Calibration of the two microphone transfer function method to measure acoustical impedance in a wide frequency range

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Introduction

In many acoustical simulations, particularly when using models with lumped elements or electrical analog circuits, the acoustical impedance of a component needs to be determined accurately. A widely used acoustical impedance measurement method is the "two microphone transfer function method", which is standardized in ISO-10534-2.

When the acoustical impedance is needed over a wide frequency band, for example from 10Hz to 10kHz, this method shows limitations. In this paper, a new calibration method has been proposed such that acoustical impedances can be measured with high accuracy over a wide frequency range. First, the estimation of the speed of sound has been eliminated. Next, in the calibration of the sensor mismatch, the deviation in sensor position after interchanging the pressure sensors has been taken into account. Finally, a recursive procedure has been proposed to refine the microphone position calibration [1].

Principle of the two microphone transfer function method

In this section, the two microphone transfer function method according to ISO 10534-2 is discussed. Figure 1 presents the set-up for acoustical impedance measurement. It consists of an acoustic wave guide with at the left end an excitation source and at the right end the impedance to be measured. This impedance includes everything present at the right side of the reference section. Two microphones at two distinct positions x_1 and x_2 measure the sound pressure inside the duct. From the transfer function between the two microphones, the reflection coefficient and consequently, the unknown connected impedance will be determined.

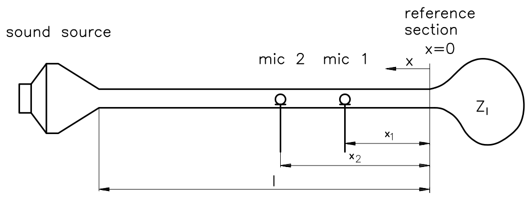

Figure 1: Wave guide closed with unknown acoustical impedance Z_l.

The transfer function T_{12} between the pressures at the positions x_1 and x_2 is:

$$T_{12} = \frac{p(x_1, k)}{p(x_2, k)} = \frac{\mathrm{e}^{j\,k\,x_1} + \Gamma_l\,\mathrm{e}^{-j\,k\,x_1}}{\mathrm{e}^{j\,k\,x_2} + \Gamma_l\,\mathrm{e}^{-j\,k\,x_2}} \quad (1)$$

From the load reflection coefficient Γ_l the load impedance Z_l is then determined:

$$Z_l = Z_0\,\frac{1+\Gamma_l}{1-\Gamma_l} = j\,Z_0\,\frac{\sin\,k\,x_1 - T_{12}\,\sin\,k\,x_2}{\cos\,k\,x_1 - T_{12}\,\cos\,k\,x_2} \quad (2)$$

Calibration of the set-up according to ISO 10534-2

Equation (2) becomes inaccurate when the reflection coefficient Γ_l approaches unity. Therefore, prior calibration of the set-up is necessary to obtain accurate results.

The ISO 10534-2 standard prescribes the following calibration actions:

- The velocity of sound needs to be determined accurately using measurements of ambient temperature and atmospheric pressure.
- The distance between the pressure sensors need to be measured accurately.
- The mismatch between the amplitude and phase of the pressure sensors needs to be calibrated. In short, the procedure is to measure the transfer function T_{12} of the two pressures at position x_1 and x_2, then interchange the two pressure sensors from location x_1 to x_2 and from x_2 to x_1 respectively, measure the transfer function T_{21} and calculate the calibration factor δ such that:

  $$\delta^2\,T_{12}\,T_{21} = 1 + 0\,j \qquad (j = \sqrt{-1}) \quad (3)$$

  This calibration factor δ is complex and frequency dependent.

The quality of the ISO calibration procedure is illustrated using a theoretical analysis wherein realistic data for the set-up are introduced.

Suppose the wave guide has a diameter of 36 mm ($Z_0 = 400\mathrm{k}\Omega$, $1\Omega = 1\mathrm{Pas/m}^3$). The distance between the closest pressure sensor and the reference section is $x_1 = 0.2$ m and between the farest pressure sensor and the reference section $x_2 = 0.5$ m. The impedance measurement range will be limited to 1000 Z_0 (60 dB). It is supposed that the sensors have different dynamics, i.e. the first sensor has an amplification $K_1 = 0.98$, a resonance frequency of $\omega_1 = 2\,\pi\,42000$ /s and a damping coefficient of $\xi_1 = 0.05$, and the second sensor $K_2 = 1.10$, $\omega_2 = 2\,\pi\,36000$ /s and $\xi_2 = 0.02$. When measuring the inverse transfer function, the interchanged pressure sensors cannot be mounted at the exact same locations. Suppose that the new locations deviate 0.2 mm from the original locations, so the new locations will be $X_1 = 0.2002$ m and $X_2 = 0.4998$ m in stead of x_1 and x_2 respectively.

The resulting calibration factor δ times the sensor mismatch after applying the calibration method according to the ISO 10534-2 standard is presented in figure 2left. This product should equal $1 + 0\,j$. In the lower frequency range, the calibration has been successfull. However, in the higher frequency range, starting from 150 Hz, the calibration is erroneous and even worse than the uncalibrated

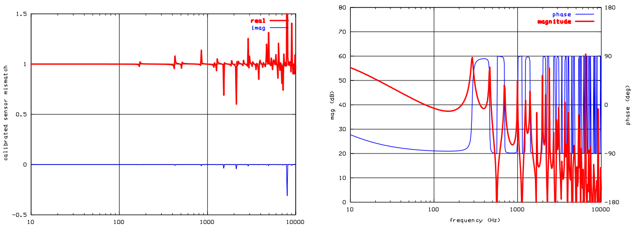

Figure 2: *left:* calibration result of sensor mismatch; *right:* resulting closed end impedance; when applying the ISO 10534-2 standard.

situation. The resulting impedance is presented in figure 2right. This should be a straight line at 60 dB, but due to the erroneous calibration, large deviations results. As result, the measurement range is limited to 40 dB around Z_0 until 400 Hz.

Improved calibration

The speed of sound is eliminated by replacing $k\, x_i$ by $\omega\, t_i$, $(i = 1, 2)$, in expression (1), wherein t_1 and t_2 are the times the wave needs to travel from the positions x_1 and x_2 to the reference section respectively.

These traveling times t_1 and t_2 are measured with the wave guide closed at the reference section. Then, these traveling times t_1 and t_2 are:

$$t_1 = \frac{1}{4\, f_1} \quad \text{and} \quad t_2 = \frac{1}{4\, f_2} \tag{4}$$

wherein the frequencies f_1 and f_2 correspond to the frequencies determined by the quarter wavelength between the reference section and the positions x_1 and x_2 respectively. In the same way, the travelling times T_1 and T_2 after exchanging the microphones are determined.

Once t_1, t_2, T_1 and T_2 are determined, the sensor mismatch can be calibrated. The pressure transfer functions are measured using microphones. So, the transfer functions between microphone output signals $u(x_1)$ and $u(x_2)$ is $T_{u12} = u(x_1)/u(x_2)$ and $T_{u21} = u(x_2)/u(x_1)$, which deviate from the pressure transfer function in amplitude and phase by a factor δ, i.e.

$$\delta^2\, T_{u12}\, T_{u21} = 1 + 0\, j \tag{5}$$

However, the interchanged sensors cannot be mounted exactly at the positons x_1 and x_2 again. Consequenctly, the transfer function $T_{u21} = u(x_2)/u(x_1)$ is not available and therefore it needs to be calculated from the transfer function $T_{U21} = u(X_2)/u(X_1)$, which is measured at the positions X_1 and X_2. After correcting the tranfer function T_{U21} and determining the correction factor δ, the unknown impedance is determined using:

$$Z_l = j\, Z_0\, \frac{\sin\, \omega\, t_1 - T_{u12}\, \delta\, \sin\, \omega\, t_2}{\cos\, \omega\, t_1 - T_{u12}\, \delta\, \cos\, \omega\, t_2} \tag{6}$$

Mostly, the travelling times t_1, t_2, T_1 and T_2 are still not sufficiently accurate when determined from the equations (4). The impedance is extremely sensitive to errors in the travelling times. Therefore, the acoustical impedance of the closed duct itself will be used to refine these travelling times.

Figure 3 present the situations wherein respectively the travelling times t_1 and t_2 will be corrected. When a pressure maximum occurs at one of the microphone positions, the gradient of the pressure is zero and a small deviation of the microphone position does not affect the accuracy of the result. The other microphone position, where the gradient is generally not zero, determines fully the observed impedance. So, the deviation of the microphone position can be accurately determined.

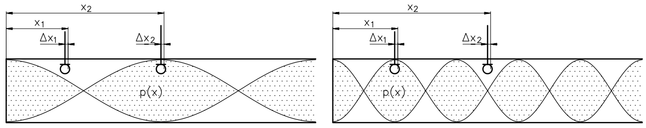

Figure 3: Refining situations for x_1 and x_2.

For example, the correction of the travelling time t_1 will be determined when a half wave length stands at position x_2 (see figure 3left). The frequency where this situation occurs is:

$$f_2 = \frac{n}{2\, t_2} \quad n = 1, 2, 3, \ldots \tag{7}$$

As a result, a load impedance appears at the reference section which is composed of the hard wall impedance and a short piece wave guide with length Δx. (Δx_1 at frequency f_2 and Δx_2 at frequency f_1) Determining the travelling time τ_1 in this short piece wave guide results in:

$$\tau_1 = \frac{t_2}{\pi}\, \arctan\, \frac{Z_0}{j\, Z_l} \tag{8}$$

which is the correction on the travelling time t_1. In the same way, the correction on the travelling time t_2 will be:

$$\tau_2 = \frac{t_1}{\pi}\, \arctan\, \frac{Z_0}{j\, Z_l} \tag{9}$$

This correction of the travelling times can be repeated until the required accuracy has been obtained. In the same way, the travelling times T_1 and T_2 will be corrected.

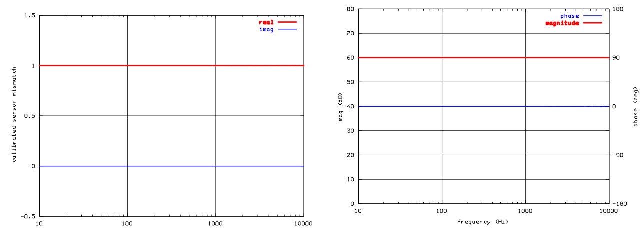

Figure 4: *left:* calibration result of sensor mismatch; *right:* resulting closed end impedance; after improved calibration.

Figure 4 presents the resulting sensor mismatch calibration and the obtianed impedance. Now, the impance measurement range is expanded to 10 kHz.

Conclusion

This theoretical investigation about the calibration of the two microphone transfer function method has demonstrated that a small distance variation between the microphone position has a large effect on the impedance accuracy. A new improved calibrition method has been proposed to calibrate the sensor position deviation. As result, the acoustical impedance can be measured accurately in a wide frequency range.

References

[1] Boonen R., Sas P., "Calibration of the two microphone transfer function method to measure acoustical impedance in a wide frequency range", proc. of the ISMA2004 Conference, (2004), Leuven, Belgium, pp. 325-336