Effects that Influence Determination of Air Column Resonance Frequencies in Cylindrical Tubes

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Introduction

The comparison of measured and calculated resonance frequencies of oscillating air column inside cylindrical tubes with one open end and the second closed showed, that the resonance frequencies calculated by simple empiric relations, e.g. relation (2), do not correspond to the measured frequencies. Therefore it is necessary to include other effects (physical processes) to these relations, which influence the determination of the resonance frequencies. It concerns especially the effects, which relate to the determination of the real speed of the sound propagation in real gas (air) inside the tube and the effect of surroundings (air) outside the tube on the radiation of the tube open end (acoustical radiator). This contribution deals with these effects.

Resonance frequencies

The resonance frequencies of the oscillating air column inside a hollow solid (resonator) can be determined from the input impedance of this solid (resonator). Then the resonance frequencies correspond to individual maxima of the modulus of the input impedance (not zero value of the input impedance phase), with respect to a possibility of comparison with measured resonance frequencies (measuring system BIAS [1]). The transversal modes of the air column inside resonator are not considered in terms of this contribution. Cited resonance frequencies are the frequencies of the axial modes, to which the distribution of the pressures and the velocities along the axes of the resonator correspond.

Input impedance of tube

In case the resonator is cylindrical tube with one open end and the second closed, input acoustic impedance is determined by relation:

$$\hat{Z}_{ainp.} = j\frac{\rho_0 c_0}{\pi R^2}\tan(kl), \qquad (1)$$

where $k = \omega/c_0$ is the wave number, l is the length of the tube, R is the radius of the tube, c_0 is the speed of the sound in free space (air) and ρ_0 is the density of air. Then resonance frequencies are determined by relation:

$$f_{nres.} = \frac{(2n-1)c_0}{4l}\ ,\ n = 1, 2, 3, \dots\ . \qquad (2)$$

These relations (1), (2) are valid on assumptions, that the losses do not occur inside the tube, plane wave is propagating through the tube and the surroundings (air) outside the tube does not influence the radiation of the tube open end (acoustical radiator).

Effect of viscosity and thermal conduction

In fact the viscothermal losses occur by the effect of viscosity and thermal conductivity of the gas (air) inside the tube. Acoustic particles of the gas (inside the tube) adjacent to the wall of the tube are inactive (at a motion of the gas) by influence of an adhesion to the wall of the tube. In consequence of that, the acoustic particle velocity inside the tube changes with the distance from the wall of the tube, see fig. 1. The friction among adjacent acoustic particles (in the radial direction) occurs with respect to this change of velocity. There is a viscous stress among adjacent acoustic particles (in the radial direction), which is proportional to the viscosity of the gas and the change of acoustic particle velocity of the gas.

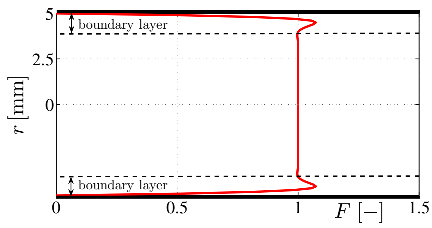

Figure 1: The course of the function F (velocity profile) indicating the distribution of the acoustic particle velocity of the gas in the tube cross-section (in case that $R = 0.005$m, $f = 100$Hz).

The motion of incompressible gas with the viscosity is described by Navier-Stokes equation [2]. In cylindrical coordinates this equation has a form (on assumption of the laminar flow of the gas inside the tube, harmonic changes of the acoustic velocity and the pressure, considering acoustic particle velocity of the gas only in the direction of (axial) z axis of the tube ($\vec{v} = \vec{e}_z v_z + \vec{e}_r 0 + \vec{e}_\varphi 0$) and considering, the component of the acoustic particle velocity v_z not depending on φ and z coordinates):

$$\mu\left(\frac{\partial^2 \hat{v}_z}{\partial r^2} + \frac{1}{r}\frac{\partial \hat{v}_z}{\partial r}\right) = \frac{\partial \hat{p}_z}{\partial z} + \rho_0\frac{\partial \hat{v}_z}{\partial t}\ , \qquad (3)$$

where r is the distance from the z axis of the tube and μ is the viscosity of the gas inside the tube [3]. The solution of this equation is relation (4), which indicates the acoustic particle velocity of the gas depending on frequency and the distance from the axis of the tube [3].

$$\hat{v}_z(f, r) = \frac{F(f, r)}{j\omega\rho_0}\frac{\partial \hat{p}_z}{\partial z} \qquad (4)$$

The function F indicates the distribution of the acoustic particle velocity of the gas in the cross section of the cylindrical tube, so it indicates so-called velocity profile, see fig. 1. It is determined by the relation:

$$F(f,r) = \frac{J_0(a\,r)}{J_0(aR)} - 1\,,\ where\ a = \sqrt{\frac{-j\omega\rho_0}{\mu}}\,. \qquad (5)$$

Due to very small acoustic particle velocity of the gas at the wall of the tube it can not be considered process of the gas happening inside the tube as purely adiabatic. There the thermal conduction among adjacent acoustic particles occurs. Owing to the effect of the viscosity and the thermal conduction, the sound inside the tube propagates the phase speed c, see fig. 2, which depends not only on climatic conditions, but also on the frequency, radius of the tube, the viscosity and the coefficient of the thermal conduction, see relation (8).

$$\hat{Z}'_a = \frac{-j\omega\rho_0}{\pi R^2}\frac{J_0(aR)}{J_2(aR)}\,,\ where\ a = \sqrt{\frac{-j\omega\rho_0}{\mu}} \qquad (6)$$

$$\hat{Y}'_a = \frac{j\omega\pi R^2}{\rho_0 c_0^2}\left(\gamma + (\gamma-1)\frac{J_2(bR)}{J_0(bR)}\right),\ where\ b = a\sqrt{Pr} \qquad (7)$$

$$k^* = \sqrt{-\hat{Z}'_a\hat{Y}'_a} = \frac{\omega}{c} - j\alpha \quad\Rightarrow\quad c = \frac{\omega}{\Re\{k^*\}} \qquad (8)$$

k^* is the complex wave number, where the imaginary part represents the losses inside the tube, $\hat{Z}'_a, \hat{Y}'_a$ are the acoustic (series) impedance and the acoustic (shunt) admittance per unit length of the tube, γ is the specific-heat ratio and $Pr = \sqrt{\mu C_p/\kappa}$ is the Prandtl number, where C_p is the specific heat of air at constant pressure and κ is the coefficient of the thermal conduction.

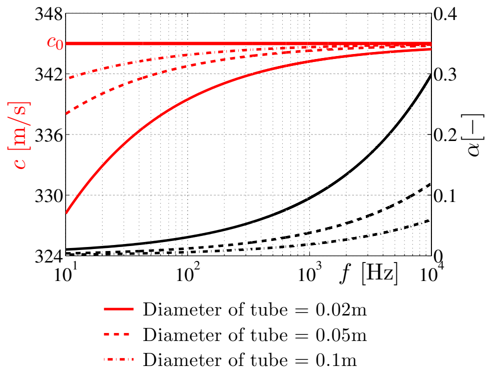

Figure 2: The courses of the attenuation coefficient (α) and the phase speed of the sound propagation (c) in the air inside the cylindrical tube in dependence on the frequency.

Effect of surroundings

The surroundings outside the tube influences the radiation of the tube open end (acoustical radiator). As a consequence there occur other losses and the change of the resonance frequencies of the oscillating gas (air) column inside the tube. This effect can be imagined as an impedance, which loads the acoustical radiator.
In case of the open end of the cylindrical tube, this effect can be simulated (approximately) by means of the radiation impedance of the equivalent pulsative sphere with the radius $R_{equ.} = R/\sqrt{2}$ or the radiation impedance of the baffled circular piston with the radius R.

Conclusion

The input acoustic impedance of a cylindrical tube with one open end and the second closed, including of the viscosity effect of the air inside the tube, the thermal conduction effect and the effect of the surroundings outside the tube on the radiation of the tube open end (acoustical radiator), is determined by the relation:

$$\hat{Z}_{ainp.} = \frac{\rho_0 c_0}{\pi R^2}\frac{\hat{z}_r\cos(k^*l) + j\sin(k^*l)}{j\hat{z}_r\sin(k^*l) + \cos(k^*l)}\,, \qquad (9)$$

where $\hat{z}_r$ is the normalized radiation impedance of the acoustical radiator. The effect of the air viscosity, the thermal conduction and the surroundings outside the tube on the determination of the resonance frequencies of oscillating air column inside the cylindrical tube is shown in figure 3. The minimal divergences (calculated resonance frequencies from measured, measured only 10 modes) are obtained presuming all above cited effects in the calculation of the input impedance of the tube, see relation (9), from which the resonance frequencies are determined. Such small divergences are obtained only in the case of the sufficiently length of the tube in proportion to the radius of the tube (typically $l > 60R$), with respect to the consideration, that the component of the acoustic particle velocity v_z does not depend on the length coordinate z, see relation (3).

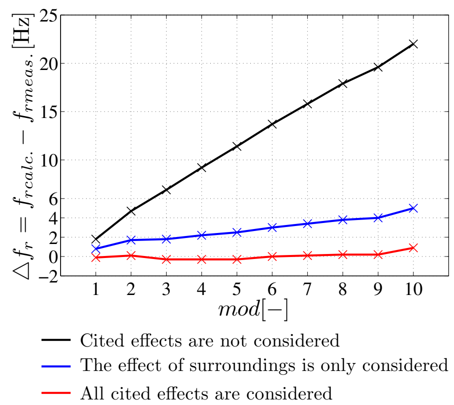

Figure 3: The comparison of the calculated and the measured resonance frequencies of the oscillating air column inside the cylindrical tube (with one open end and the second closed) with the length 0.985m and the diameter 0.0328m (the normalized radiation impedance of the equivalent pulsative sphere was used in the calculation $\hat{z}_r = jk\frac{R}{\sqrt{2}}/(1 + jk\frac{R}{\sqrt{2}})$).

References

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