Finite-Element Based Design of Structures with Shunted Piezoelectric Patches for Structural Vibration Damping

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Piezoelectric materials, such as lead zirconate titanate (PZT), are frequently used in vibration damping. For that purpose, piezoelectric patches attached to a mechanical structure are used for active control or semi-passive damping of vibrations. However, active control requires high-voltage amplifiers to drive the piezoelectric actuators leading to an expensive implementation. Furthermore, the closed loop may exhibit instability due to spillover effects. On the other hand, the shunted piezo concept is semi-passive and requires only a passive electrical network (PEN), which implies an unconditionally stable closed loop [2] and allows simple implementation. In this contribution, systematic design and analysis methods for structures with shunted piezoelectric patches (see Fig. 1) are presented based on the finite-element method (FEM). These methods are experimentally verified for a steel plate with patches of piezoelectric material, each shunted by a resistor. In contrast to frequently used analytical methods, the presented methods incorporate the stiffening effects of the applied patches.

Finite-Element Model

The finite-element discretized coupled piezoelectric and structural field equations [1] are given in terms of nodal displacements $\boldsymbol{u}$ and electric potentials $\boldsymbol{\phi}$ as

$$\begin{bmatrix} \boldsymbol{M}_{\rm ss} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{u}} \\ \ddot{\boldsymbol{\phi}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{K}_{\rm ss} & \boldsymbol{K}_{\rm s\phi} \\ \boldsymbol{K}_{\rm s\phi}^{\rm T} & \boldsymbol{K}_{\phi\phi} \end{bmatrix} \begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{\phi} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f} \\ \boldsymbol{q} \end{bmatrix}. \tag{1}$$

The right hand side of (1) represents external forces $\boldsymbol{f}$ and electric charges $\boldsymbol{q}$. The mass matrix $\boldsymbol{M}_{\rm ss}$ contains the inertia of the structure. The structural stiffness matrix is given by $\boldsymbol{K}_{\rm ss}$, whereas the matrix $\boldsymbol{K}_{\rm s\phi}$ couples piezoelectric and structural dynamics. After Eq. 1 is partitioned, the inner potential deegrees of freedom (DOF) are condensed and the potentials on the patch electrodes are set equal to $\phi_{\rm e}^{(i)}$. It follows

$$\begin{bmatrix} \boldsymbol{M}_{\rm ss} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{u}} \\ \ddot{\boldsymbol{\phi}}_{\rm e} \end{bmatrix} + \begin{bmatrix} \boldsymbol{H}_{\rm ss} & \boldsymbol{H}_{\rm sp} \\ \boldsymbol{H}_{\rm sp}^{\rm T} & \boldsymbol{H}_{\rm pp} \end{bmatrix} \begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{\phi}_{\rm e} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f} \\ \boldsymbol{q}_{\rm e} \end{bmatrix}. \tag{2}$$

Model Reduction

For fast computational implementation, a model reduction approach based on a transformation into modal space and truncation of high–frequency modes is applied. The eigenvalue problem of the electrical open–circuit case is solved numerically for m mode shapes, and these mode shapes are then arranged in the modal matrix $\boldsymbol{\Phi} \in \mathbb{R}^{n\times m}$, which is also called the reduction base. Introducing the modal coordinates $\boldsymbol{\zeta}$ according to the transformation

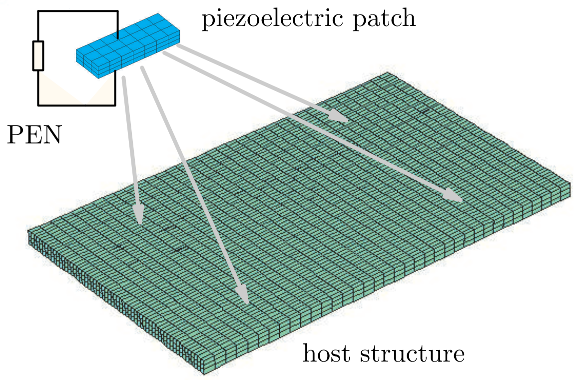

Fig. 1: The piezoelectric patch model is coupled to the structural FE model at multiple locations.

law $\boldsymbol{u} = \boldsymbol{\Phi}\boldsymbol{\zeta}$, the diagonal modal mass and stiffness matrices are obtained from

$$\boldsymbol{\Phi}^{\rm T}\boldsymbol{M}_{\rm ss}\boldsymbol{\Phi} = \boldsymbol{I}, \quad \boldsymbol{\Phi}^{\rm T}\boldsymbol{H}_{\rm ss}\boldsymbol{\Phi} = \boldsymbol{\Lambda} = {\rm diag}(\omega_{\rm r}^2), \tag{3}$$

for mass-normalized modes.

Inclusion of Passive Electrical Networks

The shunted piezo concept is based on passive electrical networks (PENs) that are connected to the piezoelectric patches and dissipate vibrational energy in their resistive elements. For reason of simplicity, only a resistor as PEN at each of the four patches i is considered in the sequel, but more complex PENs are included in a similar way. The time derivative of the charge at patch i is given by

$$\dot{q}_{\rm e}^{(i)} = \frac{\phi_{\rm e}^{(i)}}{R^{(i)}}. \tag{4}$$

The combined dynamics from the second-order structural dynamics (2) and the first-order electrical dynamics (4) without external forces is transformed to state space form

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} \\ -\boldsymbol{\Lambda} & -\boldsymbol{\Delta} & -\boldsymbol{\Phi}^{\rm T}\boldsymbol{H} \\ \boldsymbol{0} & -\boldsymbol{H}_{\rm pp}^{-1}\boldsymbol{H}^{\rm T}\Phi & -\boldsymbol{H}_{\rm pp}^{-1}\boldsymbol{R}^{-1} \end{bmatrix} \boldsymbol{x}, \tag{5}$$

with the state vector $\boldsymbol{x} = \left[\boldsymbol{\zeta}^{\rm T}, \dot{\boldsymbol{q}}^{\rm T}, \boldsymbol{\phi}^{\rm T}\right]^{\rm T}$, the matrix $\boldsymbol{R} = {\rm diag}(R^{(i)})$ and the matrix $\boldsymbol{H} = \left[\boldsymbol{H}_{\rm sp}^{(1)}, \cdots, \boldsymbol{H}_{\rm sp}^{(n)}\right]$, which consists of the column vectors $\boldsymbol{H}_{\rm sp}^{(i)}$ of each patch i. The eigenvalue problem of this system yields complex-conjugate eigenvalue pairs λ_r, λ_r^* and single eigenvalues of the electrical dynamics. The damping ratio $\Delta\delta^r$ of the mode r is given by the real and imaginary part according to $\Delta\delta^r = \Re(\lambda_r) / |\lambda_r|$.

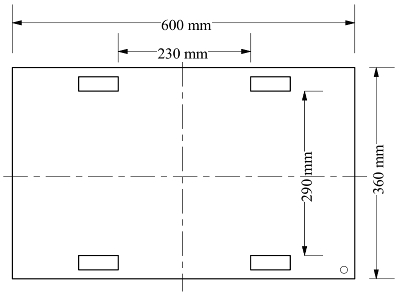

Fig. 2: Geometry of plate with piezoelectric patches.

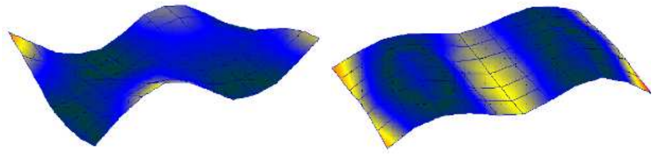

Fig. 3: Two experimentally identified modes: Modes (3,1) and (4,0) with eigenfrequencies 63.1 Hz and 92.2 Hz

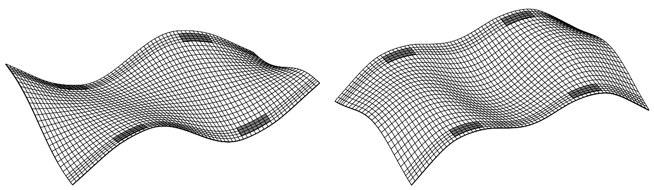

Fig. 4: Two numerically calculated modes: Modes (3,1) and (4,0) with eigenfrequencies 65.5 Hz and 89.4 Hz

Experimental Verification and Results

A plate with four patches (Fig. 2, material PIC151) and PENs with a resistor of $R = 63.5\,\mathrm{k\Omega}$ is investigated. For shunted and open-circuit piezoelectric patch electrodes, an experimental modal analysis (EMA) is performed to identify the mode shapes and the increase in modal damping caused by the shunted patches. Two numerically and experimentally determined mode shapes are presented in Figs. 3 and 4. The comparison in Fig. 5 shows good agreement of the experimental data with the numerical predictions.

The effect of different resistances on the damping of multiple modes can be seen in Fig. 6, which allows appropriate design of the PENs. The advantage of the inclusion of stiffening effects in the combined model is obvious from Fig. 7, where the obtained damping effect is plotted over variations of the patch thickness for adjusted resistance to the patch capacity C_p, i.e. $R \sim \frac{1}{C_\mathrm{p}}$. Due to the stiffening by the attached patches, the damping does not increase linearly with the thickness as indicated by simplified analytical models even if the eigenfrequencies are not altered significantly. The chosen thickness in the experiment setup is 1 mm as indicated by the vertical line.

Conclusion

Compared to analytical approaches, the presented FE-based analysis and design method based on the well-known finite-element method is not limited to simple structures or simple electrical networks. It automatically incorporates all piezoelectric coupling effects, the electrical dynamics and the mechanical stiffening effects. The method also enables further development of methods for sensitivity analysis or optimization of shunted piezoelectric structures.

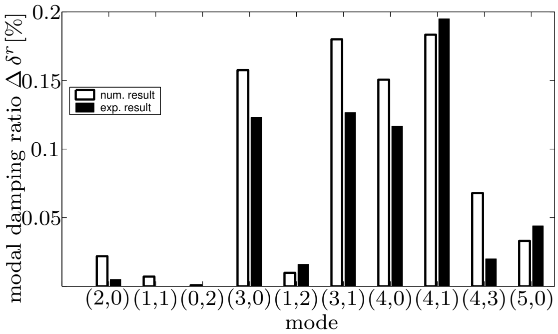

Fig. 5: Increase of modal damping ratios by PEN: Numerical predictions versus experimental results.

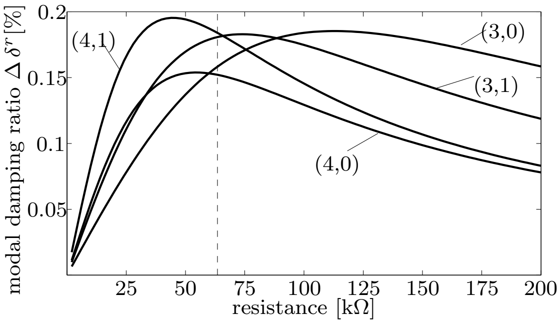

Fig. 6: Modal damping ratios for various resistance values R.

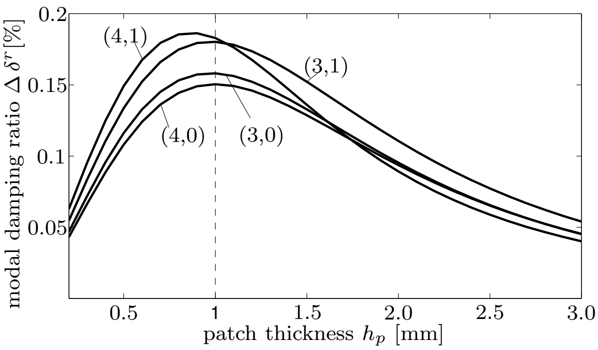

Fig. 7: Modal damping ratios for various patch thickness.

Literature

[1] Allik, H. and Hughes, T. J. Finite element method for piezoelectric vibration. *Int. J. Num. Meth. Eng.*, 2:151–157, 1970.

[2] Hagood, N. and von Flotow, A. Damping of structural vibrations with piezoelectric materials and passive electrical networks. *J. Sound and Vibration*, 197(4):243–268, 1991.