

Attenuation of acoustical waves in duct with flow

Y. Aurégan V. Pagneux

Laboratoire d'Acoustique de l'Université du Maine UMR CNRS 6613

Av. O. Messiaen, 72085 LE MANS Cedex 9, France. Email: yves.auregan@univ-lemans.fr

Introduction

A study of the attenuation of an acoustical wave in a rigid duct with shear flow is presented. The flow is considered as quasi-laminar. Two methods for solving these equations are presented: First, a perturbation expansion of the visco-thermal propagation equations is given. A second method, using the Chebyshev polynomials is also presented and compared to the perturbation expansion.

List of symbol

- $u_0(y) + \tilde{u} = c_0(M + u), \tilde{v} = c_0v$ are the axial and transverse velocities
- $p_0 + \tilde{p} = p_0(1 + p)$ is the pressure
- $\rho_0 + \tilde{\rho} = \rho_0(1 + \rho)$ is the density
- $T_0 + \tilde{T} = T_0(1 + \tau)$ is the temperature
- u, v, p, τ, ρ ; dimensionless acoustic variables

Basic equations

The linear equations governing the propagation of waves in a mean shear flow, when the diffusion coefficients are assumed to be constants, are:

$$\rho_0 \left(\frac{\partial \tilde{u}}{\partial t} + u_0 \frac{\partial \tilde{u}}{\partial x} + \frac{du_0}{dy} \tilde{v} \right) = -\frac{\partial \tilde{p}}{\partial x} + \mu_0 \Delta \tilde{u} + \left(\lambda_0 + \frac{\mu_0}{3} \right) \frac{\partial}{\partial x} \left(\frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} \right)$$

$$\rho_0 \left(\frac{\partial \tilde{v}}{\partial t} + u_0 \frac{\partial \tilde{v}}{\partial x} \right) = -\frac{\partial \tilde{p}}{\partial y} + \mu_0 \Delta \tilde{v} + \left(\lambda_0 + \frac{\mu_0}{3} \right) \frac{\partial}{\partial y} \left(\frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} \right)$$

$$\frac{\partial \tilde{p}}{\partial t} + u_0 \frac{\partial \tilde{p}}{\partial x} + \rho_0 \left(\frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} \right) = 0 \quad (1)$$

$$\rho_0 C_p \left(\frac{\partial \tilde{T}}{\partial t} + u_0 \frac{\partial \tilde{T}}{\partial x} \right) = \frac{\partial \tilde{p}}{\partial t} + u_0 \frac{\partial \tilde{p}}{\partial x} + \kappa \Delta \tilde{T} + 2\mu_0 \frac{du_0}{dy} \left(\frac{\partial \tilde{u}}{\partial y} + \frac{\partial \tilde{v}}{\partial x} \right)$$

$$p = R\rho_0 \tilde{T} + RT_0 \tilde{\rho}$$

Perturbation expansion

The dimensionless equations are developed in terms of $\Omega = \omega H/c_0$ and M (the mean Mach number) and the dimensionless wavenumber is sought under the form:

$$K = K_{00} + \Omega K_{10} + M K_{01} + \Omega M K_{11} + \Omega^2 K_{20} + M^2 K_{02} + \dots$$

First order K_{00}

At the first order, we obtain:

$$\frac{1}{\gamma} p'_{00} = 0$$

$$i\tau_{00} - i\frac{\gamma-1}{\gamma} p_{00} - \frac{1}{\sigma^2 s^2} \tau_{00} = 0$$

$$iu_{00} - \frac{1}{\gamma} iK_{00} p_{00} - \frac{1}{s^2} u_{00} = 0$$

$$v'_{10} - iK_{00} u_{00} + ip_{00} - i\tau_{00} = 0$$

where $s = \delta_{ac}/H$, δ_{ac} is the thickness of the acoustical boundary layer and σ is the Prandtl number. This set of equations can be formally written:

$$L_{K_{00}} \begin{pmatrix} p_{00} \\ \tau_{00} \\ u_{00} \\ v_{10} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

The solution of this problem gives the Zwikker and Kosten solution with $k_h^2 = -i\sigma^2 s^2$ and $k_v^2 = -is^2$:

$$p_{00} = 1$$

$$\tau_{00} = \frac{\gamma-1}{\gamma} \left(1 - \frac{\cos(k_h y)}{\cos(k_h)} \right) = \frac{\gamma-1}{\gamma} f_h$$

$$u_{00} = \frac{K_{00}}{\gamma} \left(1 - \frac{\cos(k_v y)}{\cos(k_v)} \right) = \frac{K_{00}}{\gamma} f_v$$

$$v_{10} = -i \left(\frac{K_{00}^2 - 1}{\gamma} y - \frac{K_{00}^2}{\gamma} \frac{\sin(k_v y)}{k_v \cos(k_v)} - \frac{\gamma-1}{\gamma} \frac{\sin(k_h y)}{k_h \cos(k_h)} \right)$$

and the wavenumber is given by

$$K_{00}^2 = \frac{1 + (\gamma-1) \frac{\tan k_h}{k_h}}{1 - \frac{\tan k_v}{k_v}} \quad (2)$$

The adjoint solution is given by:

$$\Phi = \begin{pmatrix} \phi_1 = -v_{10} \\ \phi_2 = f_h \\ \phi_3 = K_{00} f_v \\ \phi_4 = 1 \end{pmatrix}$$

Second order K_{10} (term in Ω)

The set of equations giving K_{10} can be written:

$$L_{K_{00}} \begin{pmatrix} p_{10} \\ \tau_{10} \\ u_{10} \\ v_{20} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{\gamma} iK_{10} p_{00} \\ iK_{10} u_{00} \end{pmatrix}$$

By use of the Fredholm theorem, this set of equations has a solution only if

$$K_{10} \int \left(\frac{1}{\gamma} p_{00} \phi_3 + u_{00} \phi_4 \right) dy = 0 \quad (3)$$

so that

$$K_{10} = 0$$

Second order K_{01} (term in M)

The set of equations giving K_{01} can be written:

$$L_{K_{00}} \begin{pmatrix} p_{01} \\ \tau_{01} \\ u_{01} \\ v_{11} \end{pmatrix} = \begin{pmatrix} 0 \\ imK_{00}\tau_{00} - iK_{00}\frac{\gamma-1}{\gamma}mp_{00} + 2\frac{\gamma-1}{s^2}m'u'_{00} \\ \frac{1}{\gamma}iK_{01}p_{00} - v_{10}m' + imK_{00}u_{00} \\ iK_{01}u_{00} + iK_{00}mp_{00} - iK_{00}m\tau_{00} \end{pmatrix} \quad (4)$$

where m is the transverse dependence of the mean velocity ($u_0(y) = Mm(y)$).

By use of the Fredholm theorem, this set of equations has a solution only if

$$\int \left(\phi_2 (imK_{00}\tau_{00} - iK_{00}\frac{\gamma-1}{\gamma}mp_{00} + 2\frac{\gamma-1}{s^2}m'u'_{00}) + \phi_3 \left(\frac{1}{\gamma}iK_{01}p_{00} - v_{10}m' + imK_{00}u_{00} \right) + \phi_4 (iK_{01}u_{00} + iK_{00}mp_{00} - iK_{00}m\tau_{00}) \right) dy$$

or

$$2 \left(\int f_v dy \right) K_{01} = - \int \left(m((\gamma-1)(f_h-2)f_h + K_{00}^2 f_v^2 + \gamma) + \frac{1}{i} m' (-\gamma f_v v_{10} + 2\frac{\gamma-1}{s^2} f_h f'_v) \right) dy \quad (5)$$

The value of K_{00} and K_{01} as a function of s is given in Figures 1 and 2 for two values of the mean flow profile $m = (2n_M + 1)(1 - y^{2n_M}) / (2n_M)$.

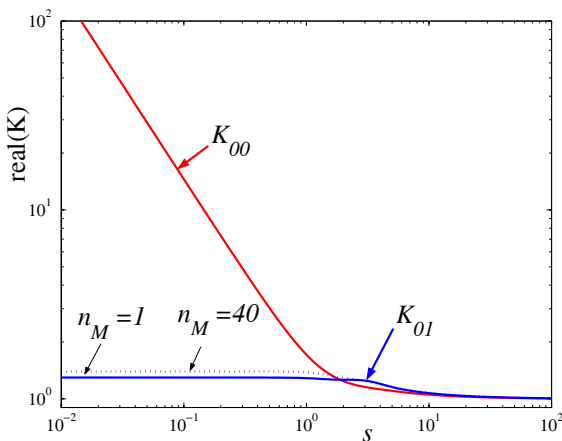


Figure 1: Real part of K_{00} and K_{01} as a function of s .

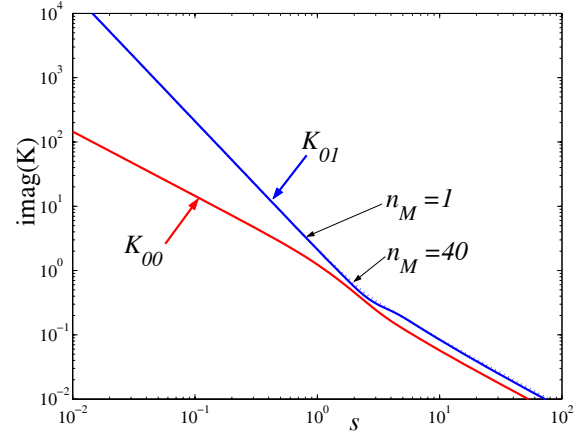


Figure 2: Imaginary part of K_{00} and K_{01} as a function of s .

Use of Chebyshev polynomials

The equations (1) can be rearranged to eliminate the variables p and ρ . The remaining variables u, v, τ have to vanish at the wall. The vector \mathbf{U} represents the value of u at the N Chebyshev points taken along the transverse direction. The equations (1) can be put under the form

$$K \begin{pmatrix} \mathbf{U} \\ \mathbf{V} \\ \mathbf{T} \\ K\mathbf{U} \\ K\mathbf{V} \\ K\mathbf{T} \\ K^2\mathbf{U} \\ K^2\mathbf{V} \\ K^3\mathbf{V} \end{pmatrix} = \mathbf{M} \begin{pmatrix} \mathbf{U} \\ \mathbf{V} \\ \mathbf{T} \\ K\mathbf{U} \\ K\mathbf{V} \\ K\mathbf{T} \\ K^2\mathbf{U} \\ K^2\mathbf{V} \\ K^3\mathbf{V} \end{pmatrix}$$

The modes can be found by computing the eigenvalues of the $9N \times 9N$ matrix \mathbf{M} . The values find by this method for the quasi-plane wave are compared to the value find by the perturbation expansion in Figure 3.

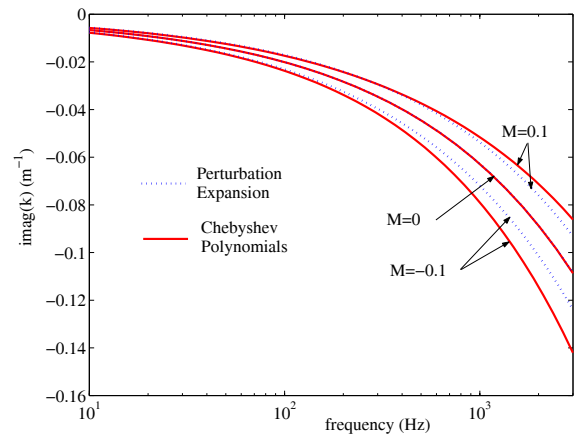


Figure 3: Imaginary part of the wavenumber k as a function of the frequency f for a channel of height 15 mm at 20 °C ($15 < s < 265$ and $0.0014 < \Omega < 0.41$).