Discretisation of complex sound sources for reproduction with Wave Field Synthesis

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Introduction

In current Wave Field Synthesis implementations only two types of sound sources are available: point sources and plane waves. These ideal source types turn out to be inadequate for a realistic representation of complex sound sources. This paper presents first steps towards an implementation of a sound source of user-defined size, form, and radiation characteristic. It studies the effects of discretisation of the source object with some theoretical considerations and in *Matlab* simulations.

Theory

According to Vogel [1] sources of an arbitrary shape can be treated as a number of point sources. When attempting to auralise arbitrary source shapes, the question then is, how to choose these point sources.

Spatial aliasing due to the discretisation of the secondary sources, i.e. the loudspeaker array, is well researched and documented in the literature (e.g. Verheijen [2]). With an arbitrary shape of the primary source, spatial aliasing can also occur due to spatially sampling the wavefield on the source surface. This sampling is not dependent on physical constraints, as in the case of the WFS reproduction with loudspeakers, where the speaker size gives a constraint. Instead the sampling distance can be chosen freely. However, for a practical implementation it is desirable to keep the calculation power and data size requirements as low as possible, and thus to choose $\Delta\xi$ as large as possible.

If we assume that for a proper reproduction at least the speakers should be driven with an alias-free signal, we can derive the following formula for the maximum sampling distance ($\Delta\xi_{max}$) of a source (in analogy to the spatial aliasing frequency for the loudspeaker distance [2]):

$$\Delta\xi_{max,1} = \frac{c}{2 f_{max} sin\alpha_{max}} \qquad (1)$$

Here c is the velocity of sound, f_{max} is the highest frequency in the source signal, and α_{max} is the largest angle that the vector between source point and a speaker makes with the normal on the source surface (see figure 1a). For the geometry as in figure 1a we can replace $sin\alpha_{max}$ with $\frac{x_{0,max}}{r_{max}}$ and we can see that $\Delta\xi_{max}$ will be larger at greater distances from the array.

Another source for spatial aliasing can be the undersampling of the amplitude variations over the source surface. Here the Rayleigh criterion (e.g. Bleistein [3]) determines the maximum sampling distance:

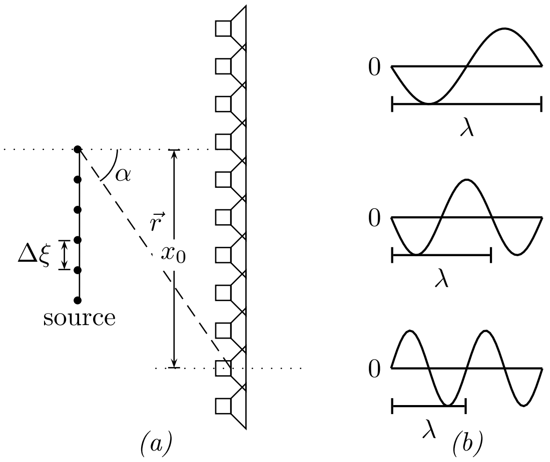

Figure 1: *a)* Geometry for the derivation of the aliasing formula. *b)* Examples of amplitude variation over the length of a line source with an indication of the corresponding λ.

$$\Delta\xi_{max,2} = \frac{\lambda_{min}}{4} = \frac{1}{4k'_{max}} \qquad (2)$$

here λ_{min} is the smallest wavelength of the amplitude variation (see figure 1b) and k_{max} can be seen as its inverse, the spatial frequency of the amplitude.

The maximum sampling distance allowed is then the minimum of equations 1 and 2.

Simulations

To study discretisation effects, some simulations were done with *Matlab*. A line source with a length of $1.0m$, $1.0m$ behind, parallel to and centered to the center of the WFS array was used as the source. The source signal was a gaussian wavelet at two different frequency bandwidths (first with center frequency $750Hz$ and maximum $3kHz$, second with $200Hz$ and $750Hz$ respectively). The source signal was extrapolated with a wave field extrapolation operator based on the Rayleigh I integral. In the simulation experiments 2 receiver distances were used: 1 and 3 meters in front of the array (see figure 2). The reference line was at 3 meters in front of the array. The WFS reproduction array was sampled at $5cm$ intervals, as were the listening positions. The corresponding aliasing frequency for this array is $3.4kHz$; for both source wavelets this is above the highest frequency component.

In figure 3 the effects of aliasing are shown. Already in the WFS reproduction signal, there are aliasing effects visible if $\Delta\xi$ is taken too large (righthand plots: $\Delta\xi = 0.2m$). The aliasing appears as sidelobes in the higher frequency parts (from ca. $1500Hz$). In the receiver area

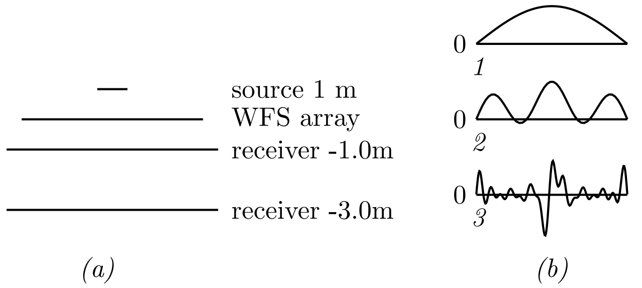

Figure 2: *a)* Geometry of the simulation experiments. *b)* Amplitude variations of the different sources, λ_{min}-values: 2.0, 0.40 and 0.069m.

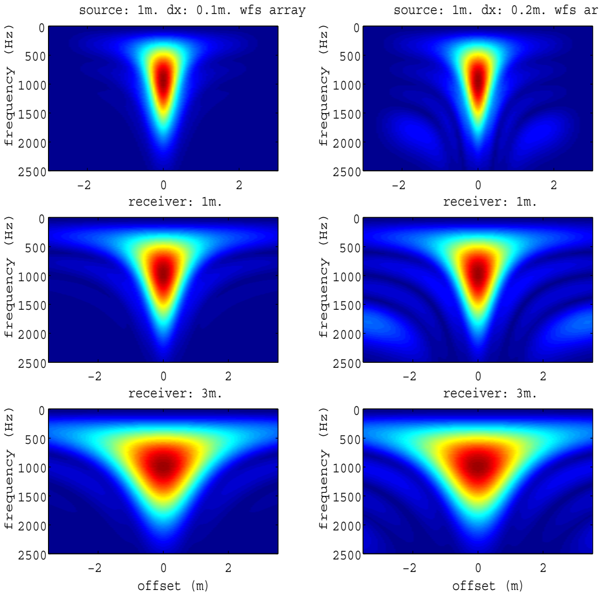

Figure 3: Aliasing effect for a source signal of type 1, with a Gaussian wavelet with center frequency at 750 *Hz* and highest frequency at 3 *kHz*. The top plots are the WFS reproduction signals, the lower plots at receiver distance 1m and 3m. Left plots are with discretisation distance 0.1m, the righthand plots with 0.2m.

this aliasing should be audible mainly towards the sides of the listening area. In the plots on the lefthand side ($\Delta\xi = 0.1m$), these effects are not visible.

In figure 4 the extrapolated field to the listening area is shown at 1m from the array, for a source 1m behind the array with an amplitude distribution of type 2 as indicated in figure 2b. For this source $\Delta\xi_{max,2} = 0.1m$. The reproduction is indeed correct for this value. At 0.2m the frequency spectrum becomes less focused and thus incorrect. In the figure 5 we see similar plots for a source of type 3 ($\Delta\xi_{max,2} = 0.017m$); here the effects are much more clear: the reproduction with $\Delta\xi = 0.01m$ is correct, but at 0.05m the reproduction is not correct anymore, even further away from the WFS array.

Conclusions and future research

Two criteria were given for finding the maximum sampling distance allowed for a proper WFS reproduction of an arbitrary source shape. In simulations the correctness of these criteria was illustrated, as well as the effects of undersampling. These criteria can easily be calculated from the source model itself if the frequency response of the surface is known (or user defined) as a function of position on the surface. The results discussed in this paper will still need to be evaluated by listening experiments.

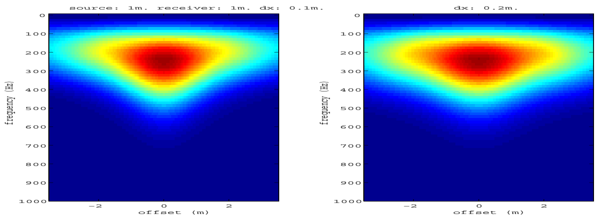

Figure 4: WFS reproduction at receiver line 1 meter in front of the array for source of type 2 (see figure 2b), low frequency wavelet, with $\Delta\xi = 0.1m$ (left) and 0.2m (right).

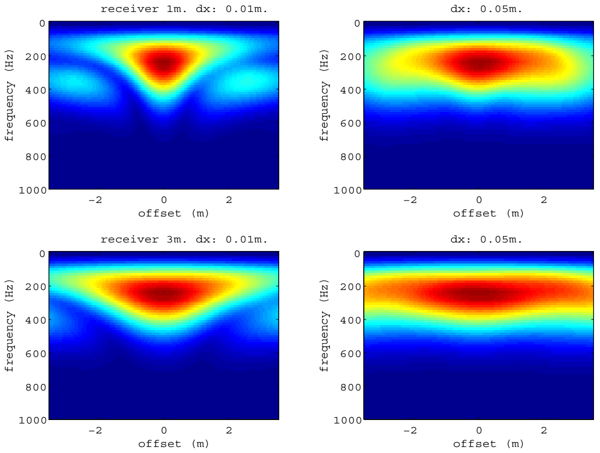

Figure 5: WFS reproduction at receiver line 1 (top) and 3 (bottom) meter in front of the array for a source of type 3 (see figure 2b), low frequency wavelet, with $\Delta\xi = 0.01m$ (left) and 0.05m (right).

Future research will focus on other problems that may occur in the auralisation of arbitrarily shaped source objects, such as attenuation errors due to the $2\frac{1}{2}$D-operator and the problem of how to auralise elevated points with a linear WFS array, which is needed for auralising 3D objects.

References

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[3] N. Bleistein, J.K. Cohen & J.W. Stockwell Jr. *Mathematics of multidimensional seismic imaging, migration, and inversion.* Springer-Verlag, New York, 2001. pp. 114, 115