Sound Source Localization in Cabins by Inverse Finite Elemente Analysis

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INTRODUCTION

New regulations and higher customer demands require quieter vehicles. To achieve this goal various measurement techniques for sound source localization have been developed, reaching from simple sound pressure level measurements to highly advanced inverse methods as Spatial Transformation of Sound Fields (STSF), Beamforming or the Inverse Boundary Element Method (IBEM). However, most if these methods do not work in vehicle interiors due to reflections and standing waves. Recently, a novel approach has been introduced using the Inverse Finite-Element-Method (IFEM). This method is especially well suited for interior problems. The IFEM method is unfortunately ill-posed and needs therefore regularization. In this paper three regularization algorithms have been compared.

Inverse Finite Element Formulation

In summery the FEM-equation in frequency domain for the sound pressure can be written as

$$\mathbf{K}_P \mathbf{p} = \mathbf{v}_P , \quad (1)$$

where $\mathbf{K}_P$ is the stiffness matrix, $\mathbf{p}$ the vector of the excess pressure and $\mathbf{v}_P$ the vector of the velocity in the sound field. By splitting the calculation domain into three regions (Fig. 1b), a measurement region (M), a border region (B) and a transition region (T), the stiffness matrix can be divided into nine sub matrices $\mathbf{K}_P^i$. Using this approach the equation

$$\begin{bmatrix} \mathbf{K}_P^2 & \mathbf{K}_P^3 \\ \mathbf{K}_P^5 & \mathbf{K}_P^6 \end{bmatrix} \begin{bmatrix} \mathbf{p}_{TU} \\ \mathbf{p}_{BU} \end{bmatrix} = \begin{bmatrix} -\mathbf{K}_P^1 \mathbf{p}_{MK} \\ -\mathbf{K}_P^4 \mathbf{p}_{MK} \end{bmatrix} , \quad (2)$$

can be derived, see [1]. The first sub index of the sound pressure $\mathbf{p}$ denotes the region of the calculation domain, whereas the second sub index denotes whether the variable is known (K) or unknown (U).

Ill-Posed Problem & Regularization

Equation (2) can be solved to determine the complete sound field in the interior using only the measurements in the inner region (M) of the sound field. For simplicity (2) is rewritten in the form

$$\mathbf{A}\mathbf{x} = \mathbf{b} . \quad (3)$$

The condition number of the matrix $\mathbf{A}$ is very high. This is a typical behaviour of ill-posed problems.

Truncated Singular Value Decomposition

The SVD of a general matrix $\mathbf{A}$ has the form

$$\mathbf{A} = \sum_{i=1}^{I} \mathbf{u}_i \, \sigma_i \, \mathbf{w}_i^T \quad (4)$$

where $\mathbf{u}_i$ is the i'th left singular vector, $\mathbf{w}_i$ is the i'th right singular vector, σ_i is the i'th singular value and I is the total number of singular values. In this case $\mathbf{A}$ is not exactly rank deficient, but numerically rank deficient. This means it has small but nonzero singular values. A common approach to regularization of numerical rank-deficient problems is to consider the given matrix $\mathbf{A}$ as a noisy representation of a mathematically rank-deficient matrix, and to replace $\mathbf{A}$ by a matrix that is close to it but mathematically exact rank-deficient. This can be done by replacing small nonzero singular values $\sigma_{k+1},...,\sigma_n$ with exact zeros. Therefore, the solution can be computed by

$$\mathbf{x}_{\alpha_{TSVD}} = \sum_{i=1}^{k} \frac{\mathbf{u}_i^T \mathbf{b}}{\boldsymbol{\sigma}_i} \mathbf{w}_i , \quad (5)$$

where $k \geq \alpha_{TSVD}$ and α_{TSVD} is the regularization parameter which determines how many singular values are taken into account.

Tikhonov Regularization

The solution of (3) is also a solution of

$$\mathbf{A}^T \mathbf{A}\, \mathbf{x} = \mathbf{A}^T \mathbf{b} \quad (6)$$

This equation, however, shows no improvements of the properties for the ill-conditioned problem. The key idea in Tikhonov's method is to incorporate a priori assumption about the size and the smoothness of the desired solution [2]. This can be achieved by using the identity matrix $\mathbf{I}$ to construct a family of neighbouring problems

$$\left(\mathbf{A}^T \mathbf{A} + \alpha_{Tik} \mathbf{I}\right) \mathbf{x} = \mathbf{A}^T \mathbf{b} \quad (7)$$

where α_{Tik} is the regularization parameter. High values of α_{Tik} cause a higher punishment of $\mathbf{x}$ and therefore all values of $\mathbf{x}$ are moved towards zero.

Conjugate Gradient Algorithm

The Conjugate Gradient (CG) algorithm is an iterative method. The original CG method was designed for solving large sparse systems of equations with a symmetric positive definite coefficient matrix [3]. In connection with discrete ill-posed problems, it is an interesting fact that when the CG algorithm is applied to (6) then the lower-frequency components of the solution converge faster than the higher-frequency components. Therefore, the CG algorithm has some inherent regularization effect where the number of iterations plays the role of the regularization parameter.

Elving [4] found experimentally that the most stable implementation -on average- is the CGLS, see [5]. The kth step of the CGLS process has the form

$$\begin{aligned} \alpha_k &= \left\|\mathbf{A}^T\mathbf{r}^{(k-1)}\right\|_2^2 \Big/ \left\|\mathbf{A}\mathbf{d}^{(k-1)}\right\|_2^2 \\ \mathbf{x}^{(k)} &= \mathbf{x}^{(k-1)} + \alpha_k \mathbf{d}^{(k-1)} \\ \mathbf{r}^{(k)} &= \mathbf{r}^{(k-1)} - \alpha_k \mathbf{A}\mathbf{d}^{(k-1)} \\ \beta_k &= \left\|\mathbf{A}^T\mathbf{r}^{(k)}\right\|_2^2 \Big/ \left\|\mathbf{A}^T\mathbf{r}^{(k-1)}\right\|_2^2 \\ \mathbf{d}^{(k)} &= \mathbf{A}^T\mathbf{r}^{(k)} + \beta_k \mathbf{d}^{(k-1)} \end{aligned} \qquad (8)$$

where $\mathbf{r}^{(k)}$ is the residual vector and $\mathbf{d}^{(k)}$ is an auxiliary iteration vector. The algorithm is initialized with the starting vector $\mathbf{x}^{(0)}$. α and β are auxiliary variables.

L-curve

One of the main difficulties in properly solving a discrete ill-posed problem is how to determine a suitable regularization parameter. A small regularization parameter gives a good approximation to the system equations and has therefore a low residual but is very sensitive to the influence of data errors. In this case the solution is said to be under-regularized or under-smoothed. Conversely, a large regularization parameter suppresses data errors but increases the approximation error. A well suited regularization parameter α can be determined from the so called L-curve [3]. The L-curve is a plot of the norm of the regularized solution $\left\|\mathbf{x}_{reg}\right\|_2$ versus the corresponding residual norm $\left\|\mathbf{A}\mathbf{x}_{reg} - \mathbf{b}\right\|_2$.

Numerical Results

In order to confirm the validity of the proposed formulation a two-dimensional test geometry has been examined. A classical forward FEM calculation has been used as an input for the IFEM algorithm. The input data has been made noisy by adding a random error on every measurement point. Figure 1a shows the modelled geometry and the imposed normal velocity boundary condition.

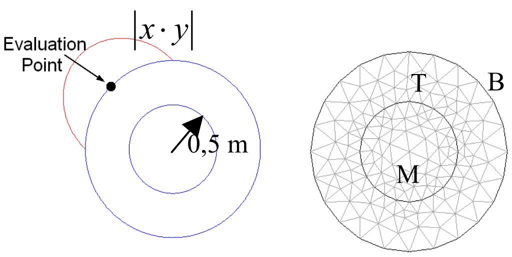

Figure 1: a) Geometry, boundary condition and evaluation Point b) FE-mesh

In a parameter study the imposed measurement error has been increased and the inverse calculation error has been calculated at the location shown in figure 1a. The FE-mesh is shown in figure 1b.

The error curves plotted in figure 2 are not smooth, since an arbitrary measurement error has been imposed. The CGLS-algorithm seems to perform best, as no peaks in the inverse error can be observed. The Tikhonov regularization shows also good results, but has peaks in the inverse error for some points. The worst implemented algorithm is the TSVD. However, all regularized solutions show a high improvement against the not regularized solution with an error of up to 17%.

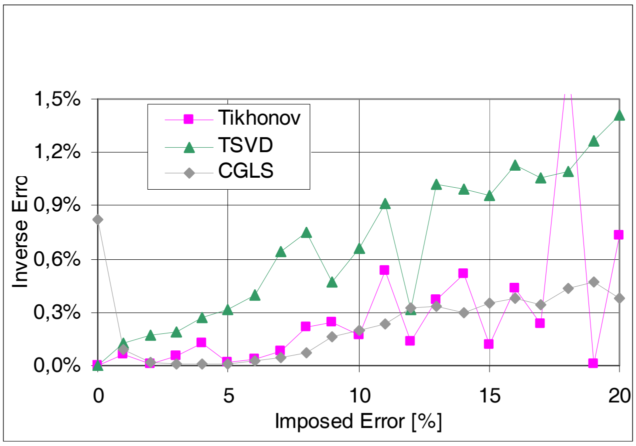

Figure 2: Error of inverse calculation

Summary

This paper has presented a formulation for the inverse finite element calculation of the acoustic field in interiors. The Tikhonov, the Truncated Singular Value Decomposition and a Conjugate Gradient Algorithm have been implemented as regularization algorithms. The inverse calculation error for a variety of measurement errors has been examined. It could be observed, that the CGLS algorithm performs best. For future work a test structure is under construction.

References

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