On Noise Production by a Limited Region of Turbulence in a Circular Rigid-Walled Pipe

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Formulation of the problem

A low Mach number flow of mean velocity U is considered in an infinite rigid pipe of a circular cross-section of radius a. A finite fluid volume V_0 is in the turbulent state, and produces noise. This noise must be found, and the relationships between its characteristics and the parameters of pipe and flow established.

The noise field is governed by the Lighthill's equation, in which the right part contains the volume quadrupoles, $\partial^2 T_{ij} / \partial y_i \partial y_j$, and surface dipoles, $\partial F_i / \partial y_i$, viz. [1]

$$\frac{\partial^2 \rho_a}{\partial t^2} - c_0^2 \nabla^2 \rho_a = \frac{\partial^2 T_{ij}}{\partial y_i \partial y_j} + \frac{\partial F_i}{\partial y_i} . \qquad (1)$$

The boundary conditions are that the radial velocity $\partial p_a / \partial r$ vanishes at the pipe wall, and all waves are outgoing at infinity. Here ρ_a and p_a are the acoustic density and pressure, which are related as $p_a = c_0^2 \rho_a$, T_{ij} and F_i the Lighthill's stresses and the *i*-th force component acting on the unit area of the pipe wall (T_{ij} and F_i vanish outside volume V_0 and bounding surface S_0), and a summation on repeated indices is assumed.

Acoustic density and pressure

The problem formulated is solved via the use of a Green's function technique and the normal mode method [1]. A final expression for the acoustic density fluctuations in the pipe is

$$\rho_a(\vec{r},t) = \int_{-\infty}^{\infty} dt_0 \iiint_{V_0} \frac{\partial^2 T_{ij}(\vec{r}_0,t_0)}{\partial y_i \partial y_j} G(\vec{r},t;\vec{r}_0,t_0) dV_0 +$$

$$+ \int_{-\infty}^{\infty} dt_0 \iint_{S_0} \frac{\partial F_i(\vec{r}_{oa},t_0)}{\partial y_i} G(\vec{r},t;\vec{r}_{0a},t_0) dS_0(\vec{r}_{0a}), \qquad (2)$$

$$dV_0(\vec{r}_0) = r_0 dr_0 d\phi_0 dz_0 , \; dS_0(\vec{r}_{0a}) = a d\phi_0 dz_0 ,$$

in which G is a Green's function given by the following series in the pipe acoustic modes $\psi_{nm}(r,\phi) = J_n(\alpha_{nm}r)\cos(n\phi)$

$$G(\vec{r},t;\vec{r}_0,t_0) = -\frac{i}{4\pi c_0^2} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{\psi_{nm}(r_0,\phi_0)}{\|\psi_{nm}\|^2} \times$$

$$\times \psi_{nm}(r,\phi) \int_{-\infty}^{\infty} \frac{e^{ik_{nm}|z-z_0|}}{k_{nm}} e^{-i\omega(t-t_0)} d\omega, \qquad (3)$$

J_n cylindrical Bessel function of n-th order, $\alpha_{nm} = \zeta_{nm}/a$ and $k_{nm} = \sqrt{k_0^2 - \alpha_{nm}^2}$ the radial and axial wavenumbers, respectively, ζ_{nm} the tabular roots of equation $J_n^/(\zeta_{nm}) = 0$, and

$$\|\psi_{nm}\|^2 = \begin{cases} \pi a^2 J_0^2(\alpha_{0m}a), n = 0 \\ \\ (\pi a^2/2) J_n^2(\alpha_{nm}a)\left[1 - (n^2/\alpha_{nm}^2 a^2)\right], n \geq 1 \end{cases}$$

The acoustic pressure is then given by formula (2) multiplied by c_0^2.

Expression (2) establishs the quantitative relation between the volume quadrupoles and surface dipoles and the acoustic density in the pipe. In addition, it also contains information about the pipe geometry and that of domains in which the noise sources are distributed.

Acoustic power

A general expression for the acoustic power $P(\omega)$ generated in case of non-uniform noise source distribution is [1]

$$P(\omega) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} P_{nm}(\omega) =$$

$$= \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{1}{4\|\psi_{nm}\|^2 k_{nm} \rho_0 \omega} \left[\iiint_{V_0} dV_0(\vec{r}_0) \times \right.$$

$$\times \iiint_{V_0} \frac{\partial^4 S_{ijkl}^T(\vec{r}_0,\vec{r}_0^/,\omega)}{\partial y_i \partial y_j \partial y_k^/ \partial y_l^/} \psi_{nm}(r_0,\phi_0)\psi_{nm}(r_0^/,\phi_0^/) \times$$

$$\times e^{-sign(z-z_0)ik_{nm}(z_0^/-z_0)} dV_0(\vec{r}_0^/) + \iint_{S_0} dS_0(\vec{r}_{0a})$$

$$\times \iint_{S_0} \frac{\partial^2 S_{ik}^F(\vec{r}_{oa},\vec{r}_{oa}^/,\omega)}{\partial y_i \partial y_k^/} \psi_{nm}(a,\phi_0)\psi_{nm}(a,\phi_0^/)$$

$$\times e^{-sign(z-z_0)ik_{nm}(z_0^/-z_0)} dS_0(\vec{r}_{0a}^/) + \qquad (4)$$

$$2\text{Re}\left(\iiint_{V_0} dV_0(\vec{r}_0) \iint_{S_0} \frac{\partial^3 S_{ijk}^{TF}(\vec{r}_0,\vec{r}_{0a}^/,\omega)}{\partial y_i \partial y_j \partial y_k^/} \iint_{S_0} \psi_{nm}(r_0,\phi_0) \right.$$

$$\left. \times \psi_{nm}(a,\phi_0^/)e^{-sign(z-z_0)ik_{nm}(z_0^/-z_0)} dS_0(\vec{r}_{0a}^/) \right],$$

where ρ_0 is the undisturbed fluid density; $sign(z-z_0)$ the sign function, S_{ijkl}^T and S_{ik}^F the cross-spectra of the Fourier images of the Lighthill's stresses, viz.

$$S_{ijkl}^{T}\delta(\omega-\omega^{/})=<\breve{T}_{ij}^{*}(\vec{r}_0,\omega)\breve{T}_{kl}(\vec{r}_0^{/},\omega^{/})>$$

and the force components, viz.

$$S_{ik}^{F}\delta(\omega-\omega^{/})=<\breve{F}_{i}^{*}(\vec{r}_{0a},\omega)\breve{F}_{k}(\vec{r}_{0a}^{/},\omega^{/})>$$

S_{ijk}^{TF} the cross-spectrum of the Fourier images of the Lighthill's stresses and the force components, viz.

$$S_{ijk}^{TF}\delta(\omega-\omega^{/})=<\breve{T}_{ij}^{*}(\vec{r}_0,\omega)\breve{F}_{k}(\vec{r}_{0a}^{/},\omega^{/})>$$

Re(...) denotes a real part of the quantity indicated in the parenthesis.

When the noise sources are distributed uniformly in their domains, formula (4) is simplified due to simplification of expressions for the spectra S_{ijkl}^{T}, S_{ik}^{F} and S_{ijk}^{TF}, which become the functions of the source separation distance (i.e., $\vec{\xi}=\vec{r}_0^{/}-\vec{r}_0$, $\vec{\xi}_{aa}=\vec{r}_{oa}^{/}-\vec{r}_{oa}$ and $\vec{\xi}_a=\vec{r}_{0a}^{/}-\vec{r}_0$, respectively) and the frequency only.

The analysis of expression (4) shows that the acoustic power, P, is independent of the axial coordinate, z, and hence, P does not decrease as the distance from the sources increases (as is to be expected in a hard-walled pipe where energy is conserved). In addition, P is a sum of powers of the pipe acoustic modes, P_{nm}, the individual mode contribution, P_{nm}, consisting of the three parts. The first part is the sound power generated by the volume quadrupoles, the second part comes from the surface dipoles, and the third part is due to interaction of quadrupoles and dipoles.

Further analysis of formula (4) shows that the relative contribution of each part depends on Mach number. When Mach number falls in the range where dipoles dominate, expression (4) is simplified significantly, viz.

$$P(\omega)=\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{1}{4\|\psi_{nm}\|^2 k_{nm}\rho_0\omega}\iint_{S_0}dS_0(\vec{r}_{0a})\times$$

$$\times\iint_{S_0}\frac{\partial^2 S_{ik}^{F}(\vec{r}_{oa},\vec{r}_{oa}^{/},\omega)}{\partial y_i\partial y_k^{/}}\psi_{nm}(a,\phi_0)\psi_{nm}(a,\phi_0^{/})\times$$

$$\times e^{-sign(z-z_0)ik_{nm}(z_0^{/}-z_0)}dS_0(\vec{r}_{0a}^{/}).$$

When Mach number is such that the noise field is dominated by the contribution from quadrupoles, we have

$$P(\omega)=\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{1}{4\|\psi_{nm}\|^2 k_{nm}\rho_0\omega}\iiint_{V_0}dV_0(\vec{r}_0)\times$$

$$\times\iiint_{V_0}\frac{\partial^4 S_{ijkl}^{T}(\vec{r}_0,\vec{r}_0^{/},\omega)}{\partial y_i\partial y_j\partial y_k^{/}\partial y_l^{/}}\psi_{nm}(r_0,\phi_0)\psi_{nm}(r_0^{/},\phi_0^{/})\times$$

$$\times e^{-sign(z-z_0)ik_{nm}(z_0^{/}-z_0)}dV_0(\vec{r}_0^{/}).$$

When the contributions of dipoles and quadrupoles are of the same order of magnitude, the acoustic power is determined by formula (4).

General comments

Expressions (2)-(4) are the general solution to the problem of noise generation by a limited region of turbulence in an infinite straight rigid-walled pipe of a circular cross-section. They establish the quantitative relations between the noise field characteristics and the parameters of pipe and flow. In using these formulas, data for the noise sources and/or their statistical characteristics can be taken either from the scientific literature or experiment, or from a solution of the appropriate fluid dynamics problem.

Conclusions

1. A general theory of noise generation by a limited region of turbulence in an infinite straight rigid-walled pipe of a circular cross-section has been developed. In this theory, turbulence is modelled by the quadrupole and dipole sources, and the cases of uniform and non-uniform source distribution are considered.

2. The noise power is a sum of powers of the pipe acoustic modes. The individual mode power consists of the three parts. The first part is the power generated by quadrupoles, the second part comes from dipoles, and the third part is due to interaction of quadrupoles and dipoles.

Acknowledgement

References

[1] Borisyuk A.O. Sound generation by a limited region of disturbed flow in a rigid channel of a circular cross-section. Part 1. A general theory. Acoustic Bulletin **6**(3) (2003), 3-9 (in Ukrainian).