Finite Element Methods for the Analysis of Smart Vibro-acoustic Systems

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Introduction

Over the last few years a lot of scientific work has been done in the field of piezoelectric smart structures. The main objective of the smart structures concept is to reduce the structural vibrations as well as the radiated sound. Considering thin-walled lightweight structures of large surface, thin piezoceramic wafers are used as ditributed actuators and sensors. Active noise reduction techniques are of great interest in the low frequency range where passive methods are less effective. The engineering design process of smart structures requires powerfull numerical analysis tools to investigate the performance of the developed system. Regarding vibro-acoustic systems the numerical model has to include the main functional parts as the passive structure, the piezoelectric sensors and actuators, the acoustic fluid and the control algorithm. Such a simulation tool based on the finite element method (FEM) is presented here. The smart structure is modelled by using active layered shell elements [3] which are coupled with 3D hexahedron elements to discretize the acostic fluid. Modal truncation is used as an appropriate model reduction technique for controller design purposes [6]. As a test example the performance of an active acoustic box is shown.

Basic Equations and Finite Element Modelling

The derivation of the finite element formulation for the piezoelectric smart structure is based on the mechanical equilibrium [2] and the linear coupled electromechanical constitutive equations

$$\boldsymbol{\sigma} = \mathbf{C}^{(E)}\boldsymbol{\varepsilon} - \mathbf{e}^T\mathbf{E}, \tag{1}$$

$$\mathbf{D} = \mathbf{e}\boldsymbol{\varepsilon} - \boldsymbol{\kappa}^{(\varepsilon)}\mathbf{E}, \tag{2}$$

with the stress vector $\boldsymbol{\sigma}$, the vector of electric displacements $\mathbf{D}$, the elasticity matrix $\mathbf{C}^{(E)}$, the piezoelectric matrix $\mathbf{e}$, the dielectric matrix $\boldsymbol{\kappa}^{(\varepsilon)}$, the strain vector $\boldsymbol{\varepsilon}$ and the electric field vector $\mathbf{E}$. For the modeling of the homogeneous and inviscid fluid the linear wave equation

$$\frac{1}{c^2}\ddot{\Phi} - \nabla \cdot \nabla^T \Phi = 0 \tag{3}$$

is considered. Apart from the speed of sound c in equation (3) the velocity potential Φ is introduced as an unknown quantity. The velocity potential is related to the fluid velocity $\mathbf{v}$ and the sound pressure p by

$$\mathbf{v} = -\nabla^T \Phi \tag{4}$$

and

$$p = \rho_0 \dot{\Phi}, \tag{5}$$

where ρ_0 denotes the fluid density. To get a weak form of the electromechanical problem we start from the principle of virtual work extended by the electrical part. Regarding the acoustic fluid it is possible to formulate a principle of virtual velocity potentials [7]. Following a standard finite element procedure the semi-discrete equations of motion of the coupled multi field problem are received as [4]

$$\begin{bmatrix} \mathbf{M}_{uu} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\rho_0\mathbf{M}_a \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}} \\ \ddot{\boldsymbol{\varphi}} \\ \ddot{\boldsymbol{\Phi}} \end{bmatrix} + \begin{bmatrix} \mathbf{C}_{uu} & \mathbf{0} & -\mathbf{C}_{uc} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\mathbf{C}_{uc}^T & \mathbf{0} & -\rho_0\mathbf{C}_a \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}} \\ \dot{\boldsymbol{\varphi}} \\ \dot{\boldsymbol{\Phi}} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{u\varphi} & \mathbf{0} \\ \mathbf{K}_{u\varphi}^T & -\mathbf{K}_{\varphi\varphi} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\rho_0\mathbf{K}_a \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \boldsymbol{\varphi} \\ \boldsymbol{\Phi} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_u \\ \mathbf{f}_\varphi \\ \mathbf{f}_a \end{bmatrix} \tag{6}$$

or

$$\tilde{\mathbf{M}}\ddot{\mathbf{r}} + \tilde{\mathbf{C}}\dot{\mathbf{r}} + \tilde{\mathbf{K}}\mathbf{r} = \tilde{\mathbf{f}}. \tag{7}$$

In equation (6) the vectors $\mathbf{u}$, $\boldsymbol{\varphi}$ and $\boldsymbol{\Phi}$ contain the nodal values of the displacement, the electric potentials and of the velocity potential. For the calculation of the vibro-acousic coupling matrix $\mathbf{C}_{uc}$ it is considered, that the sound pressure p represents an additional surface load taking effect on the structure while the acoustic fluid is excited by the normal surface velocity $\dot{u}_n$ at the same time. It is remarked that the use of the velocity potential as a nodal degree of freedom ensures the symmetry of the resulting system matrices. In this way effective numerical algorithms can be employed to solve the problem. For the modelling of active vibro-acoustic systems different types of finite elements were implemented in the software package COSAR [1]. Particularly, Semiloof-type shell elements for the discretization of laminates with piezoelectric layers were coupled to acoustic hexahedron elements.

Model Reduction

The derived finite element model contains a large number of degrees of freedom. This large number is infeasible for controller design purposes. A suitable model reduction is performed by using the modal truncation technique. This procedure is known from structural vibration analysis and can be applied to vibro-acoustic systems in the low frequency range. The behaviour of the system is described by a few pre-selected eigenmodes. At this, two different strategies are distinguished. First, the two decoupled eigenvalue problems of the structure and of the acoustic fluid

$$\left(\begin{bmatrix}\mathbf{K}_{uu} & \mathbf{K}_{u\varphi}\\ \mathbf{K}_{u\varphi}^T & -\mathbf{K}_{\varphi\varphi}\end{bmatrix} - \omega_{ui}^2\begin{bmatrix}\mathbf{M}_{uu} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\right)\hat{\mathbf{q}}_{ui} = \mathbf{0}, \tag{8}$$

$$\left(\mathbf{K}_a - \omega_{ai}^2\mathbf{M}_a\right)\hat{\mathbf{q}}_{ai} = \mathbf{0}, \tag{9}$$

are solved separately. Based on the received eigenmodes a modal coupled numerical model can be formulated. This truncated model is used to design the controller and to study the time depending behaviour of the coupled system [5]. In case of a stronger vibro-acoustic coupling behaviour it is required to solve the complex eigenvalue problem of equation (7)

$$\left(\tilde{\mathbf{A}} - \lambda_i\tilde{\mathbf{B}}\right)\hat{\mathbf{q}}_i = \mathbf{0}, \tag{10}$$

which is derived from

$$\begin{bmatrix}\tilde{\mathbf{C}} & \tilde{\mathbf{M}}\\ \tilde{\mathbf{M}} & \mathbf{0}\end{bmatrix}\dot{\mathbf{z}} + \begin{bmatrix}\tilde{\mathbf{K}} & \mathbf{0}\\ \mathbf{0} & -\tilde{\mathbf{M}}\end{bmatrix}\mathbf{z} = \tilde{\mathbf{B}}\dot{\mathbf{z}} + \tilde{\mathbf{A}}\mathbf{z} = \begin{bmatrix}\tilde{\mathbf{f}}\\ \mathbf{0}\end{bmatrix} \tag{11}$$

with

$$\mathbf{z} = \begin{bmatrix}\mathbf{r} & \dot{\mathbf{r}}\end{bmatrix}^T, \tag{12}$$

in order to increase the accuracy.

Controller Design

The k pairs of conjugate complex eigenvectors constitute the modal matrix $\mathbf{Q}$. If $\mathbf{Q}$ is ortho-normalized with

$$\mathbf{Q}^T\tilde{\mathbf{B}}\mathbf{Q} = \mathbf{I} = \operatorname{diag}(1), \quad \mathbf{Q}^T\tilde{\mathbf{A}}\mathbf{Q} = \mathbf{\Lambda} = \operatorname{diag}(\lambda_i), \tag{13}$$

equation (6) can be reduced to

$$\dot{\mathbf{q}} + \mathbf{\Lambda}\mathbf{q} = \mathbf{Q}^T\begin{bmatrix}\tilde{\mathbf{f}}\\ \mathbf{0}\end{bmatrix} \tag{14}$$

by introducing modal coordinates $\mathbf{z} = \mathbf{Q}\mathbf{q}$. For controller design equation (14) is transformed into the state space form

$$\dot{\mathbf{q}} = -\mathbf{\Lambda}\mathbf{q} + \mathbf{Q}^T\begin{bmatrix}\bar{\mathbf{B}}\\ \mathbf{0}\end{bmatrix}\mathbf{u}_c + \mathbf{Q}^T\begin{bmatrix}\bar{\mathbf{E}}\\ \mathbf{0}\end{bmatrix}\mathbf{f} = \mathbf{A}\mathbf{q} + \mathbf{B}\mathbf{u}_c + \mathbf{E}\mathbf{f} \tag{15}$$

and extended by a measurement equation

$$\mathbf{y} = \mathbf{C}\mathbf{q} + \mathbf{D}\mathbf{u}_c + \mathbf{F}\mathbf{f}, \tag{16}$$

where $\mathbf{u}_c$ represents the controller influence. The matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{E}$, $\mathbf{C}$, $\mathbf{D}$, $\mathbf{F}$ are transferred to Matlab/Simulink via a bi-directional data interface. Using the Matlab/Simulink software a time independent LQ-controller is designed, which is characterized by the controller matrix $\mathbf{R}$. The controller influence in equation (15) is calculated from

$$\mathbf{u}_c = -\mathbf{R}\mathbf{q}. \tag{17}$$

Example

As a test example the numerical model of an active acoustic box was investigated. The model consists of a simply supported elastic plate attached with two collocated piezoelectric sensor/actuator pairs and coupled with an acoustic cavity. For the controller design and the following analysis in the time domain the first five pairs of conjugate complex eigenmodes were taken into account (modes up to 175.5 Hz). The plate was excited by a harmonic force

$$F(t) = \frac{1}{2}\left\{\sin(2\pi f_1 t) + \sin(2\pi f_4 t)\right\}. \tag{18}$$

Figure 1 shows the time regime of the sound pressure in the middle of the cavity when the controller was switched on after 1.5 s. For time integration the Newmark formulas were used.

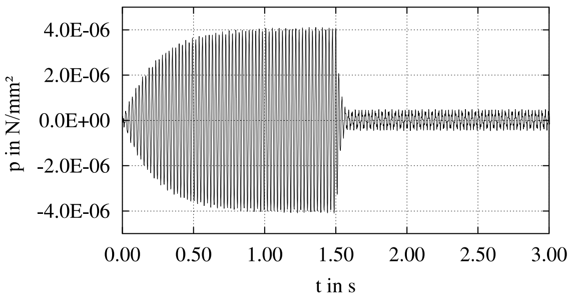

Figure 1: Resulting sound pressure.

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