h]^{-1}[\mathbf{\Phi}_h]^t. \qquad (5)$$

The generalized State residual inertia-relief attachment mode matrix $[\mathbf{\Phi}_d]$ is then defined by:

$$[\mathbf{\Phi}_d] = [\mathbf{G}_h][\boldsymbol{\mu}_B] = ([\mathbf{G}_f] - [\mathbf{G}_p])\,[\boldsymbol{\mu}_B]. \qquad (6)$$

Coupling of substructures

Subset of Ritz vectors of a substructure

If one defines the set of kept modes $[\mathbf{\Phi}_k]$ which consists of the rigid body modes and the flexible preserved modes

($[\mathbf{\Phi}_k] = [\mathbf{\Phi}_r, \mathbf{\Phi}_p]$), the Generalized unknown of the problem $\mathbf{X}$ is written as follows:

$$\mathbf{X} = \underbrace{[\mathbf{\Phi}_k, \mathbf{\Phi}_d]}_{[\mathbf{\Sigma}]} \left\{ \begin{array}{c} \mathbf{q} \\ \boldsymbol{\lambda} \end{array} \right\}. \tag{7}$$

In this decomposition, $\mathbf{q}$ is the contribution of the kept modes and will be determined after the assembling of the substructures. $\boldsymbol{\lambda}$ is the static contribution vector of the inertia relief modes. It can be shown that $\boldsymbol{\lambda}$ can be interpreted as the boundary force on the considered substructure induced by the other substructure.

Global Ritz family of the whole structure

If one considers two substructures (with subscripts α and β) each one represented by decomposition (7), it can be shown that the force compatibility relations leads to $\boldsymbol{\lambda}_\alpha = -\boldsymbol{\lambda}_\beta$ and that one can exhibit a $[\mathbf{T}]$ matrix so that

$$\boldsymbol{\lambda}_\alpha = [\mathbf{T}]\mathbf{q}_T \,,\; \mathbf{q}_T = \left\{ \begin{array}{c} \mathbf{q}_\alpha \\ \mathbf{q}_\beta \end{array} \right\}.$$

Finally one obtains:

$$\left\{ \begin{array}{c} \mathbf{X}_\alpha \\ \mathbf{X}_\beta \end{array} \right\} = \underbrace{\left(\left[\begin{array}{cc} [\mathbf{\Phi}_{k\,\alpha}] & [\mathbf{0}] \\ [\mathbf{0}] & [\mathbf{\Phi}_{k\,\beta}] \end{array} \right] + \left[\begin{array}{c} [\mathbf{\Phi}_{d\,\alpha}] \\ -[\mathbf{\Phi}_{d\,\beta}] \end{array} \right] [\mathbf{T}] \right)}_{[\mathbf{R}]} \mathbf{q}_T.$$

$[\mathbf{R}]$ is the global modal matrix of the whole substructure. Once more, the term modal refers to Ritz vectors. Matrix $[\mathbf{R}]$ is used to determine the dynamical response of the system as presented in the following case of application.

Simple case of application

The case of the coupling of a plate and a porous material and excited by a point force is considered in this paper. The configuration is depicted in Figure 1.

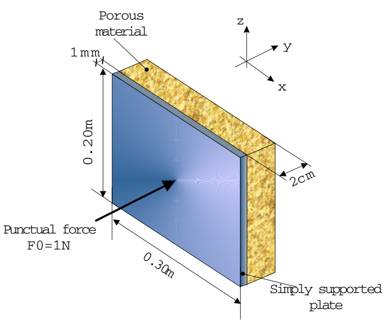

Figure 1: Configuration of the problem

The plate is in aluminum and the properties of the porous material are given in table 1

The convergence of the technique for material A is showed in Figure 2. 3512 degrees of freedom are needed to ensure the convergence of the direct solution. One can notice that the different peaks are very damped from 300 Hz to 500 Hz. Two numerical simulations are presented for the modal solution. In the first case, convergence is

$\sigma\,[kNm^{-4}s]$	$h\,[1]$	$\alpha_\infty\,[1]$	$\rho_1\,[kgm^{-3}]$	
10.9	0.99	1.02	8.8	
$\Lambda'\,[m]$	$\nu\,[1]$	$N\,[kPa]$	$\eta_s\,[1]$	$\Lambda\,[m]$
130.10^{-6}	0	80	0.17	100.10^{-6}

Table 1: Considered porous material

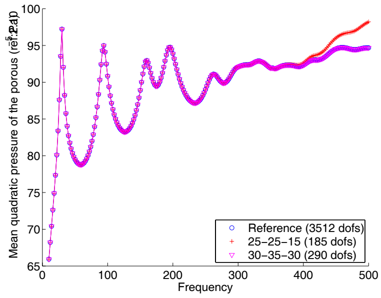

Figure 2: Convergence of the method

obtained up to 300 Hz. The reduction rate is around 20 which is quite interesting. The second simulation (triangle upside down) is a 290 degrees of freedom problem with a good agreement regarding to the reference solution. The convergence rate is her around 12.

Conclusion

This paper presented the extension of Component Mode Synthesis for Generalized Complex Modes. Different Ritz vectors were introduced and used to build a global modal basis of the whole structure. A numerical example was given to show the convergence of the method

References

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