Analytical Simulation of Silicon Condenser Microphones

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Abstract

Capacitive silicon microphones are designed for the reception of sound. For optimum performance perforated backplates are required. Characteristics cannot be explained by the behavior of the diaphragms used. In addition to the membrane, the airgap between the electrodes has to be taken into account. Furthermore dynamic effects as the mechanical force of the airgap and flow losses in the airgap as well as the acoustic holes in the backplate has to be considered. This paper describes an extended analytical model for the calculation of the receiving and transmitting sensitivity, membrane deflection and input impedance of silicon microphones.

Theory of dynamic modeling

Electrostatic models well describe the effect of the bias voltage on the membrane deflection. Existing models [1, 2] used for the calculation of microphones will be adapted. The differential equation for the membrane deflection is solved numerically in the dynamic case employing the finite difference method to get insight into the form of the membrane vibration. For a complete model of condenser microphone it is necessary to include the airgap between membrane and backplate, the acoustic holes and the backvolume into the model. The theory of the transducer model and the employed electromechanical equivalent circuit is described mainely in [3].

If a deflection in the form $s(x, y)$ is assumed, the differential equation for the membrane behavior at homogeneous static pressure P with membrane thickness d_m and the tensile stress σ is [4]:

$$\frac{\partial^2 s(x,y)}{\partial x^2} + \frac{\partial^2 s(x,y)}{\partial y^2} = \frac{P}{\sigma d_m} \quad (1)$$

In the case of dynamic modeling the equation must be extended for the description of the dynamic membrane behavior by a mass term with ρ_m as membrane density, $p(t)$ as time variable pressure [4].

$$\frac{\partial^2 s(x,y,t)}{\partial x^2} + \frac{\partial^2 s(x,y,t)}{\partial y^2} = \frac{p(t)}{\sigma d_m} + \frac{\rho_m}{\sigma}\frac{\partial^2 s(x,y,t)}{\partial t^2} \quad (2)$$

All time-dependent parameters, e.g. velocity, deflection or pressure are assumed to be harmonic. All parameters are represented by the product from complex indicators describing the place and frequency dependence. By using the complex indicators the differential equation for the description of the dynamic membrane can be written in the following form:

$$\frac{\partial^2 S(x,y,\omega)}{\partial x^2} + \frac{\partial^2 S(x,y,\omega)}{\partial y^2} = \frac{P}{\sigma d_m} + \frac{\rho_m}{\sigma}\omega^2 S(x,y,\omega) \quad (3)$$

This differential equation permits the calculation of the deflection $S(x, y, \omega)$ of a square membrane. The equation still has no damping term, which has to be taken into account in order to avoid infinite deflections at resonance. If the acoustic impedance of air R_a, is considered the differential equation becomes:

$$\sigma d_m \left[\frac{\partial^2 S(x,y,\omega)}{\partial x^2} + \frac{\partial^2 S(x,y,\omega)}{\partial y^2}\right] = P + \left(j\omega R_a - \omega^2 \rho_m d_m\right)\cdot S(x,y,\omega) \quad (4)$$

The solution of the differential equation is the behaviorial description of the membrane. Using electromechanical analogies a simplified analytical model was derived. This permits the integration of electrical and mechanical impedances into the same equivalent electrical circuit. Figure 1 shows a comparison of the differential equation an the simplified model.

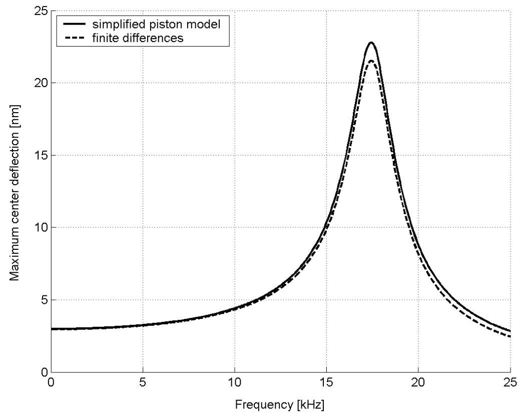

Figure 1: Comparison of differential equation and simplified piston model for membrane description (gold membrane with edge length 1.8 mm, tensile stress 30 MPa)

After simplification of the membrane description it is possible to add the dynamic effects of the microphone. The following dynamic effects are included in the complete analytical model:

- ▷ membrane: acoustic impedance of air above membrane R_a, effective membrane mass M_m, compliance of membrane C_m
- ▷ airgap: compliance of air C_g, mechanical impedance for flow towards acoustic holes R_g, mass of air inside airgap M_g
- ▷ acoustic holes: impedance for all acoustic holes R_h, inertial force for the mass of air M_h
- ▷ backvolume: compliance of air inside backchamber C_v

Figure 2 shows the complete electronic circuit with all impedances.

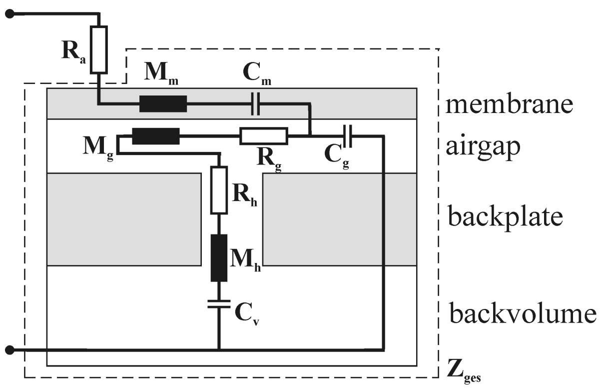

Figure 2: Transducer model with mechanical impedances for the membrane, the airgap, the acoustic holes and the backvolume

Equivalent circuit

With the capacity C_0, bias voltage U_0, airgap height d_0, membrane thickness d_m and the dielectric constant of membrane material ε_m the transformation factor between mechanical force and electrical voltage is given by

$$\Phi = \frac{C_0 U_0}{d_0 + d_m/\varepsilon_m} \tag{5}$$

With this transformation factor it is possible to build an electromechanical equivalent circuit as in Fig. 3 which additionally accounts a parasitic capacitance C_P on the electrical side. With the acoustic impedance R_a, membrane

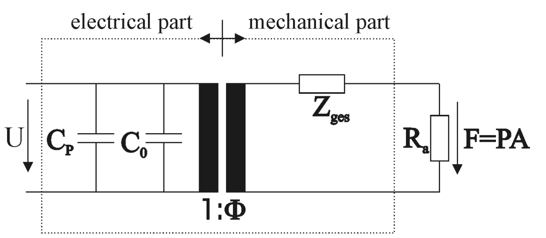

Figure 3: Electromechanical equivalent circuit [3]

area A, membrane edge length a and complete transducer impedance Z_{ges}, the circuit allows after a transformation with the transformation factor Φ the calculation of the transmitting sensitivity P_{out}/U_{in}

$$\frac{P_{out}}{U_{in}} = \frac{\Phi}{A} \cdot \frac{R_a}{Z_{ges} + R_a} \tag{6}$$

receiving sensitivity U_{out}/P_{in}

$$\frac{U_{out}}{P_{in}} = \frac{a^2 \Phi}{\Phi^2 + j\omega(C_0 + C_p) \cdot (R_a + Z_{ges})} \tag{7}$$

membrane deflection S

$$\frac{S}{U} = \frac{\Phi}{j\omega(Z_{ges} + R_a)} \tag{8}$$

and the input impedance Z_{in}:

$$Z_{in} = \frac{Z_{ges} + R_a}{\Phi^2 + j\omega(Z_{ges} + R_a) \cdot (C_0 + C_p)} \tag{9}$$

Silicon microphone

The airgap, the acoustic holes and the backvolume are strongly related to the performance of an silicon condenser microphone. These parameters must be considered during the simulation and the parameters shown in Tab. 1 are used. The relationship between the backvolume and the frequency response is shown in Fig. 4.

Parameter	Value
membrane material	polysilicon
membrane width	1,7 mm
membrane thickness	1 µm
stress of the membrane	10 MPa
airgap height	3 µm
backplate thickness	50 µm
perforation ratio	6,5 %
diameter of acoustic holes	243 µm
pitch of acoustic holes	400 µm
backvolume	1,48 and 2,43 $\cdot$ 10^{-9} m^3
sound velocity in air	330 ms^{-1}
bias voltage	10 V

Table 1: Parameters for the simulation of silicon condenser microphones

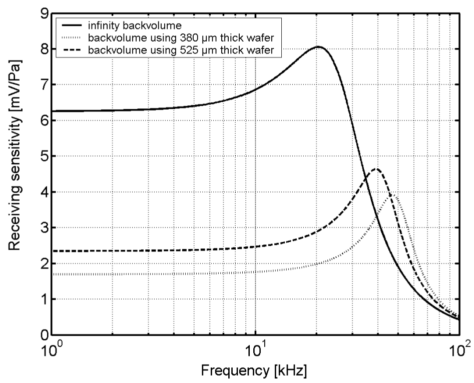

Figure 4: Frequency response depending on the backvolume using different wafer thicknesses with a 1.7 mm x 1.7 mm square membrane.

References

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