Suppression of intra-plate echoes induced by a titanium Fabry Perot: the Stokes equations linking time-reversal and the inverse filter

François Vignon[1] (francois.vignon@loa.espci.fr), Jean-François Aubry[1], Alexandre Saez[2], Mickaël Tanter[1], Didier Cassereau[1], and Mathias Fink[1].

[1] *LOA, , CNRS. UMR. 7587, ESPCI, 10 rue Vauquelin 75231 Paris cedex 05*
[2] *LBHP, Université Paris 7, 4 place Jussieu, 75251 Paris cedex 05*

Introduction

The aim is to achieve temporal focusing through a titanium plate immersed in water, which acts as a Fabry-Perot structure for ultrasonic waves. This Fabry-Perot is surrounded by two identical linear "λ" arrays of 69 ultrasonic transducers, placed at equal distance from the block, of central frequency 1.5 MHz, as shown on fig.1.

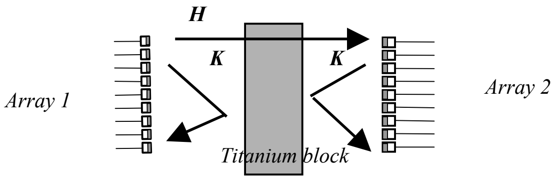

Figure 1: Description of the experiment.

The aim is to find the signals to shoot from array 1 and/or 2 to obtain a single-front plane wave on array 2 ("target signal"), represented on figure 2.

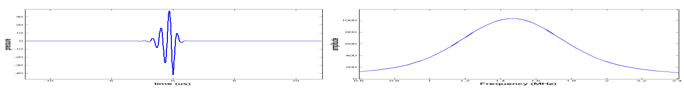

Figure 2: Temporal shape s(t) (left) and spectrum s(ω) (right) of the signals that have to be received on all elements of array 2.

First, the propagators H from array 1 to array 2, and K from array 1 to itself and from array 2 to itself, are acquired (a propagator between arrays 1 and 2 is a matrix in which (i,j) position is the fourier transform of the Green's function $h_{ij}(t)$ of the medium between element i of array 1 and j of array 2). If we define an emission vector $E(\omega)=[e_1(\omega),e_2(\omega),\dots e_{69}(\omega)]$, containing in i^{th} position the fourier transform of the signal $e_i(t)$ sent from transducer i of array 1, the fourier components of the signal subsequently received on array 2, $F(\omega)=[f_1(\omega),f2(\omega),\dots f_N(\omega)]$, are linked with E *via* the propagator by the equation, valid for each frequency:

$$F(\omega)=H(\omega)E(\omega). \quad (1)$$

The time reversal mirror and cavity

The time-reversal mirror (TRM) [1] is an adapting focusing technique which principle is the following: the target signal $S(\omega)=s(\omega)[1,1\dots,1]$ is sent from array 2 to array 1. The recorded signals on array 1 are $R(\omega)={}^tH(\omega)S(\omega)$ (${}^tH$ describes propagation from array 2 to 1). These signals are the time-reversed (equivalent to a phase conjugation in Fourier domain), so that the resent signals are then $E(\omega)=R(\omega)^*={}^tH(\omega)^*S(\omega)$ (S=S* if s(t) is a symmetric signal). The signals recorded in focal plane are finally

$$F(\omega)=H(\omega)R(\omega)^*=[H(\omega){}^tH(\omega)]^*S(\omega). \quad (2)$$

The temporal shape, and the spectrum, of the signals F received on array 2, are represented on fig.3. Due to multiple reflections of the wave in the Fabry-Perot, they present a series of echoes preceding and following the main wavefront. On the spectrum, the resonant frequencies of the Fabry-Perot are enhanced, so that the frequency contents of those signals are far from the one of the target signal S.

The Time-Reversal cavity (TRC) is the same as the TRM, but using both arrays 1 and 2 to focus on target S on array 2: S is shooted from 2, the signals R_1 and R_2 then arriving respectively on array 1 and 2 are recorded, time-reversed, and sent back. The resulting signals on array 2 are

$$F=(\omega)=[H(\omega){}^tH(\omega)^*]S(\omega)+(\omega)^*+[K(\omega){}^tK(\omega)^*]S(\omega), \quad (3)$$

the first half of the above expression describing the time-reversal mirror process in transmission (using only array 1), and the second half the TRM in reflexion (using only array 2). Whereas TRM-focused signals, in transmission and in reflection, both present echoes preceding and following the main wave front (fig.3), they interfere constructively at the arrival time of the main wave front, and destructively at the arrival times of the echoes, thus leading to main front enhancement and echoes cancellation. In the frequency domain the same frequencies that are lost in the transmission TRM process are complementary to the ones that are lost in the reflexion TRM process, and the sum of the two spectras approaches closely the target signal spectra (fig.3).

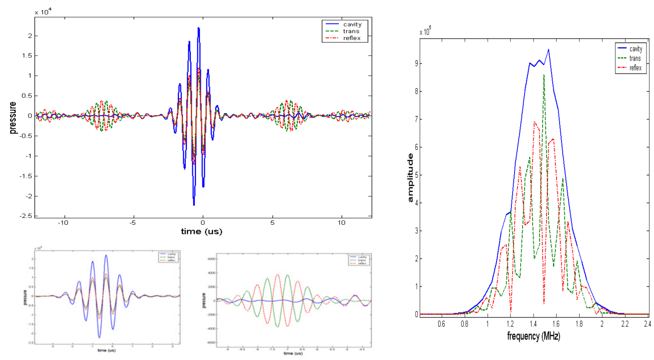

Figure 3: Temporal shape (left) and spectrum (right) of the signals received on array 2 after time-reversal processes. Plain, blue: TRC (using both arrays 1 and 2). Dashed, green: TRM in transmission (using only array 1). Dash-dotted, red: TRM in reflexion (using only array 2). The two insets on the left represent the main wavefront (left) and the first preceding echo (right)

The Inverse Filter

The inverse filter (IF) [1,2] allows to obtain such a good focusing as the one with the TRC, but sending signals only from array 1. The signals E to emit from array 1 are computed as

$$E=H^{-1}S \quad (4)$$

H^{-1} designing the pseudoinverse of the propagator from array 1 to 2 (only the highest eigenvalues of H are inversed, the others are set to 0). Inversion is performed at all frequencies. Those signals E (represented on fig.4) are sent from array 1 and the signals F subsequently received on array 2 are

$$F=[H\,H^{-1}]S \qquad (5)$$

That would be equal to S if the inversion were perfect. They are in fact close to S (echoes are cancelled, see fig.4)

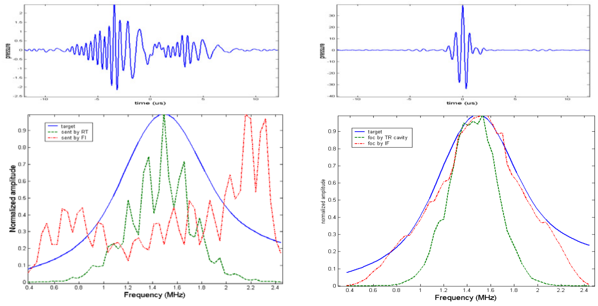

Figure 4: Top: left: the focusing signals E to be emitted from array 1 to focus on array 2, calculated by IF; right: the resulting signals F obtained on array 2. Bottom: their fourier transforms: left, the fourier transform of the signals E (red, dash-dotted), to be compared to the signals E computed by transmission TRM (green, dashed) and target (blue, plain); right, the spectras of the subsequent focalisation F (red, dash-dotted), with TRC (green, dashed) and target signal (blue, plain).

The temporal shape of the signals E calculated by IF to focus on S have the following interpretation: the main wave front is first sent, followed by a second wave front of smaller amplitude that is to interfere destructively with the first rebound of the main front in order to eliminate following echoes. In frequential domain, the IF enhances the frequences that will be further atenuated because of the presence of the Fabry-perot (see fig.4)

Iterating Time Reversal

The fact that, using the TRC, the target signals S are quite well reproduced by the focalisation signals F given by equation (4) (S≈F) is equivalent to the matricial relation

$$H\,{}^tH^* + K\,{}^tK^* \approx I \qquad \text{(6, Stokes relation)}$$

Multiplying on the left by H^{-1} it leads to

$${}^tH^* + H^{-1}\,K\,{}^tK^* \approx H^{-1} \qquad (7)$$

relation that can be physically understood as follows: the signals E_{IF} to be shot from array 1 to focus on array 2 are $E_{IF}=H^{-1}S=E_{TR}+E'$, where $E_{TR}={}^tH^*S$ are the signals to be shot from array 1 to focus on S by TRM, and $E'= H^{-1}\,K\,{}^tK^*S$ are complementary signals that try to shoot from array 1 what should have been shot from array 2 ($K\,{}^tK^*S$) if we were using the full TRC to focus properly on S, thus the presence of H^{-1} to adapt these signals to the shooting array.

Equation (7), that gives H^{-1} as a function of H^{-1}, leads to the idea of using a matrix sequence (M_n) to compute H^{-1} without performing inversion:

$$M_0=0;\ M_{n+1}= {}^tH^* + M_n\,K\,{}^tK^* \qquad (9)$$

It can be demonstrated that this sequence converges rapidly to H^{-1}, the eigenvalues of $K\,{}^tK^*$ being all less than one, as guaranteed by the Stokes relation (7). The convergence can be visualised at looking at the eigenvalues of HM_n, that should converge to identity (see fig.5). The first eigenvalues converge to 1: the physically meaning part of H is inverted.

In practice, it would be interesting to be invert H only knowing information reflection, ie K: not having to measure H would give the possibility of performing *non-invasive* IF (adaptive) focusing. However, H intervenes in equation (7). Hence the idea to remplace, using equation (7), ${}^tH^*$ by ${}^tH^*= M_n - M_{n-1}\,K\,{}^tK^*$. (7) then becomes

$$M_{n+1}= M_n - M_{n-1}\,K\,{}^tK^* + M_n\,K\,{}^tK^* \qquad (8)$$

It can be demonstrated that the limit of this sequence is $M_0+(M_1-M_0)(H\,{}^tH^*)^{-1}$, that is to say H^{-1} if we choose $M_0=0$ and $M_1={}^tH^*$:we still have to know H to be able to compute H^{-1}. Convergence is illustrated on fig.5.

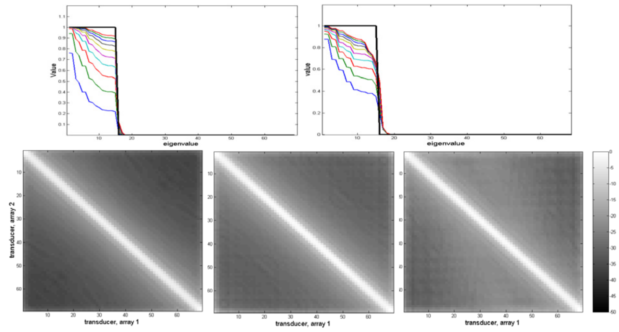

Figure 5: Top: progression of the eigenvalues of HM_n, for simple iteration method (left) and double iteration method (right). The thin, colored curves correspond to the value of the eigenvalues, for iteration orders from 1 to 10. They converge towards the black, thick line, showing the values of the eigenvaules of the HH^{-1} of the IF. Bottom: The images of HM_{10}, calculated by simple iteration method (left), HH^{-1} of the IF (center), and HM_{10}, and double iteration method (right).

Conclusions and perspectives

The limits of the time-reversal mirror to perform good temporal focalisation through a Fabry-Perot, due to the loss of certain frequencies, have been presented and explained. Those frequencies can be recovered using a time-reversal cavity, or with an inverse filter technique. From a matricial mathematical description of the time-reversal cavity process, the inverse filter strategy to try to imitate the time-reversal cavity has been enlightened, showing how the time reversal and the inverse filter are linked by the Stokes relations. Two sequences have been proposed to compute the inverse of a propagator by an iterative method, equivalent to iterating time-reversal [3].

References

[1] Tanter, Thomas and Fink "Time reversal and the inverse filter" *J. Acoust. Soc. Am* 108(1), pp. 223-234 (2000).

[2] Tanter, Aubry, Gerber, Thomas and Fink "Optimal focusing by spatio temporal inverse filter. Part I : Basic Principles", and "idem, part II: experiments. *J. Acoust. Soc. Am* 110(1), pp. 37-47 (2001).

[3] Montaldo, Tanter, and Fink "Real time inverse filter through iterative time reversal" *J.Acoust. Soc. Am* 115, 768 (2004).