Modeling the Transient Behavior of a Thermoacoustic Refrigerator

Gaelle Poignand[1], Pierrick Lotton[1], Mikhail Mironov[2], Michel Bruneau[1], Vitaly Gusev[1], Philippe Blanc-Benon[3]

[1] *Laboratoire d'Acoustique de l'Université du Maine,UMR CNRS 6613, Le Mans, France,*

[2.] *Andreev Acoustics Institute, Russian Academy of Sciences,Moscow, Russia*

[3] *Laboratoire de Mécanique des Fluides et d'Acoustique,UMR CNRS 5509, Ecole Centrale de Lyon, France*

Introduction

Nowadays, the expression of the temperature difference across the stack of a thermoacoustic refrigerator can easily be obtained in steady state, but models for analyzing the transient process are still in progress [1]. More precisely, previous models used to describe the behavior of thermoacoustic devices usually assume several approximations preventing us from describing the transient behavior of the temperature field. Particularly, heating due to viscous effects at the interfaces between the walls and the fluid in the thermoacoustic stack and thermal losses are usually neglected. Therefore, the parameters which govern the transient response, its shape and its characteristic stabilization time are not fully identified.

Thus, the aim of the present work is to investigate more deeply, analytically, the transient behavior of the thermoacoustic refrigerator, using a revisited description for the temperature variations in the stack and in the resonator due to thermoacoustic processes, by introducing new features as heat losses through the thermoacoustic system and viscous heating in the boundary layers near the walls.

Model

In the model, the resonator of the thermoacoustic refrigerator is split in three regions (as shown in Fig. 1), that are the region between the loudspeaker and the stack of plates (region 1), the stack (region 2) and the region between the stack and the end of the resonator (region 3). The conduction of heat equation is written for each region, as follows :

$$\rho_i C_i \frac{\partial T_i}{\partial t} = \lambda_i \frac{\partial^2 T_i}{\partial t_2} - \eta_i T_i \text{ , } i=1,3, \tag{1}$$

$$\rho_2 C_2 \frac{\partial T_2}{\partial t} = \lambda_2 \frac{\partial^2 T_2}{\partial t_2} - \eta_i T_2 + \partial_x q_{th} + Q_v \text{ ,} \tag{2}$$

where the transfer of thermal energy from the thermoacoustic device to the surroundings through the resonator walls is taken into account by the empirical coefficient η_i. The coefficient q_{th} represents the heat flux per unit surface due to thermoacoustic process,

$$q_{th} = a_{th}(x) + b_{th}(x)\partial_x T_2 \text{ ,} \tag{3}$$

with coefficients a_{th} and b_{th} functions of particle velocity and acoustic pressure. The coefficient Qv is the heat power per unit volume due to viscous effects at the interfaces between the walls and the fluid in the stack region,

$$Qv = \frac{1}{T_p}\int \left\langle \mu\left(\partial_y v_x\right)^2 \right\rangle dt \text{ ,} \tag{4}$$

with T_p the acoustic period, μ the viscosity coefficient, v_x the x-component of the particle velocity and <•> standing for the spatial average over a cross section of the stack.

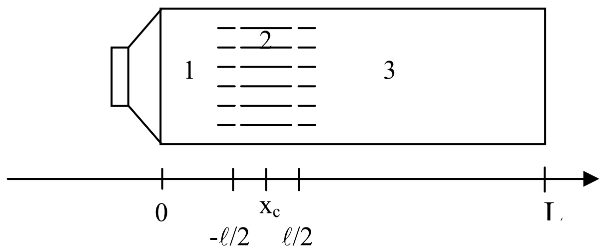

Figure 1: Thermoacoustic refrigerator

A set of differential equations, derived from the given physical situation, corresponding to initial and boundary conditions is also written

$$T_1(x_c - \ell/2) = T_2(x_c - \ell/2) \text{ , } \forall t, \tag{5}$$

$$T_2(x_c + \ell/2) = T_3(x_c + \ell/2) \text{ , } \forall t, \tag{6}$$

$$\lambda_1 \partial_x T_1(x_c - \ell/2) = \lambda_2 \partial_x T_2(x_c - \ell/2) - qth(xc - \ell/2), \forall t \text{ ,} \tag{7}$$

$$\lambda_3 \partial_x T_3(x_c + \ell/2) = \lambda_2 \partial_x T_2(x_c + \ell/2) - qth(xc + \ell/2), \forall t \text{ ,} \tag{8}$$

$$T_i(t=0) = 0, \quad \forall x \text{ ,} \tag{9}$$

where the factor q_{th} (Eq. 7 and 8) takes into account the thermoacoustic heat transfer from the fluid to the plates at both ends of the stack.

A solution can easily be written in the Laplace domain, but an analytical expression of the inverse transform of this solution can not be obtained without making some approximations. First, the stack is considered as a short one (that is its length is considered very small having regard to wavelength) and, second, the heat transfers from the region of the stack to the regions (1) and (3) are not taken into account ($\lambda_1 = \lambda_3 = 0$). Then, the expression of the temperature difference between the two ends of the stack can be written as :

$$\Delta T(t) = \frac{a_{th}}{\rho_2 C_2 L} \cdot \left\{ \frac{2\left(1 - exp(-\sqrt{\frac{\eta_2}{\alpha_2'}} L)\right)}{\sqrt{\frac{h_2}{\alpha_2'}} L \left(1 + exp(-\sqrt{\frac{\eta_2}{\alpha_2'}} L)\right)} - 8 \sum_{n=0}^{\infty} \frac{e^{-\left[\frac{(2n+1)^2 \pi^2 \alpha_2'}{L^2} + \eta_2\right] t}}{(2n+1)^2 \pi^2 + \frac{\eta_2 L^2}{\alpha_2'}} \right\} H(t) . \quad (10)$$

An example of temperature difference between the ends of the stack obtained with this model is given in Fig. 2 for a given thermoacoustic device. This theoretical result can be compared with experimental and numerical ones recently available [2,3]. A good agreement is obtained when fitting the control parameter η_2.

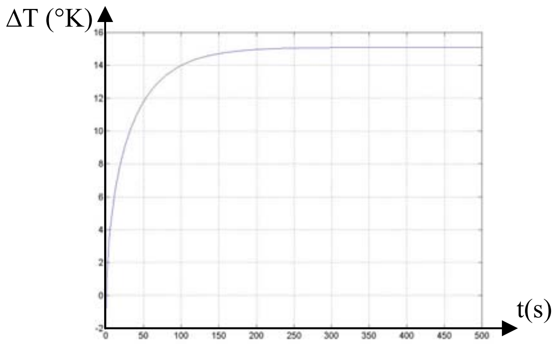

Figure 2 : Theoretical temperature evolution

Conclusion

The transient process has been modeled in terms of a system of a coupled heat conduction partial derivative equations and a set of initial and boundary conditions derived from the given physical situation. A solution has been obtained, using Laplace transform, when assuming drastic approximations. Another analytical solution which would allow to describe the evolution of the temperature at any point of the resonator is currently in progress.

References

[1] Piccolo A., Cannistraro G., "Convective heat transport along a thermoacoustic couple in the transient regime", International Journal of Thermal Sciences, vol. 41 (11), 2002, pp. 1067-1075

[2] Duffourd S., Blanc-Benon Ph., Lotton P., Bruneau M., Gusev V., "Etude temporelle du gradient de temperature dans un réfrigérateur thermoacoustique : expérience et modèle", Proc. 5ème Congrès Français d'Acoustique, Lausanne, p 226-229, 2000.

[3] Marx D., Blanc-Benon Ph., "Numerical simulation of hydrodynamic and thermal effects around a 2-D stack plate in a thermoacoustic refrigerator", Proc. 146th ASA meeting, Austin, 2003